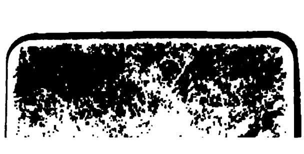

NAUTICAL ASTRONOMY

AND

NAVIGATION.

PART I.

CONTAINING RULES FOR FINDING THE LATITUDE AND LONGITUDE, AND THE VARIATION OF THE COMPASS.

With numerous Examples.

Designed for Beginners.

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PREFACE.

In the present edition the Author has adapted the rules not only to the Tables of Dr. Inman (the most comprehensive and useful yet published), but also to those in more general use, such as Riddle's, Norie's, &c. The student will therefore find now no difficulty in working out the examples contained in the book by any of the above tables. In the last edition of the Author's work on *Plane and Spherical Trigonometry*, rules are also given depending on the common tables of sines, &c., as well as on the valuable table of haversines contained in Inman's Tables.

Langstone House, Havant,
Feb. 1, 1870.

ERRATA.

p. 22, l. 19, *for* $2\frac{1}{2}$ r. N. *read* $2\frac{1}{4}$ r. N.

„ 22, *for* $2\frac{1}{4}$ r. N. or N.N.E.$\frac{1}{4}$N. *read* $2\frac{1}{4}$ r. N. or N.N.E.$\frac{1}{4}$E.

85, 4 from bottom, *for* 52·7 *read* 51·7.

90, 21 from top, *for* 2^h $26{\cdot}9^m$ *read* 2^m $26{\cdot}9^s$.

91, 9 „ „ 4^h $1{\cdot}1^s$ „ 4^m $1{\cdot}1^s$.

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NAVIGATION OR PLANE SAILING.

CHAPTER I.

(1.) Two distinct methods are used for navigating a ship from one place to another: the first is an application of the common rules of Plane Trigonometry, the necessary angles and measurements being supplied by means of the compass and log-line; the second and more exact method requires a knowledge of the rules of Spherical Trigonometry and of the principal definitions and facts in Astronomy, the necessary data being obtained by astronomical observations taken usually with the sextant. The latter method is for this reason called *Nautical Astronomy;* the characteristic name of the former being *Navigation* or *Plane Sailing.*

Before we proceed to give the rules in navigation for finding the *place* of the ship, that is, its latitude and longitude, we will reprint, for the sake of reference, the definitions and terms in pp. 143-44 of *Navigation*, Part II., and also some trigonometrical and nautical problems taken for the most part out of the author's *Trigonometry*, Part I. These problems are intended to serve as a useful introduction to navigation; at the same time they will show the student that a knowledge of the rules in plane trigonometry is nearly all that will be required to enable him to understand and work out the problems and examples in navigation or plane sailing.

Definitions in Navigation.

(2.) The following are the principal terms in Navigation: the definitions of these terms, like those in Nautical Astronomy, must be thoroughly understood and committed to memory.

Course.
Distance.
Departure.
True difference of latitude.
Meridional difference of latitude.
Difference of longitude.
Middle latitude.

Definitions of the preceding terms.

The course is the angle which the ship's track makes with all the meridians between the place left and the place arrived at.

The distance is the spiral line made by the ship's track in describing the course between the place left and the place arrived at.

The departure is the sum of all the arcs of parallels of latitude drawn between the place left and the place arrived at, through points indefinitely near to one another taken on the distance, and intercepted between the meridians passing through those points.

The true difference of latitude is the arc of a meridian intercepted between the parallels of latitude drawn through the place left and the place arrived at.

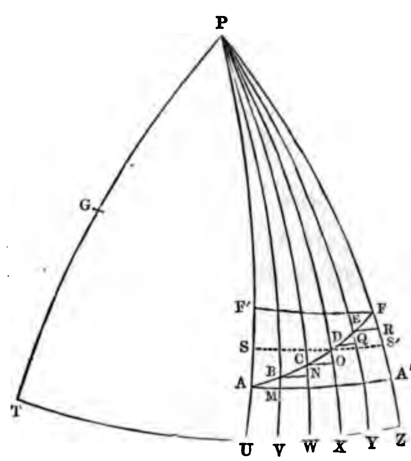
The meridional difference of latitude is the value in minutes of a great circle of the line on a Mercator's chart, into which the true difference of latitude has been expanded.

The difference of longitude is the arc of the terrestrial equator intercepted between the meridians passing through the place left and the place arrived at.

The middle latitude is the mean of the latitudes (supposed of the same name) of the place left and the place arrived at.

These definitions will be clearly understood by means of the following diagrams.

Let P represent the pole of the earth, TZ an arc of the equator, PT the meridian passing through a known place G, as Greenwich, A and F two other places on the earth (considered as a sphere), PU, PZ, their meridians.



Through the points A and F suppose a curve line AF to be drawn, cutting all the intermediate meridians PV, PW, PX, &c., *at the same angle;* that is, making the angle PAB=PBC=PCD=&c. Then this common angle PAF is called the *course.* The arc AF* is the *distance.* Draw the parallels of latitude AA′, FF′; then, since the latitude of A is the arc AU, and the longitude of A the arc TU, and the latitude of F is the arc FZ, and the longitude of F is the arc TZ; therefore the difference, or, as it is usually called, the *true difference of latitude*, between A and F is the arc AF′ or A′F, and *the difference of longitude* between A and F is the arc of the equator UZ. Again, suppose the intermediate meridians PV, PW, &c., to be

* AF is sometimes called the *rhumb line,* sometimes the *loxodromic curve,* sometimes the *equiangular spiral.*

drawn through points B, C, D, &c., taken on the line AF indefinitely near to each other; and through the points A, B, C, D, &c., the arcs of parallels of latitude AM, BN, CO, &c. On this supposition (namely that the points A, B, C, &c., are indefinitely near to each other) the elementary triangles ABM, BCN, CDO, &c., may be considered without any error to be right-angled *plane triangles.* *The departure* between A and F$=$AM$+$BN$+$CO$+ \ldots$ ER, the points A, B, C, &c., being supposed to be indefinitely near to each other.

If a parallel of latitude SS′ be drawn through the middle of AF′, then the arc of the meridian SU is called the mean or *middle latitude* between A and F. It is manifest that the arc SS′ will be nearly equal to AM$+$BN$+ \ldots$ DQ$+$ER, the departure, A and F being supposed to be on the same side of the equator. For short distances SS′ is substituted without any practical error for the departure, and one of the principal rules in Navigation is deduced from it.

(3.) There are two kinds of charts; the Plane chart, and Mercator's chart.

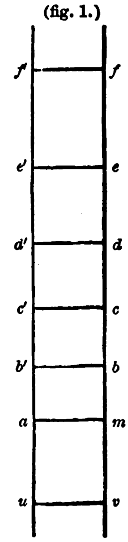
The Plane chart.

The plane chart is a representation of the earth's surface, considering it as a plane. When a small portion of the surface is concerned, this mode of representing it will lead to no practical error; hence coasting charts are usually constructed in this manner, in which the different headlands, lighthouses, &c., are laid down according to their bearings.

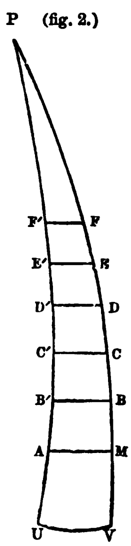
Mercator's chart.

The chart used at sea for marking down the ship's track, and for other purposes, is called Mercator's chart. It exhibits also the surface of the earth *on a plane;* but the meridians are drawn perpendicular to the equator, and therefore the arcs AM, B′B, &c., of parallels of latitude intercepted between any two meridians are increased to *am*, *b′b*, &c., and become equal to one another and to line *uv*, and therefore to the intercepted arc UV of the equator. If we wish to make the figures (supposed to be very small) *ambb′*, *b′bcc′*, &c., on the chart similar to AMBB′, B′BCC′, &c., of the globe, it is evident we must increase the sides *bm*, *bc*, &c., in the same proportion as *am*, *b′b*, &c. (that represent AM, BB′, &c.), have been increased. Let us therefore suppose the straight lines *am*, *b′b*, *c′c*, &c., have been drawn at such a distance from each other that the above similarity of figure is preserved (and

(fig. 1.)



(fig. 2.)



this can only be done by supposing the surfaces *ambb′*, *b′bcc′*, &c., *indefinitely* small, so that the surfaces AMBB′, B′BCC′, &c., may be considered as plane surfaces). Then a representation of the earth's surface, or any part of it, so constructed, is called a *Mercator's chart.*

The straight line *mf*, into which MF, the true difference of latitude between M and F, has been expanded, is called the *meridional difference of latitude* between M and F, and the values of *bv*, *cv*, &c., in minutes, are called the *meridional parts* of B, C, &c.: hence the *meridional difference of latitude* between two places is the difference of the meridional parts for the two places.

The method of constructing a Mercator's chart and laying down a ship track thereon will be given hereafter (see art. 16).

From these definitions and principles are deduced the following

ELEMENTARY RULES IN NAVIGATION.

Rule (*a*). *To find the true difference of latitude*, having given the latitude from and latitude in.*

(1.) When latitude from and latitude in have *like names*, that is, are both north or both south.

Under the latitude from, put down the latitude in, take the difference and reduce the same to minutes; place N. or S. against the result according as the latitude in is north or south of the latitude from; the remainder is the *true difference of latitude.*

(2.) When latitude from and latitude in have *unlike names*, that is, one north and the other south.

Take the sum of the two latitudes, reduce it to minutes, and attach N. or S. thereto, according as the latitude in is north or south of the latitude from; the result is the *true difference of latitude.*

EXAMPLES.

1. Find the true difference of latitude, having given latitude from 42° 10′ N., and latitude in 50° 48′ N.

lat. from	42° 10′N.
lat. in	50 48 N.
	8 38
	60
∴ T. D. lat.	518 N.

2. Find the true difference of latitude, having given latitude from 3° 42′N., and latitude in 2° 40′ S.

lat. from	3° 42′N.
lat. in	2 50 S.
	6 32
	60
∴ T. D. lat.	392 S.

* The latitude of the place left is called the latitude *from*, the latitude of the place arrived at is called the latitude *in*.

Find the true difference of latitude in each of the following examples:

	Lat. from	Lat. in	Answers.
3.	33° 42′N.	40° 40′N.	∴ T. D. lat. = 418 N.
4.	40 40 N.	33 42 N.	.. = 418 S.
5.	3 42 S.	1 40 N.	.. = 322 N.
6.	3 8 S.	14 42 S.	.. = 694 S.
7.	68 48 N.	38 30 N.	.. = 1818 S.
8.	14 14 N.	0 0	.. = 854 S.

Rule (*b*). *To find the meridional difference of latitude*, having given the latitude from and latitude in.

Take the meridional parts for the two latitudes from the table of meridional parts: subtract if the names be alike, and add if the names be unlike, the result is the *meridional difference of latitude;* N. or S. being attached thereto according as the latitude in is north or south of latitude from.

EXAMPLES.

9. Find the meridional difference of latitude, having given latitude from 42° 10′ N., and latitude in 50° 48′ N.

lat. from	42° 10′N.
lat. in	50 48 N.
mer. parts	...2795·2 N.
mer. parts	...3549·8 N.
mer. diff. lat.	...754·6 N.

10. Find the meridional difference of latitude, having given latitude from 3° 42′ N. and latitude in 7° 32′ S.

lat. from	3° 42′N.
lat. in	7 32 S.
mer. parts	...222·2 N.
mer. parts	...453·3 S.
mer. diff. lat.	...675·5 S.

Find the meridional difference of latitude in each of the following examples:

	Lat. from	Lat. in	Answers.
11.	34° 42′N.	33° 15′N.	M.D. lat. = 104·9 S.
12.	14 14 N.	30 14 N.	.. = 1041·7 N.
13.	84 10 N.	80 30 N.	.. = 1681·5 S.
14.	2 8 S.	3 10 N.	.. = 318·1 N.
15.	4 5 N.	4 5 S.	.. = 490·4 S.
16.	0 0	2 45 N.	.. = 165·1 N.

Rule (*c*). *To find the middle latitude*, having given the latitude from and latitude in.

The names being supposed to be *alike*, that is, both north or both south.

Add together the two latitudes, and take half the sum; the result is the middle latitude.

When the names are unlike, the mid. lat. (which is seldom required but for obtaining the departure) should be found by means of a table; but in

this case it may perhaps be as well to avoid the use of the middle latitude in any of the common problems in navigation.

EXAMPLES.

17. Find the middle latitude, having given latitude from 3° 42′ N., and latitude in 13° 52′ N.

lat. from	3°42′	N.
lat. in	13 52	N.
	2)17 34	
mid. lat.	8 47	N.

Find the middle latitude in each of the following examples:

	Lat. from	Lat. in	Answers.
18.	38° 42′ N.	30° 30′ N.	mid. lat. 34° 36′ N.
19.	62 17 S.	62 30 S.	.. 62 23½ S.

Rule (*d*). *To find the difference of longitude*, having given the longitude from and longitude in.

(1.) When the longitude from and longitude in have *like names;* that is, are both east or both west.

Under longitude from put longitude in, take the difference, and reduce the same to minutes; place E. or W. against the remainder according as the longitude in is east or west of longitude from; the remainder will be the difference of longitude.

(2.) When the longitude from and longitude in have *unlike names;* that is, one east and the other west.

Take the sum of the two longitudes, reduce it to minutes, and attach E. or W. thereto according as the longitude in is east or west of the longitude from; the result is the true difference of longitude.

Note. If the difference of longitude found by this rule exceed 180° it must be subtracted from 360°, and the remainder brought into minutes must be considered the difference of longitude, with the contrary letter attached to it.

20. Find the difference of longitude, having given the longitude from=110° 42′ W., and longitude in=100° 42′ W.

long. from	110° 42′	W.
long. in	100 42	W.
	10 0	
	60	
∴ diff. long.	600	E.

21. Find the difference of longitude, having given long. from 12° 10′ E., and long. in 2° 45′ W.

long. from	12° 10′	E.
long. in	2 45	W.
	14 55	
	60	
∴ diff. long.	895	W.

Find the difference of longitude in each of the following examples:

	Long. from	Long. in	Answers.
22.	33° 40′ E.	40° 10′ E.	Diff. long. 390 E.
23.	104 0 W.	110 30 W.	.. 390 W.
24.	2 45 W.	3 30 E.	.. 375 E.
25.	0 0	4 10 W.	.. 250 W.
26.	3 10 W.	3 10 E.	.. 380 E.
27.	179 0 E.	179 0 W.	.. 120 E.

Rule (*e*). *To find the latitude in*, having given the latitude from and true difference of latitude.

(1.) When the latitude from and true difference of latitude have *like names.*

To the latitude from, *add* the true difference of latitude (turned into degrees and minutes, if necessary); the sum will be the latitude in, of the same name as the latitude from.

(2.) When the latitude from and true difference of latitude have *unlike names.*

Under the latitude from put the true difference of latitude (in degrees and minutes, if necessary); take the less from the greater; the remainder, marked with the name of the greater, is the latitude in.

EXAMPLES.

28. Find the latitude in, having given the latitude from 42° 30′ N., and true diff. lat. 342′ N.

	60)342′ N.
T. D. lat.	5° 42′ N.
lat. from	42 30 N.
lat. in	48 12 N.

29. Find the latitude in, having given the latitude from 42° 30′ S., and true diff. lat. 342′ N.

	60)342′ N.
T. D. lat.	5° 42′ N.
lat. from	42 30 S.
lat. in	36 48 S.

Find the latitude in, in each of the following examples:

	Lat. from	T. D. lat.	Answers.
30.	30° 10′ N.	182′ N.	Lat. in 33° 12′ N.
31.	3 2 S.	190 N.	.. 0 8 N.
32.	2 48 S.	368 N.	.. 3 20 N.
33.	2 48 S.	288 N.	.. 2 0 N.
34.	4 48 N.	288 S.	.. 0 0
35.	0 10 N.	228 N.	.. 3 58 N.

Rule (*f*). *To find the longitude in*, having given the longitude from and the difference of longitude.

(1.) When the longitude from and diff. long. have *like names*.

To the long. from, *add* diff. long. (turned into degrees if necessary); the sum will be long. in, of the same name as long. from.

(2.) When the long. from and diff. long. have *unlike names*.

Under long. from put diff. long. (in degrees and minutes, if necessary); take the less from the greater; the remainder, marked with the name of the greater, is the long. in.

NOTE. If the long. in, found as above, exceed 180°, subtract it from 360°, and attach to the remainder the contrary name to the one directed in the Rule.

EXAMPLES.

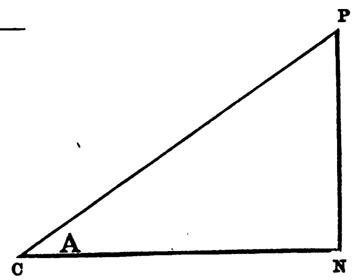
36. Find the long. in, having given long. from 38° 42′W., and diff. long. 384·5′W.

60)384·5		long. from	38° 42′W.
6° 24·5′W.		diff. long.	6 24·5 W.
		long. in	45 6·5 W.

Find the longitude in, in each of the following examples:

	Long. from	Diff. long.	Answers.
37.	62° 32′ E.	1000·5′ W.	long. in 45° 51·5′ E.
38.	2 30 E.	126·6 E.	.. 4 36·6 E.
39.	3 40 W.	220·0 E.	.. 0 0
40.	0 0	100·4 W.	.. 1 40·4 W.
41.	179 59 W.	2·0 W.	.. 179 59·0 E.

NAUTICAL PROBLEMS SOLVED BY TRIGONOMETRY.



(4.) It is shown in *Trigonometry*, Part I. p. 2, that if in the right-angled triangle PCN the angle PCN (opposite the perpendicular PN) be denoted by A, then,

$\frac{\text{Perp.}}{\text{Hyp.}}$ or $\frac{PN}{CP}$=sine of the angle A or PN=CP sin. A.

$\frac{\text{Hyp.}}{\text{Perp.}}$ or $\frac{CP}{PN}$=cosecant of the angle A or CP=PN cosec. A.

$\frac{\text{Perp.}}{\text{Base}}$ or $\frac{PN}{CN}$=tangent of the angle A or PN=CN tan. A.

$\frac{\text{Base}}{\text{Perp.}}$ or $\frac{CN}{PN}$=cotangent of the angle A or CN=PN cot. A.

$\frac{\text{Hyp.}}{\text{Base}}$ or $\frac{CP}{CN}$=secant of the angle A or CP=CN sec. A.

$\frac{\text{Base}}{\text{Hyp.}}$ or $\frac{CN}{CP}$=cosine of the angle A or CN=CP cos. A.

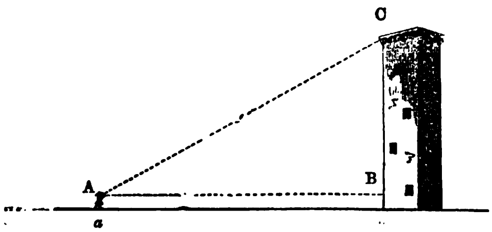
These six equations or formulæ enable us to work out all the ordinary questions in Navigation involving right angles. Before, however, the student proceeds with the following problems he should make himself also acquainted with the other rules in Plane Trigonometry (see *Trigonometry*, Part I. Rules 1, 2, and 3).*

NAUTICAL PROBLEMS.

42. Wishing to find the height of a tower, I observed with a sextant the angle of elevation of its top above the level of my eye to be 32° 14′. I

* In this edition the rules will be adapted to the common table of sines, &c., as well as to the table of haversines; but preference will be given to the use of the latter, as it diminishes considerably the labour of calculation.

then measured the distance from the place of observation to the base of the tower, and found it to be 142 feet. Required the height of the tower.



Let CB* represent the tower, A the place of observer. Draw the horizontal line AB at a height above the ground ab equal to the height of the eye, and join AC.

Then in right-angled triangle ABC are given AB=142, angle CAB=32° 14′ and B=90°: to find CB, the height of the tower, above the horizontal line AB.

(Mark the figure in the usual way. See *Trigonometry*, Part I., rule for right-angled plane triangles.)

Since $\frac{CB}{AB}$=tan. A.

∴ CB=AB tan. A.

∴ log. CB=log. AB+log. tan. A−10.

(*Trig.* Part I. art. 31.)

Calculation.

log. AB.....2·152288

„ tan. A.....9·799717

„ CB.....1·952005

∴ CB= 89·5 feet.

To the value of CB must be added the height of the eye Aa: the result will be the height of the tower required.

43. Wishing to find the height of a tower, I observed the angle of elevation of its top above the level of my eye (supposed to be 5 feet above the ground) to be 47° 56′. I then measured the distance from the place of observation to the base of the tower, and found it to be 190·4 feet. Required the height of the tower. *Ans.* 216 feet.

44. On the opposite bank of a river to that on which I stood is a tower known to be 216 feet high: with a pocket sextant I ascertained the angle between a horizontal line drawn from my eye (supposed to be 5 feet above the ground) and its top to be 47° 56′. Required the distance across the river from the place where I stood to the bottom of the tower.

Let cb (fig. to Prob. 1) represent the height of tower=216 feet; Aa the

* The figures or diagrams of the following problems are not drawn accurately to scale; the student should endeavour to draw them as neatly as he can by the eye, so as to indicate the form without regard to the exact value of the sides and angles in the problems to which they refer. Problems will be given hereafter (see Art. 9), which are solved not only by logarithms but by *construction;* that is, by using mathematical instruments. The practice of using instruments thus obtained will form a proper introduction to the construction of charts, and the tracing the ship's track thereon.

height of spectator's eye=5 ft.; and AB or ab width of river. Suppose AB parallel to horizontal line ab: join CA, then CAB the angle of elevation = $47^\circ 56'$, and BC=height of tower−5=211 feet are given: to find AB or ab the width of river.

Since $\frac{AB}{BC}=\text{cot. } CAB$

$\therefore AB = BC \text{ cot. } CAB$

$\therefore \text{log. } AB = \text{log. } BC + \text{log. cot. } CAB - 10$

$= \text{log. } 211 + \text{log. cot. } 47^\circ 56' - 10$

Calculation.

log. 211 ....2·324282

„ cot. 47° 56′ .9·955453

„ AB ....9·279735

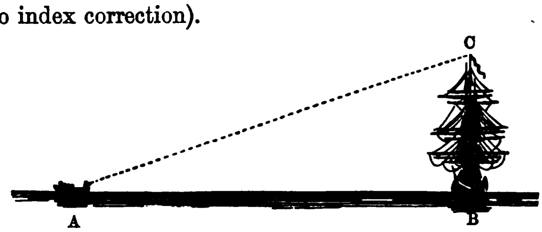
$\therefore AB = 190{\cdot}4$ width of river.

45. On the opposite bank of a river to that on which I stood is a tower known to be 94.5 feet high: with a pocket sextant I ascertained the angle between a horizontal line drawn from my eye (supposed to be 5 feet above the ground) and its top to be 32° 14′. Required the distance across the river from the place where I stood to the bottom of the tower.

Ans. 142 feet.

46. Being ordered to place a target at 500 yards from the ship, and knowing that the height of the truck above the water-line was 213 feet: it is required to find what angle the height will subtend on my sextant when I am at the required distance (before allowing for index correction).

Let BC represent the ship's mast, A the required place of target: then the angle BAC is the angle which must be read off on the sextant (supposing it to have no index correction).



In right-angled triangle are given the side BC=213 feet, AB=500 yards =1500 feet, and B=90°, to find the angle A.

Since $\text{tan. } A = \frac{CB}{AB}$

$\therefore \text{log. tan. } A - 10 = \text{log. } CB - \text{log. } AB$

$\therefore \text{log. tan. } A = 10 + \text{log. } CB - \text{log. } AB$

Calculation.

log. CB+10...12·328380

„ AB......... 3·176091

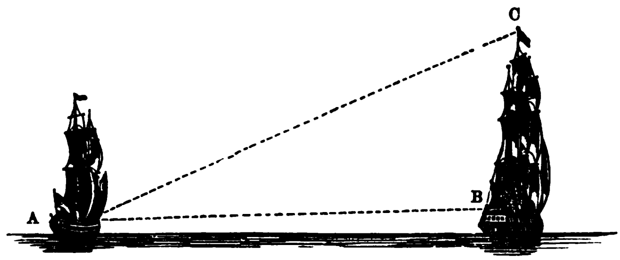
„ tan. A..... 9·152289

$\therefore$ angle on sextant=8° 5′.

47. Sailing in company with another ship, and being ordered to keep at the distance of 500 yards from her, and knowing that the height of her mast

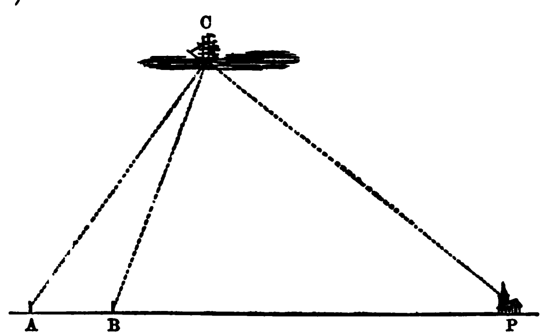
above the hammock-nettings was 198 feet, it is required to find what angle on my sextant will indicate the proper distance (see fig.).

Ans. 7° 31′ 15″.



48. Being on Southsea Common, and wishing to find my distance from a ship at anchor at Spithead, I observed with a sextant the angle between the ship and the steeple of St. Thomas's Church to be 72° 42′. I then walked 500 yards in a direct line towards the church, and again took the angle between the ship and the steeple, and found it to be 82° 45′. Required my distance from the ship at each observation.

Let C be the ship at anchor, A my first station, B the second, and P the church. Then the angle CAP=72° 42′, the angle CBP=82° 45′ and the line AB=500 yards, are given: to find BC and AC my distances at each observation. Since CBP=82° 45′ ∴ adjacent angle ABC=97° 15′, and the angle ACB=10° 3′ (since the three angles of a plane triangle are together equal to 180°).



(1.) To find BC (by *Trigonometry*, Part I. Rule 11).

In triangle AB=500	log. AB .... 2·698970
CAB=72° 42′	" sin. A...... 9·979895
ACB=10 3	12·678865
BC : AB : : sin. CAB : sin. ACB	" " ACB... 9·241814
	" " BC .... 3·437051
	∴ BC=2735 yards.

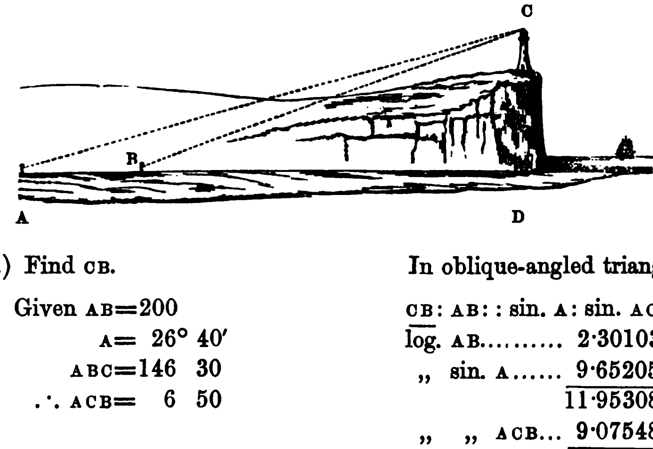
(2.) To find AC.

In triangle, AB=500	log. AB......... 2·698970
ABC=97° 15′	„ sin. ABC... 9·996514
ACB=10 3	12·695484
AC : AB : : sin. ABC : sin. ACB	„ „ ACB... 9·241814
	„ AC .... 3·453670
	∴ AC=2841 yards.

49. Two ships sailing in company, in order to determine nearly their distance from an object C on the shore, are separated from each other two nautical miles AB (fig. somewhat similar to the one to last problem), the angle is then observed from each ship between the object and the other ship: at A the angle is 85° 10′, at B the angle ABC is 82° 45′. Required the distance of each ship from the object C on shore.

Ans. 9·478 and 9·52 miles.

50. Wishing to determine the height of a lighthouse C on the summit of a cliff on the seashore, I observed the angle of elevation CAD of its top above the level sand to be 26° 40′; then, measuring AB=200 yards on the sand in a direct line towards it, I again observed the angle of elevation CBD of its top, and found it to be 33° 30′. Required the height CD of the lighthouse above the shore.



(1.) Find CB.	In oblique-angled triangle ABC,
Given AB=200	CB : AB : : sin. A : sin. ACB
A= 26° 40′	log. AB.......... 2·301030
ABC=146 30	„ sin. A...... 9·652052
∴ ACB= 6 50	11·953082
	„ „ ACB... 9·075480
	„ CB .... 2·877602
	∴ CB=754·4 yards.

(2). Find CD.

In right-angled triangle BCD are given CB (its log. 2·877602); angle CBD =33° 30′ and D=90°, to find CD.

Since $\frac{\text{CD}}{\text{CB}}=\sin.\ \text{CBD}$

$\therefore \text{CD}=\text{CB}\sin.\ \text{CBD}$

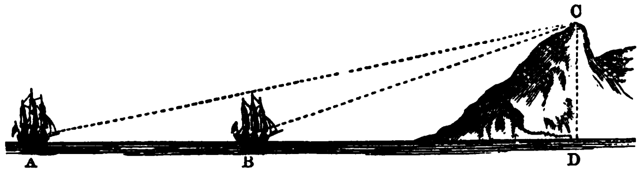
$\therefore \log.\ \text{CD}=\log.\ \text{CB}+\log.\ \sin.\ \text{CBD}-10$

log. CB	2·877602
„ sin. CBD...	9·741889
„ CD	2·619491

$\therefore$ CD=416·3 yards the height of lighthouse.

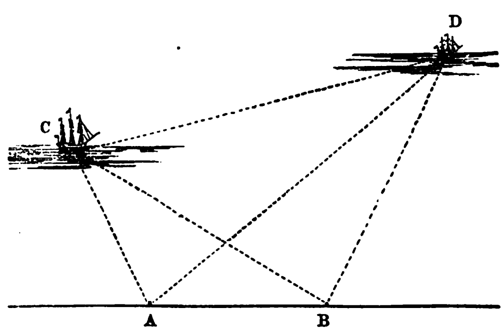
51. Standing in for the land, I observed the summit of a lofty mountain near the shore. I took the angle of elevation of the peak, and found it to be 12° 25′; after having run $3\frac{1}{2}$ miles directly towards it, I again took the angle of elevation, which was 30° 13′. Required the height of the mountain, and its distance from the second station (see fig.).

Ans. Height 1·24 miles, BD=2·13 miles.



52. Wishing to know the distance between two ships at anchor at C and D, I measured on the shore a base-line AB=735 yards, and with a sextant observed the following angles:

At A,	At B,	
CAD=63° 30′	DBC=80° 16′	Required the distance CD.
DAB=35 10	CBA=28 20	



(1.) Find AC. (2.) Find AD. (3.) Find CD.

(1.) To find AC.

In triangle ACB,

AB=735

CBA=28° 20′

CAB=63 30 +35° 10′

=98 40

and $\therefore$ ACB=53

AC : AB : : sin. CBA : sin ACB.

log. AB.........	2·866287
„ sin. CBA...	9·676328
	12·542615
„ sin. ACB...	9·902349
„ AC...	2·640266

$\therefore$ AC=436·8.

(2.) To find AD.

In triangle ABD,	AD : AB : : sin. ABD : sin. ADB.
AB=735	log. AB.........2·866287
DAB=35° 10′	„ sin. ABD...9·976702
ABD=28 20 +80° 16′	12·842989
=108° 36′	„ sin. ADB...9·771643
and ∴ ADB=36° 14′	„ AD...3·071346
	∴ AD=1178·5.

(3.) To find CD (by Rule 4, second method, *Trig.* Part I.).

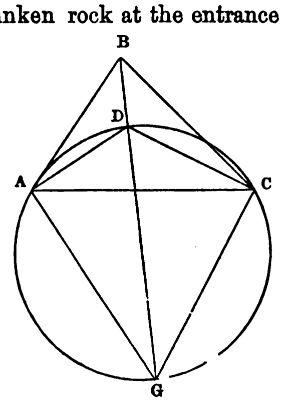
In triangle ACD,		
AC= 436·8	log. (AD+AC)...3·208253...	log (AD+AC)...3·208253
AD=1178·5	„ (AD−AC)...2·870226	„ sin. ½ CAD...9·721162
∴ AC+AD=1615·3	0·338027	2·929415
AC−AD= 741·7	„ tan. ½ CAD...9·791563	„ sin. arc.......9·904757
CAD=63° 30′	„ tan. arc.......10·129590	„ CD............3·024658
∴ ½ CAD=31 45		∴ the distance CD=1058·3 yards.

53. Wishing to know the distance between two redoubts C and D by which the entrance into a harbour is defended, a boat is placed at A with its head towards a tree seen at E (produce the line AB to some point E) in the direction AB, and the angles CAD=22° 17′ and DAE=48° 1′ were observed. The boat is then moved to B, a distance of 1000 yards, directly towards the tree, and the angles CBD=53° 15′ and DBE=75° 43′ are observed. Required the distance between the redoubts C and D.

Ans. 1290 yards.

As the two following problems are of great use in Marine Surveying, we will solve them by logarithms, and also by a geometrical construction. In Problem 98 of the volume of Astronomical Problems, analytical solutions of the same problems are also given.

54. Wishing to determine the position of a sunken rock at the entrance of a bay, and the water being smooth, I anchored a boat upon it, and measured with a sextant the angles which three objects, A, B, and C, on the shore subtended at the boat. They were as follows: the angle between A and the object B to the right was 26° 27′, and the angle between B and the object C to the right was 34° 12′. On my chart of the bay I carefully measured with compasses the distances between the three objects, and found AB=5 miles, BC=6 miles, and AC=7 miles. Required the distance of the rock from A, B, and C.



(1.) *By Construction.*

By means of a scale of equal parts make the triangle ABC, having the side AB=5, BC=6, and AC=7. At the point C, on the side of AC farthest from the boat, make the angle ACD=26° 27′, the angle observed between the other two objects A and B; and at the point A, on the farthest side also from the boat, make the angle CAD=34° 12′, the angle between the other two objects B and C: produce the sides AD and CD till they meet in D. Then describe a circle to pass through the three points A, D, and C; and the position of the rock will be somewhere in the circumference of that circle. To find that position, join BD, and produce it to the circumference in G; then G will be the station sought, or the position of the rock.

For, the angles in the same segment of a circle being equal (*Euclid*, b. iii.), therefore AGB=ACD=26° 27′, and CGB=CAD=34° 12′; and these were the angles observed at the boat. Hence G must be the position of the boat; and GA, GB, and GC measure respectively the distance of the rock from each of the objects A, B, and C.

(2.) *By Trigonometry.*

Assume any point G to be the position of the boat, and let A, B, and C be the objects. Describe roughly a circle passing through the three points A, G, C. Join GA, GB, and GC. Then GA, GB, and GC are the distances required. Draw AD, CD to the point of intersection D. Then, by Geometry, since the angles in the same segment of a circle are equal, ∴ CAD=CGB=34° 12′, and ACD=AGB=26° 27′.

[1.] Find AD, having given in the triangle ADC the side AC=7, the angle ACD=26° 27′, and ADC=180°−(34° 12′+26° 27′)=119° 21′.

[2.] Find angle BAC, having given the three sides of the triangle ABC.

[3.] Find angle ABD, having given AB, AD, and angle BAD (=BAC−CAD).

[4.] Find GA, GB, and angle BAG, having given in the triangle ABG the side AB and the angles AGB and ABG.

[5.] Find GC, having given in the triangle AGC the side AC, the angle AGC, and the angle CAG=(BAG−BAC).

Calculation.

[1.] To find AD.

AC : AD : : sin. ADC : sin. DCA

AC=7		0·845098
ADC=119°	21′	9·648766
DCA= 26	27	10·493864
		9·940338
		0·553526

∴ AD=3·577

[2.] To find angle BAC

AB=5	7	9·154902
BC=6	5	9·301030
AC=7	2	0·602060
	6	0·301030
	8	9·359022
	4	BAC=57° 7′ 15″
	4	CAD=34 12 0
	2	∴BAD=22 55 15

[3.] To find angle ABD.

AB+AD : AB−AD : : tan. $\frac{1}{2}$ (ADB+ABD) : tan. $\frac{1}{2}$ (ADB−ABD)

AB=5	0·153205
AD=3·577	10·692995
8·577	10·846200
1·423	0·933335
BAD= 22° 55′ 15″	9·912865
180	39° 17′ 15″
157 4 45	78 32 22
78 32 22	ADB=117 49 37
	ABD= 39 15 7

[4.] To find GA, GB, and the angle BAG.

AB : GA : : sin. AGB : sin. ABG

AB : GB : : sin. AGB : sin. BAG

AB=5		
AGB= 26° 27′		
ABG= 39 15	0·698970	0·698970
65 42	9·801201	9·959711
180	10·500171	10·658681
BAG=114 18	9·648766	9·648766
	0·851405	1·009915
	GA=7·103	GB=10·23

[5.] To find GC.

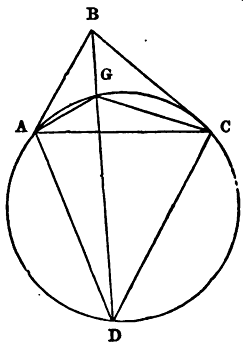
AC : GC : : sin. AGC : sin. CAG

AC=7	0·845098
AGC= 60° 39′	9·924491
BAG=114 18	10·769589
BAC= 57 7	9·940338
∴ CAG= 57 11	0·829251
	∴ GC=6·75

55. Sailing in a deep and unknown bay, I suddenly found the soundings decrease; and suspecting I was on a coral reef, I hauled off, having anchored a boat on it, from which the following angles were taken between three remarkable objects, A, B, and C, that appeared on the distant shores, namely: between A and B, a high-pointed rock to the right of A, the angle was 116° 40′; between B and C, a bluff summit to the right of B, the angle was 112° 30′. I obtained afterwards the distances between the three points A, B, and C, by measuring on the shore convenient base-lines, and taking angles, as pointed out in Example, p. 14. The distance from A to B was 5·75 miles, from B to C 7·5 miles, and from A to C 8·25 miles. It is required to find the position of the reef.

(1.) *By Construction.*

Construct the triangle ABC from the scale of equal parts, by taking AB= 5·75, BC=7·5, and AC=8·25. At the point C, in the straight line AC, make ACD=63° 20′, the supplement of the angle subtended by the other two points A and B. Again, at the point A, in the straight line AC, make CAD=67° 30′, the supplement of the angle subtended by the other two points B and C: produce the lines AD and CD to meet in the point D. About the triangle ADC describe a circle; then the place of the reef will be somewhere in the circumference of this circle. To find it, join BD; and the point of intersection G is the position of the reef required.



For since the angles in the same segment of a circle are equal (*Euclid*, b. iii.), therefore AGD=ACD=63° 20′; therefore the angle AGB=116° 40′. Again, CGD=CAD=67° 30′; therefore the angle BGC=112° 30′. And these were the angles observed at the boat; therefore G must be the place of the boat, or position of reef.

(2.) *By Trigonometry.*

Assume any point G as the position of the reef, and let A, B, and C be the objects on shore. Describe a circle passing through the three points A, G, and C. Join BG, and produce it to meet the circle in D. Join GA and GC. Then GA, GB, and GC are the distances required. Join AD and CD. Then, by Geometry, since the angles in the same segment are equal, ∴ angle DAC =DGC=180°−BGC=180°−112° 30′=67° 30′, and angle ACD=AGD=180° −AGB=180°−116° 40′=63° 20′.

[1.] Find AD, having given in the triangle ADC, AC=8·25, angle ACD= 63° 20′, and angle ADC=180°−(67° 30′+63° 20′)=49° 10′.

[2.] Find the angle BAC, having given the three sides of the triangle ABC.

[3.] Find the angle ABD, having given AB, AD, and the angle BAD= (BAC+CAD).

[4.] Find GA, GB, and the angle BAG, having given in the triangle ABG the side AB and the angles AGB and ABG.

[5.] Find GC, having given in the triangle AGC the side AC, the angle AGC, and the angle CAG=BAC−BAG.

Calculation.

[1.] To find AD.

AC : AD : : sin. ADC : sin. DCA

AC=8·25	0·916454
ADC=49° 10′	9·951159
DCA=63 20	10·867613
	9·878875
	0·988738
	∴ AD=9·744

[2.] To find the angle BAC.

AB=5·75	8·25	9·083546
AC=8·25	5·75	9·240332
BC=7·5	2·50	0·698970
	7·5	0·397940
	10·0	9·420788
	5·0	BAC= 61° 46′ 15″
	5·0	CAD= 67 30 0
	2·5	BAD=129 16 15

[3.] To find the angle ABD.

AD+AB : AD−AB : : tan. ½ (ABD+ADB) : tan. ½ (ABD−ADB)

AD= 9·744	0·601408
AB= 5·750	9·675890
15·494	10·277298
3·994	1·190164
180° 0′ 0″	9·087134
BAD=129 16 15	6° 58′
50 43 45	25 22
25 21 52	∴ ABD=32 20

[4.] To find GA, GB, and the angle BAG.

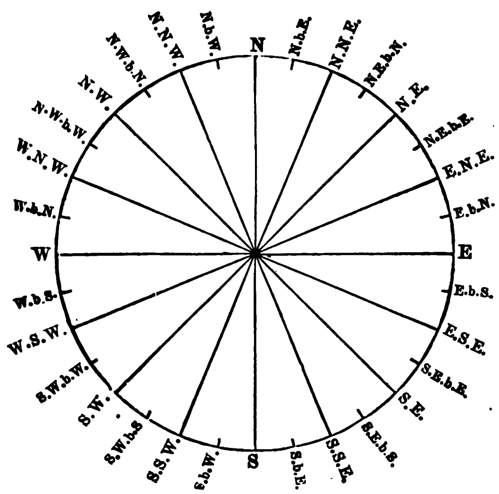
AB=5·75	AB : GA : : sin. AGB : sin. ABG	
AGB=116° 40′	AB : GB : : sin. AGB : sin. BAG	
ABG= 32 20	0·759668	0·759668
149 0	9·728227	9·711839
180	10·487895	10·471507
BAG= 31 0	9·951159	9·951159
	0·536736	0·520348
	∴ GA=3·44	∴ GB=3·31

[5.] To find GC.

AC=8·25	AC : GC : : sin. AGC : sin. CAG	
AGB=116° 40′	BAC=61° 46′	0·916454
BGC=112 30	BAG=31 0	9·708882
229 10	∴ CAG=30 46	10·625336
360		9·878875
∴ AGC=130 50		0·746461
		∴ GC=5·58

The problems about to follow require a knowledge of the several points of the Mariner's Compass. We will therefore first show how the compass card is constructed and an expeditious method of learning the bearings of its points and quarter-points from the meridian. These points should be thoroughly known and committed to memory.

THE COMPASS.



The compass card is represented above: each quadrant is divided into eight equal parts, called *points;* each point therefore contains the eighth part of 90°, or 11° 15′. The four cardinal points are the North, South, East, and West points; the intermediate points are formed and named as follows:

The middle point between N. and E. is ............................N.E.
(formed simply by putting these letters together).

Similarly:

The middle point between N. and N.E. is ..........................N.N.E.
„ „ E. „ N.E. is..........................E.N.E.
Again, one point from N. towards E. is N. by E. or.......N.b.E.
„ „ E. „ N. is E. by N. or.......E.b.N.
„ „ N.E. „ N. is N.E. by N. or.......N.E.b.N.
„ „ N.E. „ E. is N.E. by E. or.......N.E.b.E.

The other three quadrants of the compass are divided and named in a similar manner.

Before the student proceeds further, he should form as neatly as he can, in the manner pointed out above, and without the aid of instruments, the above compass, writing it down *several times,* until he is thoroughly ac-

quainted with its construction and the 32 parts or points into which it is thus divided.

(6.) *Bearings or Angular Distances by Compass.*

The points of the compass are frequently referred to with respect to their position or bearing to the *right* or *left* of the cardinal point towards which the spectator is looking: thus, suppose the compass card to represent the horizon, and the spectator to be placed in the center of the card and looking towards the north, then any point as N.E. is said to be 4 points to the right of N. (written thus—4 r. N.): E.b.N. is 7 points right of N. or 7 r. N. If the spectator is looking towards the east, then N.E. is 4 left of E. or 4 l. E., E.b.N. is 1 left of E. or 1 l. E., and so on.

EXAMPLES.

56. Required the bearings of the following points—first, from the north; second, from the east:

N.N.E. N.E.b.N. N.b.E. N.N.W. N.W. W.b.N.

Answer.

Bearings from	N.N.E. or	N.E.b.N. or	N.b.E. or	N.N.W. or	N.W. or	W.b.N. or
North......	2 r. N.	3 r. N.	1 r. N.	2 l. N.	4 l. N.	7 l. N.
East	6 l. E.	5 l. E.	7 l. E.	10 l. E.	12 l. E.	15 l. E.

57. Required the bearings of the following points from the north, east, south, and west respectively.

S.b.E. S.E.b.S. S.E.b.E. S.S.W. W.b.S. E.S.E.

Answer.

Bearings from	S.b.E. or	S.E.b.S. or	S.E.b.E. or	S.S.W. or	W.b.S. or	E.S.E. or
North......	15 r. N.	13 r. N.	11 r. N.	14 l. N.	9 l. N.	10 r. N.
East	7 r. E.	5 r. E.	3 r. E.	10 r. E.	15 r. E.	2 r. E.
South......	1 l. S.	3 l. S.	5 l. S.	2 r. S.	7 r. S.	6 l. S.
West	9 l. W.	11 l. W.	13 l. W.	6 l. W.	1 l. W.	14 l. W.

58. Required the compass bearings of the following points:

2 r. N. 5 l. N. 3 r. S. 12 r. S. 5 r. E. 4 l. W.

Answer.

2 r. N. or N.N.E.	3 r. S. or S.W.b.S.	5 r. E. or S.E.b.S.
5 l. N. or N.W.b.W.	12 r. S. or N.W.	4 l. W. or S.W.

Each point of the compass, moreover, is subdivided into *quarter points*, and named from the adjacent points; thus $2\frac{1}{2}$ points to the right of north is N.N.E.$\frac{1}{2}$E.; $7\frac{3}{4}$ points to the left of north is W.b.N.$\frac{3}{4}$W., or rather W.$\frac{1}{4}$N.

EXAMPLES.

59. Required the bearings of the following points—first, from the north; second, from the east:

N.N.E.$\frac{1}{4}$E. N.$\frac{3}{4}$E. E.b.N.$\frac{1}{2}$N. N.E.$\frac{1}{2}$N. N.W.$\frac{1}{2}$W. N.$\frac{3}{4}$W.

Answer.

Bearings from	N.N.E.$\frac{1}{4}$E. or	N.$\frac{3}{4}$E. or	E.b.N.$\frac{1}{2}$N. or	N.E.$\frac{1}{2}$N. or	N.W.$\frac{1}{2}$W. or	N.$\frac{3}{4}$W. or
North...	$2\frac{1}{4}$ r. N.	$\frac{3}{4}$ r. N.	$6\frac{1}{2}$ r. N.	$3\frac{1}{2}$ r. N.	$4\frac{1}{2}$ l. N.	$\frac{3}{4}$ l. N.
East	$5\frac{3}{4}$ l. E.	$7\frac{1}{4}$ l. E.	$1\frac{1}{2}$ l. E.	$4\frac{1}{2}$ l. E.	$12\frac{1}{2}$ l. E.	$8\frac{3}{4}$ l. E.

60. Required the bearings of the following points—first, from the south; second, from the west:

S.b.E.$\frac{1}{2}$E. S.E.b.S.$\frac{1}{4}$S. S.S.E.$\frac{3}{4}$E. S.$\frac{3}{4}$W. W.S.W.$\frac{1}{4}$S. W.S.W.$\frac{1}{4}$W.

Answer.

Bearings from	S.b.E.$\frac{1}{2}$E. or	S.E.b.S.$\frac{1}{4}$S. or	S.S.E.$\frac{3}{4}$E. or	S.$\frac{3}{4}$W. or	W.S.W.$\frac{1}{4}$S. or	W.S.W.$\frac{1}{4}$W. or
South...	$1\frac{1}{2}$ l. S.	$2\frac{3}{4}$ l. S.	$2\frac{3}{4}$ l. S.	$\frac{3}{4}$ r. S.	$5\frac{3}{4}$ r. S.	$6\frac{1}{4}$ r. S.
West ...	$9\frac{1}{2}$ l. W.	$10\frac{3}{4}$ l. W.	$10\frac{3}{4}$ l. W.	$7\frac{1}{4}$ l. W.	$2\frac{1}{4}$ l. W.	$1\frac{3}{4}$ l. W.

61. Required the compass bearings of the following points:

$2\frac{1}{2}$ r. N.	$1\frac{3}{4}$ l. N.	$10\frac{1}{4}$ r. S.	$7\frac{3}{4}$ l. N.	$3\frac{1}{2}$ r. S.	$3\frac{1}{2}$ l. S.
$6\frac{1}{2}$ r. S.	$10\frac{1}{2}$ l. S.	$14\frac{3}{4}$ r. N.	8 r. N.	8 l. S.	$15\frac{1}{2}$ r. N.

Answers.

$2\frac{1}{4}$ r. N. or N.N.E.$\frac{1}{4}$N.	$10\frac{1}{4}$ r. S. or W.N.W.$\frac{1}{4}$N.	$3\frac{1}{2}$ r. S. or S.W.$\frac{1}{2}$S.
$6\frac{1}{2}$ r. S. or W.S.W.$\frac{1}{2}$W.	$14\frac{3}{4}$ r. N. or S.b.E.$\frac{1}{4}$E.	8 l. S or East.
$1\frac{3}{4}$ l. N. or N.N.W.$\frac{1}{4}$N.	$7\frac{3}{4}$ l. N. or W.$\frac{1}{4}$N.	$3\frac{1}{2}$ l. S. or S.E.$\frac{1}{2}$S.
$10\frac{1}{2}$ l. S. or E.N.E.$\frac{1}{2}$N.	8 r. N. or East.	$15\frac{1}{2}$ r. N. or S.$\frac{1}{2}$E.

(7.) Attached to the compass card, and coinciding with the line N.S., is a magnetic bar of steel, by means of which the card, when balanced on a fine point near its center, will indicate the compass bearing or direction of any object beyond it. Thus, the compass being placed near the helm, the bearing of the ship's head is seen at once, and the direction in which the ship is steered is readily noted.

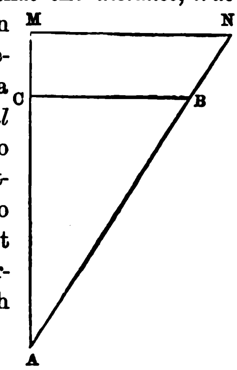
The Log-line.

(8.) The log is a flat piece of thin wood of a quadrantal form, loaded in the circular side with lead sufficient to make it swim upright in the water; to this is fastened a line about 150 fathoms long, called the *log-line*, which is divided into certain spaces called *knots;* the length of each knot is sup-

posed to be the same part of a nautical mile (about 6080 feet) that half a minute is of an hour; hence 1 knot$=\frac{6080}{120}=51$ feet nearly. If, therefore, 1 knot runs out in half a minute (shown by a half-minute glass), the rate of the ship is supposed to be 1 mile an hour; if 2 knots, the rate is 2 miles an hour, and so on. The length of the knot is very rarely so much as 51 feet, and the hour-glass used is not always a half-minute glass: various modifications of the two instruments are made, to render this method of measuring the ship's way tolerably correct; these will be more clearly understood in the use of the instruments themselves.

NAUTICAL PROBLEMS SOLVED BY TRIGONOMETRY AND ALSO BY CONSTRUCTION.

(9.) It is proved in *Navigation*, Part II. p. 151, that the *distance*, *true difference of latitude*, *departure*, and *course* between any two places on the earth may be correctly represented by the three sides and one of the angles of a right-angled plane triangle; and that the *meridional difference of latitude* and *difference of longitude* by two sides of a triangle which is similar to the same right-angled plane triangle. Thus, let A and B be the two places, AB a straight line joining them, and AC that part of the meridian passing through A that is intercepted between A and a straight line BC drawn through B perpendicular to AC: then



AC	will represent	the true	difference of latitude.
AB	,,	,,	distance.
BC	,,	,,	departure.
angle CAB	,,	,,	course from A to B.

Again, if AC be produced to M, so that AM may be equal to the meridional difference of latitude between A and B, and MN be drawn parallel to CB to meet AB produced to N: then

AM will represent the meridional difference of latitude, and MN the difference of longitude between the two places A and B.

The line AN is not used in navigation.

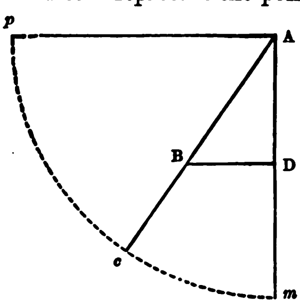
We thus see that questions in navigation or plane sailing may be much simplified by considering the above six terms as forming parts of two similar right-angled plane triangles connected together as shown in the above figure; for then we can solve nearly all the questions in plane sailing by the simple application of the rules in Trigonometry for right-angled plane triangles.

We will proceed to exemplify this by means of a few useful problems in sailing, and will at the same time show how these problems may be solved by *construction*; that is, by measuring with mathematical instruments the several lines and angles given by the problem, limiting ourselves at present to questions that require a knowledge only of the several parts of the smaller triangle ABC.

62. A ship from latitude 47° 30′ N. has sailed S.W.b.S. 98 miles: find by construction, and by calculation, the latitude in and departure.

(1.) *By Construction.*

Let A represent the point the ship departed from, AD the meridian, and



Ap, drawn at right angles to it, the parallel of latitude of the ship. At the point A, with a chord of 60°, describe the quadrant mp, and cut off mc=S.W.b.S. or 33° 45′=the course; and through c draw a line AB. From a scale of equal parts take AB=98 miles, the distance; and through B draw BD parallel to Ap, meeting AD in D. Then B is the place the ship has arrived at, AD is the difference of latitude, and BD is the departure. If AD and BD are measured by the same scale of equal parts, it will be found that the difference of latitude AD is about 81 miles, and the departure BD about 54 miles. The figure may be more easily laid off by means of a protractor (see any work on Practical Geometry).

(2.) *By Trigonometry.*

In the right-angled triangle ABD are given the course DAB=33° 45′, and distance AB=98 miles; to find the difference of latitude AD, and departure BD.

$$\text{By Rule, p. 9, } \frac{\text{AD}}{\text{AB}}=\cos.\ \text{DAB} \ \therefore\ \text{AD}=\text{AB}\cos.\ \text{DAB}$$

$$\text{By same Rule, } \frac{\text{BD}}{\text{AB}}=\sin.\ \text{DAB} \ \therefore\ \text{BD}=\text{AB}\sin.\ \text{DAB}.$$

Reducing these formulæ to logarithms, we have:

$$\log.\ \text{AD}=\log.\ \text{AB}+\log.\ \cos.\ \text{DAB}-10$$
$$\log.\ \text{BD}=\log.\ \text{AB}+\log.\ \sin.\ \text{DAB}-10$$

AB=98 DAB=33° 45′

log. AB.........1·991226	log. AB.........1·991226
„ cos. DAB...9·919846	„ sin. DAB...9·744739
„ AD..........1·911072	„ BD ....1·735965
∴ AD=81·5=1° 21′ 30″S.	∴ BD=54·4

Lat. from...47 30 0 N.
Lat. in......46 8 30 N. and dep. 54·4 W.

The Traverse Table.

Questions involving two or more parts of a right-angled plane triangle are often very easily solved by means of a table called the Traverse Table, which contains the difference of latitude and departure calculated for any course and distance, so that when any two of these quantities are given the other two may be found *by inspection.* In last example are given the course and distance to find diff. lat. and departure. Entering the table, therefore, with the course S.W.b.S. = 3 points and distance 98 miles, we find the corresponding diff. lat. and dep. to be 81·5 and 54·4 respectively.

63. A ship from latitude 20° 30′ N. has sailed W.S.W. 120 miles: find by construction, by logarithms, and by the traverse table the latitude she is in and the departure she has made.

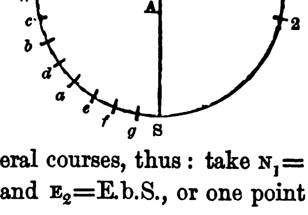
Ans. Lat. in. 19° 44′ 6″ N., dep. 110′·9.

When a ship has described more than one course during the day, and it is required to show by a diagram the latitude she has arrived at, we may proceed as in the following example.

64. A ship in latitude 47° 30′ N. sailed N.N.W. 90 miles, and E.b.S. 60 miles: find latitude in and departure.

(1.) *By Construction.*

Let A represent the place the ship left, and with any convenient radius describe the circle NWSE to represent the horizon of the ship. Draw two diameters NS and WE at right angles to each other. Let NS represent the meridian, and WE the parallel of latitude the ship departed from. To mark off the several courses and distances during the day, we may proceed as follows:



[1.] Divide one of the quadrants, WS, into eight equal parts, to form a scale of points. This may be done by bisecting WS in a, and then Wa in b, and Wb in c: the other points, $defg$, may then be readily filled in.

[2.] Mark off on the circumference the several courses, thus: take N_1= N.N.W., or two points from the scale in WS; and E_2=E.b.S., or one point from the east.

[3.] Through A_1 draw the straight line AB, and make AB=90 miles by a scale of equal parts; and through B, parallel to a line passing through A_2, draw BC=60 miles. The point C represents the place the ship has arrived at. Join AC, and through C draw CD parallel to WE, meeting AN produced in D. Then AD is the difference of latitude, and DC the departure made

good during the day. Also the angle CAD and line AC represent the direct course and distance from A to C.

If we measure AD by the scale of equal parts, we shall find the difference of latitude AD about 71 miles to the north, and the departure DC about 24 miles to the east of the place the ship left. The latitude arrived at is found thus:

Lat. A.........47° 30′N.
Diff. lat....... 1 11 N.
Lat. in.........48 41 N. and dep. 24 E.

(2.) *By Trigonometry and Traverse Table.*

To calculate the difference of latitude and departure between A and C, we must proceed as follows:

Through B draw BF parallel to WE, meeting the meridian produced in F. Then in the right-angled triangle ABF are given the course BAF=2 points, and distance AB=90 miles, to calculate AF the diff. lat. and BF the departure. Again, through B draw BG parallel to the meridian NS; and through C draw CG parallel to WE, meeting BG in G. Then in the triangle BGC are given the course GBC=7 points, and BC=60 miles, to calculate BG =FD the diff. lat. and GC the departure. By performing the calculation, we find that AF=83·2 miles to the north, and BG=11·7 to the south; so that the diff. lat.=83·2 N.−11·7 S.=71·5 miles. Similarly may be found GC=58·8 to the east, and FB or DG=34·4 to the west; so that the departure =58·8 E.−34·4 W.=24·4 E.

This method of computing the diff. lat. and departure separately for every course is in practice avoided by making use of the Traverse Table. The diff. lat. and departure, when taken out of the table, are arranged under proper heads in the following form:

Points.	Courses.	Dist.	Diff. lat.		Dep.	
			N.	S.	E.	W.
2	N.N.W.	90	83·2	—	—	34·4
7	E.b.S.	60	—	11·7	58·8	—
			83·2	11·7	58·8	34·4
			11·7		34·4	
			71·5 N.		24·4 E.	

Lat. from.........47° 30′ N.
Diff. lat.......... 1 11·5 N.=71·5′N.
Lat. in...........48 41·5 N. and dep. 24·4 E.

65. A ship in latitude 50° 30′ N. has sailed during the day N.N.E. 100 miles and W.b.S. 70 miles: what latitude is she in, and what departure has she made? *Ans.* Lat. in 51° 48′·7 N., dep. 30′·4 W.

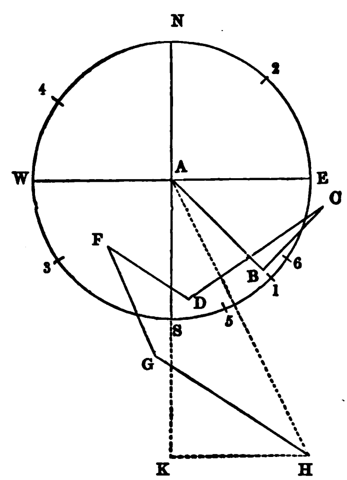
66. A ship from latitude 50° 48′ N. has sailed during the day on the following courses. Required the latitude in, and departure, and the direct course and distance.

1. S.E..........40 miles.	4. N.W.b.W....30 miles.
2. N.E.........28 „	5. S.S.E.........36 „
3. S.W.b.W...52 „	6. S.E.b.E......58 „

(1.) *By Construction.*

Let A be the place sailed from, and NWSE the horizon of the ship. Draw the meridian NS, and parallel of latitude WE.

Divide one of the quadrants into eight equal parts for a scale of points, as in the last example, and by means of this scale mark off the circumference the several courses, viz. S_1=S.E., N_2=N.E., S_3=S.W.b.W., N_4=N.W.b.W., S_5=S.S.E., and S_6=S.E.b.E.



Through A draw AB=40 by a scale of equal parts; through B, and parallel to A_2, draw BC=28 miles; through C, and parallel to A_3, draw CD=52 miles; through D, and parallel to A, draw DF=30 miles; through F, and parallel to A_5, draw FG=36 miles; and lastly, through G, and parallel to A_6, draw GH=58 miles. The point H is the place the ship has arrived at. Join AH, and through H draw HK parallel to WE and meeting the meridian NS produced in K. Then AK is the difference of latitude, and KH the departure made good during the day. Also the angle KAH represents the direct course, and the line AH the direct distance from A to H.

If we measure AK by the scale of equal parts, we shall find the difference of latitude AK about 86 miles to the south, and the departure KH about 42 miles to the east of the place the ship left. The latitude arrived at is found thus:

Lat. A............	50° 48′ N.
Diff. lat..........	1 26 S.
Lat. in...........	49 22 N. and dep. 42′ E.

(2.) *By Trigonometry and Traverse Table.*

The diff. lat. and departure for each course and distance may be computed as in Example, p. 25. But to avoid this tedious operation, the several

quantities may be taken out of the Traverse Table by inspection, as follows:

Points.	Courses.	Dist.	Diff. lat.		Dep.	
			N.	S.	E.	W.
4	S.E.	40	—	28·3	28·3	—
4	N.E.	28	19·8	—	19·8	—
5	S.W.b.W.	52	—	28·9	—	43·2
5	N.W.b.W.	30	16·7	—	—	24·9
2	S.S.E.	36	—	33·3	13·8	—
5	S.E.b.E.	58	—	32·2	48·2	—
			36·5	122·7	110·1	68·1
				36·5	68·1	
			Diff. lat.=86·2		42·0=dep.	

Lat. from.........50° 48′ 0″ N.
Diff. lat........... 1 26 12 S.=86·2 S.
Lat. in............49 21 48 N. and dep. 42′ E.

To calculate the direct course from A to H, or the angle KAH, and the direct distance A H, we have in the plane right-angled triangle AKH, AK=86·2, and KH=42; to find the course KAH, and distance AH.

[1.] To find the direct course KAH, tan. $\text{KAH}=\frac{\text{KH}}{\text{AK}}$

∴ log. tan. KAH−10=log. KH−log. AK
or log. tan. KAH=10+log. KH−log. AK

KH=42 log. KH+10....11·623249
AK=86·2 „ AK.......... 1·935507
„ tan. KAH... 9·687742
∴ the direct course, or KAH=S.25° 58′ 30″ E.

[2.] To find the distance AH.

$\frac{\text{AH}}{\text{AK}}$=sec. KAH, or AH=AK sec. KAH

∴ log. AH=log. AK+log. sec. KAH−10
log. AK. ....1·935507
„ sec. KAH−10...0·046247
„ AH................1·981754
∴ distance, or AH=95·8 miles.

The two following examples, taken out of that valuable old work, Robertson's *Elements of Navigation*, are given for practice in construction,

and for the singularity of the form of the diagrams resulting from the several courses and distances.

67. A ship sails from a place in latitude 40° N. on the following courses. Required the latitude arrived at (by Construction).

Course.		Course.	
1....S.E.b.S	29′	13....West	62′
2....N.N.E	10	14....North	10
3....E.S.E.	50	15....West	8
4....E.N.E.	50	16....South	10
5....S.S.E.	10	17....West	62
6....N.E.b.N.	29	18....South	7
7....West	25	19....E.¾S.	62
8....S.S.E.	10	20....South	110
9....W.S.W.½W.	42	21....W.N.W.½W.	42
10....North	110	22....N.N.E.	10
11....E.¾N	62	23....West	25
12....North	7		

Ans. The ship returns to the place sailed from.

68. Two ships, A and B, part company in lat. 31° 31′ N., and meet again at the end of two days, having run as follows:

The ship A. Course.		The ship B. Course.	
1....N.N.E	96′	1....N.N.W	96′
2....W.S.W	96	2....E.S.E.	96
3....E.S.E.	96	3....W.S.W	96
4....N.N.W	96	4....N.N.E.	96

Required the latitude arrived at, and the direct course and distance of each ship (by Construction).

Ans. Direct course due north, and distance 104 miles. Lat. in 33° 15′ N.

CORRECTIONS IN PLANE SAILING.

(10.) Three corrections are sometimes necessary to be applied to the course steered by compass, to reduce it to the true course; and the converse. These are called:

(1.) The correction for variation of the compass.

(2.) The correction for deviation of the compass.

(3.) The correction for leeway.

(1.) *The Correction for Variation of the Compass.*

(11.) The magnetic needle seldom points to the true north. Its deflection to the east or west of the true north is called the *variation of the compass;* it is different in different places, and it is also subject to a slow change in the same place. The variation of the compass is ascertained at sea by observing the magnetic bearing of the sun when in the horizon, or at a given altitude above it. From this observation the true bearing is found by rules given in nautical astronomy. The difference between the true bearing and the observed bearing is the variation of the compass.

The method of correcting the course for variation will be more readily understood by means of a few examples.

Suppose the variation of the compass is found to be two points to the east, that is, the needle is directed two points to the right of the north point of the heavens; then the N.N.W. point of the compass card will evidently point to the true north, and every other point on the card will be shifted round two points. If, therefore, a ship is sailing *by compass* N.N.W., or, as it is expressed, the compass course is N.N.W., her true course will be north; that is, *two points to the right of the compass course.* In a similar manner it may be shown that, when the variation is two points westerly, the true course will be *two points to the left of compass course.* Hence this rule:

To find the true course, the compass course being given.

Easterly variation allow to the right.
Westerly „ „ left.

From the preceding considerations it will be easy to deduce the converse rule, namely:

To find the compass course, the true course being given.

Easterly variation allow to the left.
Westerly „ „ right.

EXAMPLES.

69. Find the true course, having given the compass course N.W.$\frac{1}{2}$W. and variation 3$\frac{1}{4}$W.

	pts.	qrs.	
Compass course......	4	2	left of N.
variation	3	1	left.*
true course	7	3	left of N.=W.$\frac{1}{4}$N.

* When names are alike (that is, both left or both right), *add:* when unlike, *subtract*, marking remainder with the name of the greater.

70. Find the compass course, having given the true course W.$\frac{1}{4}$N. and variation $3\frac{1}{4}$W.

	pts.	qrs.
True course	7	3 left of N.
variation	3	1 right.
compass course	4	2 left of N. = N.W.$\frac{1}{2}$W.

Find the true course in each of the following examples:

	Compass course.	Var.	Answers.
71.	N.N.E.	$2\frac{1}{4}$W.	N.$\frac{1}{4}$W.
72.	N.W.	$1\frac{3}{4}$E.	N.N.W.$\frac{1}{4}$W.
73.	S.W.$\frac{3}{4}$W.	$1\frac{1}{2}$E.	W.S.W.$\frac{1}{4}$W.
74.	S.	2W.	S.S.E.
75.	W.	$2\frac{1}{2}$E.	N.W.b.W.$\frac{1}{2}$W.

Find the compass course in each of the following examples:

	True course.	Var.	Answers.
76.	N.N.E.$\frac{1}{2}$E.	$\frac{1}{4}$W.	N.N.E.$\frac{3}{4}$E.
77.	N.	$1\frac{1}{2}$E.	N.b.W.$\frac{1}{2}$W.
78.	S.S.W.	2W.	S.W.
79.	S.W.	0	S.W.
80.	N.b.W.$\frac{1}{4}$W.	$1\frac{1}{4}$W.	N.

(2.) *The Correction for Deviation of the Compass.*

(12.) This correction of the compass arises from the effect of the iron on board ship on the magnetic needle, in deflecting it to the right or left of the magnetic meridian. The increased quantity of iron used in ships has caused this correction to be attended to now more than formerly, as its effects and magnitude have become more perceptible. The amount of the deviation arising from this local cause varies as the mass of iron changes its position with respect to the compass. When a fore and aft line coincides with the direction of the magnetic meridian, the iron in the ship may be supposed to be nearly equally distributed on both sides of the needle, and its effect in deflecting the needle may be inappreciable. In other positions of the ship with respect to the magnetic meridian, the iron may produce a sensible deflection of the needle; and this deflection or deviation will in general be the greatest when the ship's head points to the east or west.

Various methods are used to determine this correction. The one usually adopted is to place a compass on shore, where it may be beyond the influence of the iron of the ship, or any other local disturbing force, and to take the bearing of the ship's compass, or some object in the same direction therewith; at the same time, the observer on board takes the bearing of the shore compass; then if 180° be added to the bearing at the shore compass, so as

to bring it round to the opposite point, the difference between the result and the bearing at ship's compass will be the amount of the deviation of the compass for that position of the ship.

The ship is then swung round one or two points, and a similar observation made; and thus the local deviation found for a second position of the ship. This being repeated for every point or two points of the compass, the deviation is thus known for all positions of the ship. A table, similar to the one below, is then formed, and the courses corrected for this deviation by the following rules; which resemble those already given for correcting for variation.

Deviation of Compass of H.M.S. ——, for given positions of the ship's head.

Direction of ship's head.	Deviation of compass.		nearly	Direction of ship's head.	Deviation of compass.		nearly
N.	E.	2° 45′	or $\frac{1}{4}$ pt.	S.	W.	3° 0′	or $\frac{1}{4}$ pt.
N.b.E.	E.	4 57	or $\frac{1}{2}$ „	S.b.W.	W.	4 20	or $\frac{1}{2}$ „
N.N.E.	E.	7 30	or $\frac{3}{4}$ „	S.S.W.	W.	5 0	or $\frac{1}{2}$ „
N.E.b.N.	E.	9 0	or $\frac{3}{4}$ „	S.W.b.S.	W.	6 7	or $\frac{1}{2}$ „
N.E.	E.	10 0	or $\frac{3}{4}$ „	S.W.	W.	7 0	or $\frac{1}{2}$ „
N.E.b.E.	E.	10 55	or 1 „	S.W.b.W.	W.	7 27	or $\frac{3}{4}$ „
E.N.E.	E.	10 40	or 1 „	W.S.W.	W.	7 50	or $\frac{3}{4}$ „
E.b.N.	E.	9 55	or $\frac{3}{4}$ „	W.b.S.	W.	8 20	or $\frac{3}{4}$ „
E.	E.	8 50	or $\frac{3}{4}$ „	W.	W.	8 50	or $\frac{3}{4}$ „
E.b.S.	E.	7 15	or $\frac{3}{4}$ „	W.b.N.	W.	8 10	or $\frac{3}{4}$ „
E.S.E.	E.	5 35	or $\frac{1}{2}$ „	W.N.W.	W.	6 50	or $\frac{1}{2}$ „
S.E.b.E.	E.	3 40	or $\frac{1}{4}$ „	N.W.b.W.	W.	5 40	or $\frac{1}{2}$ „
S.E.	E.	1 50	or $\frac{1}{4}$ „	N.W.	W.	4 50	or $\frac{1}{2}$ „
S.E.b.S.	E.	0 20	or 0 „	N.W.b.N.	W.	3 20	or $\frac{1}{4}$ „
S.S.E.	W.	0 56	or 0 „	N.N.W.	W.	1 40	or 0 „
S.b.E.	W.	2 20	or $\frac{1}{4}$ „	N.b.W.	E.	1 10	or 0 „

(13.) *To find the true course,* having given the compass course and the deviation.

Easterly deviation allow to the right.
Westerly „ „ left.

EXAMPLES.

81. Correct the compass course W.b.S. for deviation $\frac{3}{4}$W. (known from above table).

	pts.	qrs.	
Compass course.......	7	0	right of S.
deviation...............	0	3	left.
true course.............	6	1	right of S.

or W.S.W.$\frac{1}{4}$W.

82. Correct the compass course N.W.$\frac{1}{2}$W. for deviation $\frac{1}{2}$W. (from deviation table, p. 32), and also for variation of compass 3$\frac{1}{4}$W.

	pts.	qrs.	
Compass course..............	4	2 l.	N.
deviation...........0 2 l.			
variation...........3 1 l.			
	3	3 l.	
true course..................	8	1 l.	N.
	16		
or true course..............	7	3 r.	S.=W.$\frac{1}{4}$S.

Find the true course in each of the following examples, by correcting for deviation from table, p. 32, and for variation:

	Compass Course.	Var.	Answers.
83.	N.N.E.	2$\frac{1}{4}$W.	N.$\frac{1}{2}$E.
84.	N.W.	1$\frac{3}{4}$E.	N.N.W.$\frac{3}{4}$W.
85.	S.W.$\frac{3}{4}$W.	1$\frac{1}{2}$E.	S.W.b.W.$\frac{3}{4}$W.
86.	S.	2W.	S.S.E.$\frac{1}{4}$E.
87.	W.	2$\frac{1}{2}$E.	W.N.W.$\frac{1}{4}$W.
88.	W.$\frac{3}{4}$N.	1$\frac{1}{2}$W.	W.S.W.$\frac{1}{2}$W.

(14.) *To find the compass course*, having given the true course and deviation.

Easterly deviation allow to the left.
Westerly „ „ right.

Note.—The true course should first be corrected for variation (if any) by Rule, p. 30 (which is similar to the above), so as to get a compass course nearly, and then this course for deviation, from table, p. 32.

EXAMPLES.

89. Required the compass course, the true course being W.S.W.$\frac{1}{4}$W., variation 0, and deviation $\frac{3}{4}$W. (see table).

	pts.	qrs.	
True course	6	1 r.	S.=compass course nearly.
deviation	0	3 r.	
compass course...	7	0 r.	S., or W.b.S.

90. Required the compass course, the true course being S.W., variation of compass 2$\frac{1}{4}$E., and deviation as in table, p. 32.

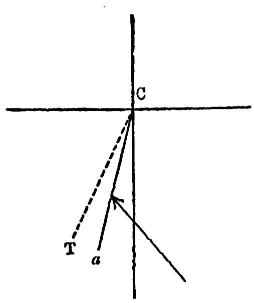
	pts.	qrs.	
True course	4	0	r. S.
variation	2	1	l.
compass course nearly...	1	3	r. S., or S.b.W.$\frac{3}{4}$W.
deviation	0	2	r.
compass course...........	2	1	r. S.=S.S.W.$\frac{1}{4}$W.

Required the compass course in each of the following examples (for deviation, see table, p. 32):

	True course.	Var.	Answers.
91.	N.$\frac{1}{2}$E.	2$\frac{1}{4}$W.	N.N.E.
92.	N.N.W.$\frac{3}{4}$W.	1$\frac{3}{4}$E.	N.W.
93.	S.W.b.W.$\frac{3}{4}$W.	1$\frac{1}{2}$E.	S.W.$\frac{3}{4}$W.
94.	S.S.E.$\frac{1}{4}$E.	2W.	S.
95.	W.N.W.$\frac{1}{4}$W.	2$\frac{1}{2}$E.	W.
96.	W.S.W.$\frac{1}{2}$W.	1$\frac{1}{2}$W.	W.$\frac{3}{4}$N.

(3.) *The Correction for Leeway.*

(15.) This correction is the angle which the ship's track makes with the direction of a fore and aft line: it arises from the action of the wind on the sails, &c. not only impelling the ship forwards, but pressing against it sideways, so as to cause the actual course made to be to *leeward* of the apparent course, as shown by the fore and aft line. The amount of leeway differs in different ships, depending on their construction, on the sails set, the velocity forwards, and other circumstances. Experience and observation, therefore, usually determine the amount of leeway to be allowed.



The method of correcting for leeway will be best seen by the following example:

Suppose the apparent course is S.S.W.$\frac{1}{2}$W., and leeway two points, the wind being S.E., required the correct course.

Draw two lines at right angles to each other towards the cardinal points of compass, and a line, as Ca, to represent (roughly) the course of the ship, and another to represent the direction of the wind (as the arrow in fig.); then it will be seen that the corrected course, as CT, will be to the *right* of the apparent course; *the observer being always supposed to be at the center* C,

and looking towards the cardinal point from whence the course is measured; hence

	pts.	qrs.
Apparent course......	2	2 r. S.
leeway................	2	0 r.
corrected course......	4	2 r. S.=S.W.½W.

EXAMPLES.

Correct the following courses for leeway, so as to find the true courses:

	Apparent course.	Wind.	Leeway.	Answers.
97.	N.N.E.	W.N.W.	1½	N.E.½N.
98.	N.W.	N.N.E.	2	W.N.W.
99.	E.S.E.	S.	2½	E.½N.
100.	E.	N.b.E.	¾	E.¾S.

Correct the following compass courses for deviation, variation, and leeway, so as to find the true courses. The deviation is found in table, p. 32, and the variation of compass is supposed to be in each example 2½ W.

	Course.	Wind.	Leeway.	Answers.
101.	N.W.¼W.	W.S.W.	2½	N.W.¾W.
102.	S.E.½.E.	E.N.E.	2¼	S.E.½E.
103.	W.¼S.	S.S.W.	2	W.S.W.½W.
104.	N.¾W.	W.b.N.	1½	N.b.W.¾W.

These examples may be worked out in the following manner:

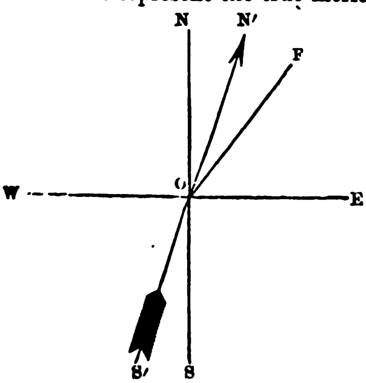
			pts.	qrs.
Ex. 101.	Compass course.........		4	1 l. N.
	deviation.........0	2 l.		
	variation.........2	2 l.		
			3	0 l.
			7	1 l. N.
	leeway....................		2	2 r.
	true course..............		4	3 l. N.=N.W.¾W.

In the preceding examples the courses, both true and compass, are corrected for variation and deviation by a formal rule. The student, however, should also know how to make these corrections by means of a construction, as in the following examples:

105. Given the true course=N. 42° 28′ E., and the variation of the compass=$1\frac{1}{2}$ points easterly; construct a figure to show the compass course.

Construction.

Let NS represent the true meridian; and since the variation of the compass is $1\frac{1}{2}$ points E., draw N′S′ $1\frac{1}{2}$ points, or 16° 52′, to the east of the true meridian; then N′S′ will represent the direction of the magnetic meridian, and the angle NON′ the variation of the compass. At the point O, in the straight line NO, make the angle NOF=42° 28′; then NOF will represent the true course, N. 42° 28′ E., and N′OF will therefore be the compass course; and it is evident by the figure that



N′OF=NOF−NON′,

or compass course=true course−variation

=42° 28′−16° 52′=25° 36′;

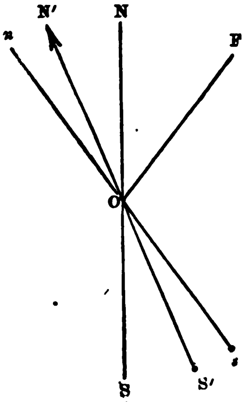
and since this angle is to the right of the magnetic meridian,

∴ the compass course=N. 25° 36′ E.

106. Given the true course=N. 25° 36′ E., the variation=2 points W., and deviation on account of local attraction=7° 20′ E.; to find the corrected compass course (by construction).

Construction.

Let NS represent the true meridian; and since the variation of the compass is 2 points westerly, draw *ns* 2 points, or 22° 30′, to the west of the true meridian; then *ns* will represent the direction of the magnetic meridian, and the angle NO*n* the variation of the compass. But the needle is deflected 7° 20′ to the east of the magnetic meridian; draw, therefore, N′S′ 7° 20′ to the right of *ns*; then N′O*n*=deviation of compass, and N′S′ will represent the position of the needle. At the point O, in the straight line NO, make the angle NOF=25° 36′; then NOF will represent the true course, N. 25° 36′ E., and N′OF the corrected compass course required.



By the figure, N′OF=NOF+(NOn−nON′)
=25° 36′+(22° 30′−7° 20′)
=25 36+15 10
=40 46

∴ the corrected compass course=N. 40° 46′ E.

By practical rule (p. 34),

True course	25° 36′	r. N.
variation	22 30	r.
compass course nearly	48 6	r. N.
deviation	7 20	l.
∴ compass course	40 46	r. N.=N. 40° 46′ E.

EXAMPLE FOR PRACTICE.

107. Given the true course, S. 15° 58′ E.; variation of compass, 2¼ points W.; deviation, 4° 20′ W. Construct figure, and find compass course.

Ans. Compass course, S. 13° 40′ W.

CONSTRUCTION OF A MERCATOR'S CHART.

(16.) A Mercator's chart represents the surface of the earth as a plane (p. 3), and is constructed as follows:

Draw at the bottom of a sheet of paper a straight line to represent the most southern parallel of latitude required for the chart; divide it into equal parts, as degrees, &c., regulating the length of each degree according to the number required in the chart and the size of the paper: or if the chart is to be drawn to any given scale, as one inch or ·7 of an inch, &c., make the length of each degree of longitude on the scale one inch or ·7 inch accordingly. The line so drawn at the bottom of the paper we may call the *longitude line;* at each extremity of this line erect a perpendicular: these perpendiculars are the *graduated meridians,* on which must be marked the length of each degree of latitude.

To obtain the linear measure of the degrees of latitude.

(17.) Write down on a slip of paper, in a vertical column, the degrees of latitude which the chart is to contain, beginning with the highest degree: take out from the Table of Meridional Parts the meridional parts for each degree, and write them down opposite their corresponding latitudes; take the successive differences between the first and second, second and

third, &c., of these meridional parts, and thus make a second vertical column. Then, to find the points on the graduated meridians through which each parallel of latitude is to be drawn, transfer these meridional differences of latitude to the graduated meridians, by measuring along the longitude line at the bottom of the chart the number of minutes, &c. contained in each meridional difference of latitude taken in order,* making a dot on the graduated meridians at the extremity of each measure; connect these dots by straight lines: these will be the parallels of latitude required. The intermediate meridian lines are then to be filled in, by drawing lines through the divisions of the base-line, or through every fifth degree, or through as many as may be considered sufficient; a compass should then be drawn on the chart (or more than one, if the chart is large); this will be useful to determine the bearings of different points, or for more conveniently finding the latitude and longitude of the ship when her course and distance run are given. To construct the compass, take some convenient intersection of a meridian and parallel as a center, and describe a circle with any suitable radius; mark the points of the circumference cut by the meridian with the letters N. and S., and complete the compass by inserting the other points.

To lay down upon the chart a point whose latitude and longitude are given.

(18.) Lay the edge of a ruler (or doubled edge of paper) along the given parallel of latitude; measure off the degrees, &c. between the given longitude and the longitude of the nearest meridian line drawn on the chart; apply this difference to the edge of the ruler in the proper direction, and the point on the chart whose latitude and longitude are given will be found.

EXAMPLE.

108. Let it be required to lay down on the chart a point whose latitude is 50° 48′ N., and longitude 22° 10′ W.

Place the edge of the ruler over latitude 50° 48′ N. in the chart, and with a pair of compasses, or otherwise, take 2° 10′ (the difference between 22° 10′ and 20°, assuming that the meridian line of 20° is the nearest on the chart), and lay it along the ruler towards the left from the meridian of 20°: the position of the required point will then be determined.

In this manner a ship's daily track is usually pricked off; for the latitude and longitude being known at noon, her place is given at that time, and the entire track during the voyage can be seen by connecting, by straight lines, the places or points on the chart thus found.

* This may be easily done by placing along the graduated longitude line the doubled edge of a piece of paper, and transferring to it the required lengths; or by taking the proper distance by a pair of compasses.

To copy a chart on a different scale.

(19.) Having drawn the meridians and parallels as pointed out in p. 37, find the latitude and longitude of the prominent points in the chart, and transfer these points to the new chart, p. 38; then sketch in neatly with the hand the outline of the coast between the assumed points, and insert all the other necessary parts of the chart, as rocks, shoals, islands, &c., as accurately as possible.

(20.) *To find the course and distance between two given places on the chart.*

1. *To find the course.* Place the edge of a parallel ruler over the two places on the chart, and keeping one part of the parallel ruler firm, move the other till the edge passes through the center of the compass described on the chart: the edge thus lying on the compass will point out the course between the two places. It also may be found by means of the small semicircular protractor contained in most cases of mathematical instruments in the following manner. Place the straight edge of the protractor against the edge of the ruler as it lies upon the two places, and slide it along till the center of the protractor is on one of the meridian lines; then the course will be seen on that point in the circumference of the protractor through which the meridian line passes. A rectangular protractor will determine this with equal facility.

2. *To find the distance.* The distance is found (nearly) by transferring the space or interval between the two places as it appears on the chart to the side line, or graduated meridian, *as nearly opposite the two places as possible*; the degrees, &c. (turned into minutes) which this space measures on the graduated meridian will be the distance required. If the places have the same latitude, the distance is found more accurately as follows: Take half the space or interval between them; apply it to the graduated meridian above and below the parallel on which the places are situated: the difference between the degrees of the extreme points (turned into minutes) will be the distance required (nearly). If the places have the same longitude, it is evident the difference of their latitudes (or the sum, if they are on different sides of the equator) will be the distance between the two places.

To find the latitude and longitude in by the chart, having given the course and distance from a given place.

(21.) Lay down the course on the chart in the manner pointed out in pages 25-27, or by any other method suited to the instruments at hand. To the line (or edge of the ruler) thus lying in the direction of the

course, apply the distance run (turned into degrees and minutes if necessary), measured from that part of the graduated meridian which is adjacent to the given place, and to that to which the ship is sailing: this distance so taken along the line or edge of the ruler from the place sailed from will determine the position of the ship on the chart, that is, its latitude and longitude in.

(22.) The following example of constructing a Mercator's chart and of tracing the ship's track thereon is similar to the one given at the Naval College at the monthly examination of sub-lieutenants.

109. Construct a Mercator's chart on a scale of one inch extending from 54° N. to 58° N., and from long. 178° E. to 178° W., and lay down thereon the ship's track, namely the several true courses and distances from the following sailings, thus forming a track chart:

Compass courses and	S.	E.b.S.	N.b.E.	N.½W.	N.W.	S.W.b.W.
dist. on each course.	90′	100′	65′	60′	80′	75′

Variation of the compass, 1 point E.

Correcting compass courses to get the true courses, we have

True courses	S.b.W.	E.S.E.	N.N.E.	N.½E.	N.W.b.N.	W.S.W.
and distance.	90′	100′	65′	60′	80′	75′

First, to construct the chart within the given limits.

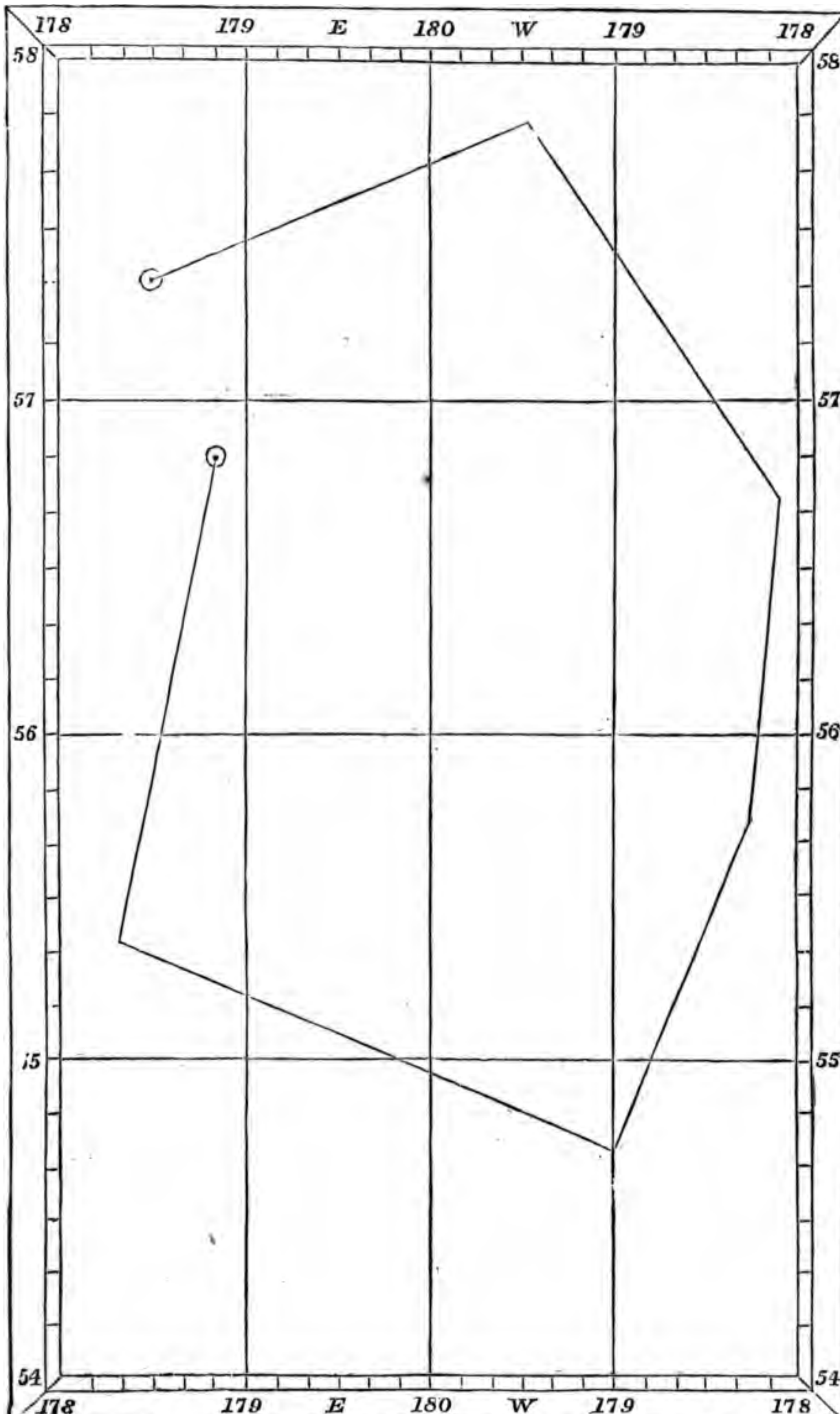
This is done by following the directions given in Arts. 16, 18, pp. 37, 38, as follows:

1. At the bottom of the paper draw the longitude line, and divide it into four equal parts each one inch long, to contain the four degrees of longitude, namely, from 178° E. to 178° W., and erect the graduated meridians at each extremity of the longitude line.

2. Subdivide each degree of the longitude line into 6 or any other convenient number of equal parts: if into 6, as in the diagram, then each subdivision will be 10′. This longitude line may now be used as a scale of equal parts from which to set off any longitude distances on the chart; but it will be better to make a scale of equal parts on a separate piece of paper by drawing a straight line and dividing it into several equal parts each one inch, and one of these equal parts again to subdivide into 6 or more equal parts.

3. Write down the meridional parts for each degree of latitude so as to get the M. D. lat. for each degree, beginning with the highest degree: thus

lat. 58...merid. parts...4294
„ 57... „ ...4183...M. D. lat=111′ between 57° and 58°
„ 56... „ ...4074... „ =109 „ 56 „ 57
„ 55... „ ...3968... „ =106 „ 55 „ 56
„ 54... „ ...3865... „ =103 „ 54 „ 55



Ans. Lat. in 57° 21′ N., long. in 178° 31′ E.

4. Transfer the quantities 103, 106, 109, 111, taken off the scale or from the longitude line to the graduated meridians; draw the parallels of latitude and intermediate meridians and mark them with their proper degrees (see preceding page), and the chart will be ready to lay down on it the track of the ship, that is, the several true courses and distances made during the day. This is done as follows:

Second, to lay down the ship's track on the chart.

1. Find the point on the chart corresponding to the place the ship sailed from: thus

Lay the edge of a ruler (or a doubled edge of a piece of paper) over lat. 56° 54′ N.; and since the longitude is 178° 50′ E.—that is, 10′ to the left of 179° E.—with a pair of compasses or otherwise take a distance of 10′ and lay it along the ruler to the left from meridian 179° E.; make a small dot at the point, and the position of the ship, namely lat. 56° 54′ N. and long. 178° 50′ E., is determined.

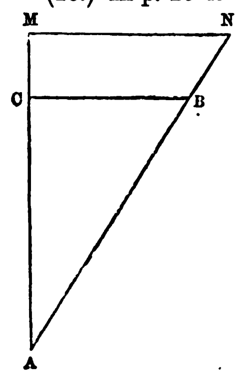
2. Draw the several lines to represent the ship's track: thus (Art. 21)

From the point thus found draw a line S.b.W. (or 1 point west of meridian 179° E.) and equal to 90′, remembering to take all the *distances from that part of the graduated meridians adjacent to the respective courses.* From the southern extremity of this line draw a line E.S.E. 100′, and proceed in the same manner to lay down the several other courses given in the question; when it will be found that the ship has arrived at a place in lat. 57° 21′ N. and long. 178° 31′ E.

3. Finish the chart off neatly by rubbing out pencilled and superfluous lines, and surrounding it with parallel lines, &c., as a boundary (see track chart on preceding page).

Fundamental Formulæ for Plane Sailing.

(23.) In p. 23 it is shown that the six terms—distance, tr. diff. latitude, departure, meridional difference of latitude, diff. long., and course—may be correctly represented by two similar right-angled plane triangles ABC, AMN.



If any two or more of these terms contained in either of the triangles are known, the others may be found by means of the trigonometrical ratios (Art. 4, p. 9).

Thus, in triangle ABC, let AB the distance and BAC the course be given: to find BC the departure, and AC the true diff. lat.

By p. 9, $\frac{\text{BC}}{\text{AB}}=\sin.\ \text{A}$ or $\frac{\text{dep.}}{\text{dist.}}=\sin.$ course $\therefore$ dep. $=$ dist. sin. course (1).

" $\frac{\text{AC}}{\text{AB}}=\cos.\ \text{A}$ or $\frac{\text{T. D. lat.}}{\text{dist.}}=\cos.$ course

$\therefore$ T. D. lat. $=$ dist. cos. course (2).

In triangle AMN, given AM the M. D. lat. and MAN the course, to find MN the diff. long.

By p. 9, $\frac{\text{MN}}{\text{AM}}=\tan.\ \text{A}$ or $\frac{\text{diff. long.}}{\text{M. D. lat.}}=\tan.$ course

$\therefore$ diff. long $=$ M. D. lat. tan. course (3).

To obtain the formulæ for proving the rules in parallel sailing, and middle lat. sailing, we must proceed as pointed out in p. 163, *Nav.* Part II. These formulæ are

In parallel sailing......dist. $=$ diff. long. $\times$ cos. lat..........(4).
In middle lat. sailing...dep. $=$ diff. long. $\times$ cos. mid. lat....(5).

Collecting these formulæ for the sake of reference,

Departure $=$ distance $\times$ sin. course....................(1).
True diff. lat. $=$ distance $\times$ cos. course...............(2).
Diff. long. $=$ meridional diff. lat. $\times$ tan. course...(3).
In parallel sailing, distance $=$ diff. long. $\times$ cos. lat...........(4).
In mid. lat. sailing,
departure (nearly) $=$ diff. long. $\times$ cos. mid. lat....(5).

(This latter formula is only approximately correct: see *Nav.* Part II. p. 145.)

The mathematical student will no doubt use these formulæ and figure to solve questions in Navigation, rather than follow the formal rules we are now about to give for that purpose. These rules are, in fact, simply the above formulæ expressed in words.

RULES IN NAVIGATION.

Rule 1. *To find the course and distance from one place to another,* having given the latitudes and longitudes of the two places (by using meridional parts, called Mercator's method).

(1.) Find true difference of latitude, meridional difference of latitude, and difference of longitude; reduce the true difference of latitude and difference of longitude to minutes, attaching thereto the proper letters. Rules (*a*), (*b*), (*d*).

(2.) *To find the course.* From the log. difference of longitude (increased by 10) subtract the log. mer. diff. latitude; the remainder is the log. tan. course, which find in the tables, and place before it the letter of the true difference latitude, and after it the letter of the difference longitude, to indicate the direction of course. At the same opening of the tables, take out the log. secant course.

(3.) *To find the distance.* Add together log. secant course and log. true difference latitude; the sum (rejecting 10 in the index) will be the log. distance, which find in the tables.

110. Required the course and distance from A to B.

lat. A 45° 15′ N. long. A 35° 26′ W.
" B 47 10 N. " B 32 15 W.

	M.P.	
lat. A...45° 15′ N.	3051·2 N.	long. A...35° 26′ W.
" B...47 10 N.	3217·4 N.	" B...32 15 W.
1 55	M. D. lat. 166·2 N.	3 11
60		60
T. D. lat. 115 N.		diff. long. 191 E.
log. diff. long + 10..12·281033		log. sec. course 0·182767
" M. D. lat....... 2·220631		" T. D. lat...2·060698
" tan. course......10·060402		" dist.........2·243465
∴ course N. 48° 58′ E.		∴ distance 175′.

Required also the *compass* course in the above example: var. of compass being 2 points W., and deviation on account of local attraction as in table (p. 32). See Rule, p. 33.

	pts.	qrs.	
True course......48° 58′ r N. or	4	1	r. N.*
variation............................	2	0	r.
compass course nearly	6	1	r. N.=E.N.E.¼E.
deviation............................	1	0	l.
∴ compass course.................	5	1	r. N.=N.E.b.E.¼E.

* Degrees are converted into points, or the converse, by means of the table for that purpose in the nautical tables.

Examples in Navigation are usually worked without attaching to each logarithm taken out its name or designation, as in the following example:

111. Required the course and distance from A to B.

lat. A...51° 31′ N.	long. A...0° 6′ W.
„ B...54 33 N.	„ B...3 5 E.

		M.P.	
	51° 31′ N.	3618	0° 6′ W.
	54 33 N.	3921	3 5 E.
	3 2	M. D. lat. 303	3 11
	60		60
diff. lat.	182 N.	diff. long.	191 E.
	12·281033		0·072650
	2·481443		2·260071
	9·799590		2·332721
	∴ N. 32° 13′ 30″ E.=course.		215·1=dist.

Required the course and distance from A to B in each of the following examples, by Rule 1 or Mercator's method.

	Lat. from and lat. in.	Long. from and long. in.	Answers. Course and distance.
112.	lat. A 49° 52′ S.	long. A 17° 22′ W.	course N. 26° 36′ E.
	lat. B 42 13 S.	long. B 11 50 W.	dist. 513·3 miles.
113.	lat. A 49 10 N.	long. A 29 17 W.	course N. 37° 48′ W.
	lat. B 56 45 N.	long. B 39 5 W.	dist. 576 miles.
114.	lat. A 50 48 N.	long. A 1 10 E.	course N. 41° 55′ W.
	lat. B 52 35 N.	long. B 1 25 W.	dist. 144 miles.
115.	lat. A 58 24 N.	long. A 4 12 W.	course N. 32° 34′ E.
	lat. B 63 17 N.	long. B 2 13 E.	dist. 347·6 miles.
116.	lat. A 2 37 N.	long. A 110 42 W.	course S. 75° 12′ W.
	lat. B 0 0	long. B 120 36 W.	dist. 614·4 miles.
117.	lat. A 3 30 N.	long. A 33 40 E.	course S. 42° 32′ E.
	lat. B 4 10 S.	long. B 40 42 E.	dist. 624 miles.

Required also the compass courses in Examples 115, 116, and 117, the variation of compass being 2 points E., and deviation as in table (p. 32). See Rule, p. 33.

ANSWERS.

115.	compass course	N.½E. nearly.
116.	„ „	S.W.b.W.¼W. nearly.
117.	„ „	E.S.E.¼E. „

Rule 2. *To find the latitude and longitude in, having given the course and distance* (by Mercator's method).

(1.) *To find latitude in.* Add together log. cos. course* and log. distance, the sum (rejecting 10 in the index) will be log. true difference latitude, which find in the tables; reduce to degrees and minutes, and place the letter N. or S. against it, according as course is northward or southward.

(2.) Apply true difference latitude to latitude from, so as to get the latitude in. Rule (*e*).

(3.) *To find longitude in.* Take out the meridional parts for the two latitudes, and get M. D. lat. Rule (*b*).

(4.) Add together log. tangent course and log. meridional difference latitude; the sum (rejecting 10 in the index) will be the log. difference longitude, which find in the tables; reduce to degrees and minutes, and place the letter E. or W. against it, according as the course is eastward or westward.

(5.) Apply difference longitude to longitude from, so as to get longitude in. Rule (*f*).

EXAMPLES.

118. Sailed from A, N. 37° 10′ E., 472·6 miles; required the latitude and longitude in.

lat. A 27° 20′ N.		long. A 25° 12′ E.
log. cos. course 9·901394		log. tan. course 9·879740
„ dist.........2·674494		„ M. D. lat. 2·641474
„ T. D. lat. 2·575888		„ diff. long. 2·521214
∴ T. D. lat. 376·6′		diff. long. 332·1′
or 6° 17′ N.	M.P.	or 5° 32′ E.
lat. from 27 20 N.	1706 N.	long. from 25 12 E.
„ in. 33 37 N.	2144 N.	„ in 30 44 E.
	M. D. lat. 438	

119. A ship in latitude 27° 0′ S. and longitude 123° W. sailed S.S.E. ½ E. (or S. 28° 7′ 30″ E.) 150 miles: required the latitude and longitude in.

9·945430		9·727957
2·176091		2·176091
2·121521		1·904048
6,0)13,2·3		80′·1
diff. lat....... 2° 12′ 18″ S.	M.P.	1° 20′ 6″ E.
lat. from ....27 0 0 S.	1683 S.	123 0 0 W.
„ in ....29 12 18 S.	1833 S.	121 39 54 W.
	150	Long. in.

* Take out, at same opening of tables, log. tan. course and place it a little to the right.

Required the latitude and longitude in, by Rule 2 or Mercator's method, in each of the following examples, having sailed from A as follows:

	Course and dist. from A.		Lat. A.	Long. A.	Answers. Lat. in.	Long. in.
120.	N. 26°36′E.	513·5′	49°52′S.	17°22′W.	42° 13′S.	11°50′W.
121.	S. 48 58 W.	175·2	47 10 N.	32 15 W.	45 15 N.	35 26 W.
122.	N. 29 10 E.	373·4	52 10 N.	17 32 W.	57 36 N.	12 15 W.
123.	S. 37 7 E.	370·0	70 14 S.	25 30 E.	75 9 S.	38 5 E.
124.	N. 47 47 E.	272·4	50 15 S.	15 10 E.	47 12 S.	20 16 E.

Rule 3. *To find the course and distance* (*middle latitude method*), having given the latitudes and longitudes of the two places.

(1.) Find the true difference latitude, middle latitude, and difference longitude by Rules (*a*), (*c*), (*d*).

(2.) *To find the course.* Add together log. cos. mid. lat. and log. diff. long., and from the sum subtract log. true difference latitude; the remainder is the log. tan. course, which find in the tables, and mark it with the same letters as the true difference latitude and difference longitude. From the same opening take out the log. secant of course.

(3.) *To find distance.* To the log. secant course just found add the log. true difference latitude; the sum (rejecting 10 in index) will be the log. distance.

EXAMPLES.

125. Required the course and distance from A to B, by middle latitude method.

lat. A 50°25′N. long. A 27°15′W.
„ B 47 12 N. „ B 30 20 W.

lat. A 50°25′N..........50°25′N. long. A 27°15′W.
„ B 47 12 N..........47 12 N. „ B 30 20 W.
3 13 2)97 37 3 5
60 mid. lat. 48 48 60

T. D. lat. 193 S. diff. long. 185 W.
log. cos. mid. lat. 9·818681 log. sec. course 0·072849
„ diff. long. ... 2·267172 „ T. D. lat...2·285557
12·085853 „ dist.........2·358406
log. T. D. lat. ... 2·285557 ∴ dist. 228·2′
„ tan. course... 9·800296 ∴ course S. 32°16′W.

Required the course and distance from A to B in each of the following examples, by middle latitude method:

	Lat. from and lat. in.	Long. from and long. in.	Answers. Course and dist.
126.	lat. A 49°52′S.	long. A 17°22′W.	N. 26°40′E.
	lat. B 42 13 S.	long. B 11 50 W.	513·6
127.	lat. A 21 15 S.	long. A 0 30 W.	S. 14°37′E.
	lat. B 30 27 S.	long. B 2 10 E.	570·5
128.	lat. A 60 15 S.	long. A 14 55 E.	S. 32°50′E.
	lat. B 65 36 S.	long. B 22 30 E.	382

Rule 4. *To find the latitude and longitude in* (by middle latitude method), having given the course from a given place, and distance.

(1.) *To find latitude in.* Add together log. cos. course* and log. distance; the sum (rejecting 10 in the index) is the log. true difference latitude, which find from tables, and mark N. or S. according as the course is northward or southward.

Apply true difference latitude (turned into degrees and minutes, if necessary) to the latitude from, and thus get latitude in. (Rule *e.*) Find the middle latitude. Rule (*c*).

(2.) *To find longitude in.* Add togther log. sin. course, log. distance, and log. secant middle latitude; the sum (rejecting 20 in the index) is the log. difference longitude, which find in tables, and mark E. or W. according as the course is eastward or westward. Apply the difference longitude (in degrees and minutes) to the longitude from, and thus get longitude in. Rule (*f*).

129. Sailed from A, S. 37° 10′ W., 472·6 miles; required lat. in and long. in (by middle lat. method).

lat. A 27°20′S.	long. A 25°12′W.
log. cos. course...9·901394	log. sin. course...9·781134
„ dist. ....2·674494	„ dist. ....2·674494
„ T. D. lat. ...2·575888	„ sec. mid. lat. 0·064531
∴ T. D. lat. 376·6′	„ diff. long. ...2·520159
or 6°17′S.	∴ diff. long. 331·3′
lat. from 27 20 S.	or 5°31′W.
„ in. 33 37 S.	long. from 25 12 W.
2)60 57	„ in 30 43 W.
mid. lat. 30 28	

* Take out at the same opening log. sin. course, and put it down a little to the right.

Required the latitude and longitude in, by middle latitude method, in each of the following examples, having sailed from A as follows:

	Course and dist. from A.	Lat. A.	Long. A.	Answers. Lat. in.	Long. in.
130.	N. 25°42′W. 427·3′	64°10′N.	40°15′W.	70°35′N.	48°17′W.
131.	S. 48 58 W. 175·2	47 10 N.	32 15 W.	45 15 N.	35 26 W.
132.	N. 34 48 W. 383·7	50 25 N.	3 40 E.	55 40 N.	2 24 W.

PARALLEL SAILING.

Rule 5. *To find the course and distance,* having given the latitude of the two places, and their longitudes.

(1.) Find the difference longitude.

(2.) The *course* is evidently due east or due west, according as the longitude in is to the east or west of longitude from.

(3.) *To find the distance.* Add together log. cos. latitude and log. difference longitude; the sum (rejecting 10 in index) is the log. distance, which find in the table.

133. Required the course and distance from A to B.

lat. A...80°N. long. A...3° 50′E.
,, B...80 N. ,, B...6 10 W.

long. from.......3° 50′ E.
,, in..........6 10 W.
10 0
60
600 W.
∴ the course is west.

log. cos. lat.......9·239670
,, diff. long.....2·778151
,, dist...........2·017821
∴ dist. 104·2′.

134. Required the *compass* course and distance from A to B.

lat. A ......50° 48′ N. long. A ......100° 0′ E.
,, B ......50 48 N. ,, B ......101 0 E.

Variation of the compass two points E., and deviation as in table, p. 32.

long. A..100° E. 9·800737
,, B..101 E. 1·778151
1 1·578888
60 37·9=dist.
60 E.

	pts.	qrs.	
True course...............	8	0	r of N.
variation.................	2	0 l	
compass course nearly	6	0	r of N=E.N.E.
deviation.................	1	0 l	
compass course.........	5	0	r of N. or N.E. by E.

Required the true course and distance from A to B in each of the following examples:

	Lat. A and B.	Long. A.	Long. B.	Answers. Course and dist.
135.	70° 10′ S.	15° 10′ E.	22° 15′ E.	East 144·2′
136.	50 48 N.	5 0 W.	5 0 E.	East 379·2
137.	50 10 N.	40 25 W.	50 10 W.	West 374·7
138.	48 10 N.	100 0 W.	110 0 W.	West 400·2
139.	75 13 N.	15 20 E.	0 0 E.	West 234·7
140.	80 15 N.	179 0 E.	176 0 W.*	East 50·8

Rule 6. *To find the longitude in*, having given the course and distance, and latitude and longitude from.

Add together log. sec. lat. and log. distance; the sum (rejecting 10 in the index) will be the log. difference longitude. Find the natural number thereof, and turn it into degrees, and mark it E. or W. according as the course is E. or W. Apply difference longitude to longitude from, and thus find longitude in.

The latitude in is the same as the latitude from.

EXAMPLE.

141. Sailed from A due east 1000 miles, required the latitude and longitude in. Lat. A...32° 10′ S.; long. A...28° 42′ W.

lat. in=lat. from=32° 10′ S.
log. sec. lat..................0·072372
" dist......................3·000000
" diff. long. ....3·072372
∴ diff. long. 1181′, or 19° 41′ E.
long. from....................28 42 W.
∴ long. in .... 9 1 W.

Required the latitude and longitude in, in each of the following examples:

	Course and dist.	Lat. from.	Long. from.	Answers. Lat. in.	Long. in.
142.	East 492·5′	52° 10′ N.	0° 29′ W.	52° 10′ N.	12° 54′ E.
143.	East 1752	60 0 N.	5 10 W.	60 0 N.	53 14 E.
144.	East 560	57 32 N.	13 5 W.	57 32 N.	4 18 E.
145.	West 740	60 0 N.	50 0 W.	60 0 N.	74 40 W.

* In this example it is evident we must modify the general rule; for the diff. long. is never considered to be greater than 180°. When, therefore, the above rule gives the diff. long. greater than 180°, subtract it from 360°, and apply thereto a contrary letter to the one directed by the rule; the result will be the diff. long. to be used.

Application and use of formulæ in p. 43.

(24.) The preceding rules are the principal ones used in Navigation. It would be easy for the mathematical student to make for himself others, by means of the relations between the several terms course, dist., dep., &c., as shown by the formulæ and diagram in p. 43: he would find then no difficulty in solving problems similar to the following:

146. Sailed from A, in long. in 3° 10′ W., 300 miles due east, and altered my longitude 10 degrees; required the latitude and longitude in.

Thus, by form (4)...dist.=diff. long. × cos. lat.

$\therefore$ cos. lat. $=\frac{\text{dist.}}{\text{diff. long.}}=\frac{300}{600}=\frac{1}{2}$ $\therefore$ lat. in=60°, and long. in=6° 50′ E.

147. Wishing to make a small island, I took the ship to windward of it in the same latitude with the island, namely 50° 48′ N. The longitude of the ship by chronometer was 20° 35′ W., and the long. of the island was 23° 50′ W. What was my distance from the island?

In this example of parallel sailing we have given lat. 50° 48′, and diff. long. 3° 15′, or 195′, to find distance.

By form (4)...dist.=diff. long. × cos lat.
log. diff. long..........2·290035
,, cos. lat............9·800737
,, dist................2·090772 $\therefore$ dist. 123·2 miles.

148. What course must be steered so that the departure may be one-third the distance?

In fig. p. 42, we have given the relation between the departure CB and distance AB; that is

$$\frac{\text{dep.}}{\text{dist.}}=\text{sin. course}$$

$$\text{and by the question, } \frac{\text{dep.}}{\text{dist.}}=\tfrac{1}{3}$$

$$\therefore \text{ sin. course}=\tfrac{1}{3} \text{ and course}=19^\circ\ 28'.$$

149. Sailed between the N. and E. 100 miles, and altered my latitude 1° 10′: required the course.

In fig. p. 42, AC=T. D. lat.=1° 10′=70′; AB=distance=100′

and cos. course $=\frac{\text{AC}}{\text{AB}}=\frac{70}{100}=\frac{7}{10}$ $\therefore$ course=N. 45° 35′ E.

To find the course and distance from one place to another, as from A to B, having given T. D. lat., mid lat., and diff. long. By fig. p. 42.

150. Find course and distance from A to B.

lat. A......58° 24′ N.	long. A.... 4° 12′ W.
„ B......63 17 N.	„ B.... 2 13 E. (Ex. 82).

	M. P.	
lat. A...58° 24′ N.	4339·8	long. A...4° 12′ W.
„ B...63 17 N.	4942·6	„ B...2 13 E.
4 53	M. D. lat....602·8=AM	6 25
60		60
T. D. lat......293=AC		diff. long.....385=MN

To find the course.	To find the distance.
(By fig.)...tan. course$=\frac{MN}{AM}$	distance AB=AC sec. course.
log. MN+10...12·585461	log. AC............ 2·466868
„ AM......... 2·780173	„ sec. course...10·074293
„ tan. course 9·805288	„ dist.......... 2·541161
∴ course=N. 32° 34′ E.	∴ distance=347′·6.

To find the latitude and longitude in, having given the latitude and longitude from, the course, and distance. By fig. p. 42.

151. Required the latitude and longitude in, having sailed from A, in lat. 52° 10′ N., long. 17° 32′ W. (see fig.), N. 29° 10′ E., 373·4 miles. (See Ex. 88.)

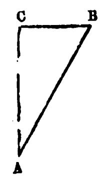
In triangle CAB, CA=AB cos. A, or T. D. lat.=dist. × cos. course; from which T. D. lat. may be found, and therefore M. D. lat. and lat. in.

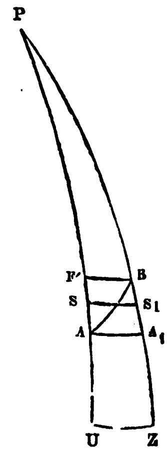
In triangle AMN, MN=AM tan. A, or diff. long.=M. D. lat. × tan. course; from which diff. long. is found, and therefore long. in.

To find T. D. lat.	To find diff. long.
By fig., T. D. lat. =dist. × cos. course.	By fig., diff. long.=M. D. lat. × tan. course.
log. dist............2·572174	log. M. D. lat..........2·754195
„ cos. course...9·941117	„ tan. course.........9·746726
„ T. D. lat......2·513291	„ diff. long. ....2·500921
∴ T. D. lat.=326	„ diff. long........316·9
or 5° 26′ N.	5° 17′ E.
lat. from ......52 10 N....3681·5	long. from...17 32 W.
„ in ....57 36 N....4249·3	„ in......12 15 W.
M. D. lat.= 567·8	

A great variety of useful examples may be worked out in middle latitude sailing and parallel sailing by means of the two figures in p. 154, *Nav.* Part II.: thus

152. Required the course and distance from A to B by middle latitude sailing: lat. A=58° 24′ N., lat B=63° 17′ N., long. A=4° 12′ W., long. B 2° 13′ E. (Ex. 82).





In the figures let A and B be the two places; then UZ=diff. long., AC=T. D. lat., SU the middle lat. ∴ SS_1=departure (nearly)=CB. (See *Nav.* Part II. p. 150.)

1. $\frac{SS_1}{UZ}$=cos. SU, or dep.=diff. long. × cos. mid. lat.

2. $\frac{CB}{AC}$=tan. A, or dep.=T. D. lat. × tan. course.

Equating, we have

T. D. lat. tan. course=diff. long. × cos. mid. lat.

∴ tan. course$=\frac{\text{diff. long.} \times \text{cos. mid. lat.}}{\text{T. D. lat.}}$, which determines the course;

and $\frac{AB}{BC}$=sec. A, or distance=T. D. lat. × sec. course;

whence distance is known.

lat. A......58° 24′ N.	58° 24′ N.	long. A......4° 12′ W.
„ B......63 17 N.	63 17 N.	„ B......2 13 E.
4 53	2)121 41	6 25
60	mid. lat....60 50 30″	60
T. D. lat. 293 N.		diff. long. 385 E.

To find course. To find distance.

By above formula,

tan. course$=\frac{\text{diff. long.} \times \text{cos. mid. lat.}}{\text{T. D. lat.}}$ dist.=T. D. lat. × sec. course.

log. cos. mid. lat.... 9·687842	log. sec. course...0·074616
„ diff. long........ 2·585461	„ T. D. lat.....2·466868
12·273303	„ distance......2·541484
„ T. D. lat....... 2·466868	∴ dist.=348′.
„ tan. course..... 9·806435 ∴ course=N. 32° 38′ E.	

NOTE. SS_1, used above as the departure, is the departure *nearly:* this method, therefore, is only approximately correct, but it will be found near enough for all practical purposes.

153. Required the latitude and longitude in, having sailed from A in lat. 52° 10′ N. and long. 17° 32′ W., N. 29° 10′, E. 373·4 miles (Ex. 88), by middle latitude method (see last figures).

1. $\frac{AC}{AB}$=cos. A ∴ T. D. lat.=dist. × cos. course,

which determines diff. lat. and ∴ the latitude in.

2. $\frac{CB}{AB}$=sin. A ∴ departure=dist. × sin. course.

3. $\frac{UZ}{SS_1}$=sec. SU ∴ diff. long.=dep. × sec. mid. lat.

=dist. × sin. course × sec. mid. lat.

To find T. D. lat.

T. D. lat.=dist. × cos. course.

log. dist. ....2·572174
„ cos. course...9·941117
2·513291

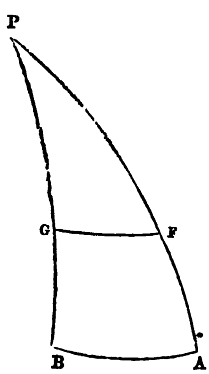
log. T. D. lat.=326
5°26′N.
lat. from ...52 10 N.
„ in.......57 36 N.
2)109 46
mid. lat.....54 53

To find diff. long.

diff. long.=dist. × sin. course × sec. mid. lat.

log. distance.......2·572174
„ sin. course....9·687842
„ sec. mid. lat. 0·240149
2·500165

∴ diff. long.=316′
or 5°16′E.
long. from...17 32 W.
„ in......12 16 W.



154. Find the course and distance from G to F: lat. G=lat. F=50° 48′ N., long. G=5° W., long. F=5° E. (Ex. 102). This is an example in parallel sailing.

long. G... 5°W.
„ F... 5 E.
10=600′=diff. long.

GB=50° 48′, and GF is the arc of the parallel described by ship. Then $\frac{GF}{AB}$=cos. GB, or dist.=diff. long. × cos. lat.

log. diff. long....2·778151
„ cos. lat......9·800737
„ dist.........2·578888

The course is evidently *East*.

∴ distance=379·2.

155. Required the latitude and longitude in, having sailed due east 560 miles from a place G (see fig.) in lat. 57° 32′ N. and long. 13° 5′ W. (Ex. 110).

lat. GB=57° 32′, dist. GF=560	log. dist.......2·748188
and AB=diff. long. required	„ sec. lat...0·270180
By fig., $\frac{AB}{GF}$=sec. GB	„ diff. long.3·018368
	∴ diff. long=1043
∴ diff. long.=dist. sec. course	or 17°23′E.
	long. from ....13 5 W.
	∴ „ in......... 4 18 E.

The examples from 110 to 145 may be worked in a similar manner (by making a figure to suit each case) as Examples 146 to 155.

EXAMPLES.

By Construction and Traverse Sailing.

A. A ship sails from A, in latitude 24° 32′ N., on the following courses and distances: required latitude in and direct course and distance. (1.) S.W.b.W. 45′; (2.) E.S.E. 50′; (3.) S.W. 30′; (4.) S.E.b.E. 60′; (5.) S.W.b.S.¼W. 63′. *Ans.* Lat. 22° 3′, south, 149·2′.

B. A ship sails from A, in lat. 28° 32′ N., on the following courses and distances: required lat. in and direct course and distance. (1.) N.W.b.N. 20′; (2.) S.W. 40′; (3.) N.E.b.E. 60′; (4.) S.E. 55′; (5.) W.b.S. 41′; (6.) E.N.E. 66′. *Ans.* Lat. 28° 32′ N., east, 70·2′.

C. Since yesterday at noon we have run the following courses: required diff. lat. and departure, and direct course and distance. (1.) S.W.b.S. 20′; (2.) W. 16′; (3.) N.W.b.W. 28′; (4.) S.S.E. 32′; (5.) E.N.E. 14′; (6.) S.W. 36′. *Ans.* Diff. lat. 50·7′, dep. 50·7′, S.W., 71·7′.

EXAMPLES.

By Construction and Trigonometry.

A′. Two ships, A and B, sail from two islands bearing the one from the other N.E. and S.W., their distance being 76′. A sails S.b.E., and B E.b.S.: at last they meet. How far has each sailed?

Ans. A sails S.b.E. 68·4′, B sails E.b.S. 68·4′.

B′. Coasting along shore, a headland bore N.E.b.N.; then, having run E.b.N. 15′, the headland bore W.N.W.: required the distance from headland at each observation. *Ans.* 8·5′ and 10·8′.

C′. Yesterday noon we were in lat. 33° 15′ N., and bound to a port in latitude 28° 35′ N., lying 196′ to the west; and this day at noon we were in lat. 30° 20′ N., having made departure 168′ west: required the direct course and distance to the port. *Ans.* S. 14° 55′ W., dist. 108·8′.

THE DAY'S WORK.

(25.) *To find the place of the ship at noon*, that is, its latitude and longitude, having given the latitude and longitude at the preceding noon, the compass courses, and distances run in the interval, the deviation of the compass for each course on account of local attraction, the variation of the compass, the leeway, the velocity and direction of current (if any), &c., constitutes what is called the Day's Work.

The Day's Work.

Rule 7. (1.) Correct each course for variation, deviation, and leeway; thus get the true courses, and arrange the same in a tabular form, as in the example, p. 28. Add together the hourly distances sailed on each course, and insert the same in table opposite the true course.

(2.) Take out of the traverse table the true difference latitude and departure for each course and distance, putting them down in the columns headed with the same letters as in course. Previously to opening the traverse table, fill up the columns of true difference latitude and departure not wanted by drawing horizontal lines; this will frequently prevent mistakes.

(3.) If the ship does not sail from a place whose latitude and longitude are known, her bearing and distance from some near object, as a church-spire, &c., must be ascertained, and also its latitude and longitude. Then the ship is supposed to sail from this known object to her anchorage, her course being the opposite to the bearing of the object from the ship. This course must be corrected like the rest for variation and deviation, and inserted in the table as an actual course, with the distance of the object as a distance.

(4.) If a current sets the ship in any ascertained direction, and with a known velocity, these also may be conceived to be an independent course and distance, and must be corrected for variation, and should be for deviation also, if the latter correction is appreciable, which is rarely the case.

(5.) *To find the latitude in.* The quantities in the four columns of true difference latitude and departure being added up separately, the difference between the north difference of latitude and south difference of latitude, with the name of the greater, will give the true difference of latitude made at the end of the day. The departure is found in a similar manner. Apply true difference latitude to latitude from, so as to obtain the latitude in.

(6.) *To find the longitude in.* Add together log. sec. mid. lat. and log. departure, the result (rejecting 10 in the index) is the log. difference longitude. Find this in table, and thus the longitude in is found.*

The following example, worked out in detail, will perhaps be sufficient to explain the operations directed in the above general rule.

EXAMPLE.

156. April 27th, 1852, at noon. A point of land in latitude 36° 30′ S. and longitude 110° 20′ W. bore by compass E.b.N.½N. (ship's head being S.E. by S. by compass), distant 14 miles; afterwards sailed as by the following log account; required the latitude and longitude in, on April 28th, at noon.

Hours.	Knots.	Courses.	Winds.	Lee-way.	Devi-ation.	Remarks.
1	2·5	S.W.½W.	S.b.E.	2¼	½ l.	P.M.
2	3·4					
3	2·3					
4	3·2	W.b.S.½S.	S.b.W.	2½	¾ l.	
5	4·4					
6	2·3					Variation of compass 1¾E.
7	2·3	W.b.N.¾N.	S.W.	2	¾ l.	
8	3·3					
9	4·0					
10	5·4					
11	4·2					
12	4·4					
1	3·3	N.W.½W.	W.b.S.¾S.	2½	½ l.	A.M.
2	3·3					
3	3·5					
4	4·2					A current set the ship the last 8 hours, by compass, E.½S., 2 miles an hour.
5	6·3	W.b.S.	S.½W.	1½	¾ l.	
6	3·7					
7	2·5					
8	5·0					
9	5·2	S.W.	S.b.E.	2½	½ l.	
10	3·4					
11	6·3					
12	5·4					

(1.) The column in the above table headed deviation should be formed from the general table of deviations (p. 32) previously to correcting courses.

* Or thus: To find diff. long., add together log. M. D. lat. and log. dep., and from the sum subtract log. T. D. lat.; the remainder is the log. diff. long., which find in the tables.

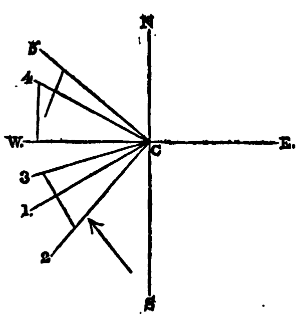
Thus, in the first course in the preceding table, the ship's head is S.W.½W.; looking in the deviation table, we see that the corresponding correction is ½ W. or ½ l, and so for the others.

(2.) Form a table such as below, by writing down the headings, points, courses, &c., over the seven columns which are to be filled in with the corrected courses, &c.

Points.	Courses.	Distance.	Diff. lat.		Departure.	
			N.	S.	E.	W.

(3.) *To correct the courses.*

The courses are more readily corrected by drawing two lines at right angles, to represent the N., S., E., and W. points of the compass, and then a line to represent (roughly) the compass course of the ship. The direction in which the correction for leeway must be applied will then be easily seen.



After some experience in correcting courses, they can be made mentally, and the diagram dispensed with.

To correct the departure course which is W.b.S.½S. (the opposite bearing to E.b.N.½N.).

Draw a line roughly in the fig. W.b.S.½S. as c 1; it is then seen that

	pts.	qrs.	
Compass course	6	2 r. S.	
variation	1	3 r.	
ship's head S.E.b.S. ∴ dev.	0	0	(See table, p. 32.)
	1	3 r.	
true course	8	1 r. S.	

or 7 pts. 3 qrs. left of N., or W.¼N. dist. 14′.

Insert this course and distance in table below.

Points.	Courses.	Dist.	Diff. lat. N.	Diff. lat. S.	Departure. E.	Departure. W.
$7\frac{3}{4}$	W.$\frac{1}{4}$N.	14·0	0·7	...	...	14·0
8	W.	8·2	...	...	...	8·2
6	W.N.W.	9·9	3·8	...	...	9·2
$3\frac{1}{4}$	N.W.$\frac{3}{4}$N.	23·6	19·0	...	...	14·1
$\frac{3}{4}$	N.$\frac{3}{4}$W.	14·3	14·1	...	...	2·1
$6\frac{1}{2}$	W.b.N.$\frac{1}{2}$N.	17·5	5·1	...	...	16·7
$7\frac{3}{4}$	W.$\frac{1}{4}$S.	20·3	...	1·0	...	20·3
$5\frac{3}{4}$	S.E.b.E.$\frac{3}{4}$E.	16·0	...	6·8	14·5	...
			42·7	7·8	14·5	84·6
			7·8			14·5
		T. D. lat. 34·9 N.				Dep. 70·1 W.

First Course.—S.W.$\frac{1}{2}$W.

Draw a line in fig. S.W.$\frac{1}{2}$W. as c 2; then

	pts.	qrs.	
Compass course...........	4	2 r.	S.
variation.........1 3 r.			
deviation.........0 2 l.			
	1	1 r.	
	5	3 r.	S.
leeway (wind S.b.E.).....	2	1 r.	
true course................	8	0 r.	S. or due W. 8·2′.

The distance 8·2′ is found by adding up the hourly distances until the course is altered, at 4 o'clock. Insert this course and distance in the table.

Second Course.—W.b.S.$\frac{1}{2}$S.

Draw a line in fig. W.b.S.$\frac{1}{2}$S. as c 1.

	pts.	qrs.	
Compass course...........	6	2 r.	S.
variation..........1 3 r.			
deviation.........0 3 l.			
	1	0 r.	
	7	2 r.	S.
leeway....................	2	2 r.	
true course..............	10	0 r.	S.
or	6	0 l.	N.
	=W.N.W.9·9′		

Insert this course and distance in the table.

Third Course.—W.b.N.$\frac{3}{4}$N.

Draw a line W.b.N.$\frac{3}{4}$N. as c 4.

		pts.	qrs.
Compass course		6	1 l. N.
variation	1 3 r.		
deviation	0 3 l.		
		1	0 r.
		5	1 l. N.
leeway		2	0 r.
true course		3	1 l. N.
	or N.W.$\frac{3}{4}$N. 23·6′.		

Insert course and distance in table.

Fourth Course.—N.W.$\frac{1}{2}$W.

Draw a line N.W.$\frac{1}{2}$W. as c 5.

		pts.	qrs.
Compass course		4	2 l. N.
variation	1 3 r.		
deviation	0 2 l.		
		1	1 r.
		3	1 l. N.
leeway		2	2 r.
true course		0	3 l. N.
	or N.$\frac{3}{4}$W. 14·3′.		

Insert course and distance in table.

Proceed with the 5th and 6th courses in the same manner, thus :

Fifth Course.

	pts.	qrs.
W.b.S.	7	0 r. S. as c 3.
1 3 r.		
0 3 l.		
	1	0 r.
	8	0 r. S.
	1	2 r.
	9	2 r. S.
or	6	2 l. N.
=W.b.N.$\frac{1}{2}$N. 17·5′.		

Sixth Course.

	pts.	qrs.
S.W.	4	0 r. S.
1 3 r.		
0 2 l.		
	1	1 r.
	5	1 r. S.
	2	2 r.
	7	3 r. S.
or W.$\frac{1}{4}$S. 20·3′.		

	pts.	qrs.
Current course E.$\frac{1}{2}$S.	7	2 l. S.
variation	1	3 r.
true course	5	3 l. S.
or S.E.b.E.$\frac{3}{4}$E. 16·0′.		

Previously to opening the traverse table to take out the difference latitude and departure corresponding to each course and distance in the above table, fill the columns not wanted : thus in the first course W.$\frac{1}{4}$N. the N. and W. columns will be wanted; fill up the S. and E. columns by drawing a line under S. and E. In the second course W., the three columns N., S., and E., will not be wanted; fill them up with lines. In the same manner proceed with the other courses.

(4.) *To find difference latitude and departure* for each course and distance, by traverse table.

Enter traverse table, and take out the difference latitude and departure corresponding to $7\frac{3}{4}$ points, and distance 14·0. (Look out rather $7\frac{3}{4}$ points and 140 distance, the diff. lat. and dep. for which are 6·9 and 139·8; move the decimal points one place to the left,) and put down the result to the nearest tenth, which are ·7 and 14·0. Insert them in the spaces left unmarked under N. and W.

The second course being due W. 8·2′, the departure will be 8·2 (the same as the distance).

With third course 6 points and distance 9·9 (looking for 99, and making the proper change in decimal points), the diff. lat. is 3·8′ and dep. 9·2′.

In a similar manner find difference latitude and departure for the other courses.

When the four columns are added up, it appears that the ship has sailed N. 42·7′ and S. 7·8′; therefore upon the whole the true difference latitude is 34·9′ N.; and her departure has been 14·5′ E. and 84·6′ W.; hence the departure made good in the 24 hours is 70·1 W.

(5.) *To find the latitude in*, apply the true difference latitude to the latitude from, in the usual manner, to obtain the latitude in.

(6.) *To find the longitude in.** With the latitude from and latitude in, find middle latitude. Add together log. secant mid. lat. and log. departure; the result (rejecting 10 in index) is the log. difference longitude, which, found in the tables, and applied to the longitude from, gives the longitude in. Thus:

To find latitude in.

T. D. lat....	0°	34′	54″	N.
lat. from....	36	30	0	S.
„ in.......	35	55	6	S.
2)	72	25	6	
mid. lat.....	36	12	33	

To find longitude in.

log. sec. mid. lat.	0·093148
„ departure......	1·845718
„ diff. long......	1·938866
∴ diff. long.	87′
	or 1° 27 W.
long. from...	110 20 W.
„ in......	111 47 W.

---

* Or thus: To find long. in (by inspection).

Since $\frac{\text{dep.}}{\text{dist.}}$=sin. course

and $\frac{\text{dep.}}{\text{diff. long.}}$=cos. mid. lat.=sin. complement mid. lat.

If, therefore, the traverse table is entered with complement of mid. lat. as a course, and with the given departure, the distance corresponding thereto will be the difference of longitude nearly.

NAUTICAL ASTRONOMY.

CHAPTER II.

ASTRONOMICAL AND NAUTICAL TERMS AND DEFINITIONS.

(26.) NAUTICAL Astronomy teaches the method of finding the *place* of a ship, that is, its latitude and longitude, by means of astronomical observations.

(27.) The following are the principal terms in Nautical Astronomy; they are fully explained in *Navigation*, Part II., to which the student is referred: they are inserted in this place for the sake of reference. The definitions of these terms should be thoroughly understood and carefully committed to memory.

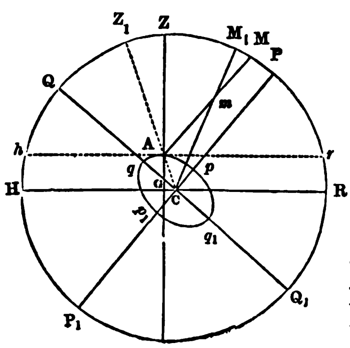
True place of a heavenly body.
Apparent place of a heavenly body.
Axis of the earth.
Terrestrial equator.
Poles of the earth.
Axis of the heavens.
Celestial equator.
Poles of the heavens.
The ecliptic.
Obliquity of the ecliptic.
True latitude of spectator.
Reduced or central latitude of spectator.
Meridians of the earth.

True zenith.
Reduced zenith.
Visible horizon.
Rational horizon.
Poles of the horizon.
Vertical circles, or circles of altitude.
Celestial meridian.
North and south points.
Prime vertical.
East and west points.
Circles of declination.
Circles of latitude.
Right ascension of a heavenly body.
Declination of a heavenly body.
Longitude of a heavenly body.
Latitude of a heavenly body.
Altitude of a heavenly body.
Azimuth or true bearing of a heavenly body.
Amplitude of a heavenly body.
Hour-angle of a heavenly body.
Solar year.
Sidereal year.
Mean solar year.
Sidereal day.
Apparent solar day.
Mean sun.
Mean solar day.
Sidereal time.
Apparent solar time.
Mean solar time.
Equation of time.
Sidereal clock.
Mean solar clock or chronometer.

Definitions of the preceding Terms in Nautical Astronomy.

(28.) To a spectator on the earth, the sun, moon, and stars seem to be placed on the interior surface of a hollow sphere of great but indefinite magnitude. The interior surface of this sphere is called the *celestial concave*, the center of which may be supposed to be the same as that of the earth.

(29.) The heavenly bodies are not in reality thus situated with respect to the spectator; for they are interspersed in infinite space at very different distances from him; the whole is an optical deception, by which an observer, wherever he is placed, is induced to imagine himself to be the center of the universe. For let us suppose the elliptical figure $p\ q\ p_1\ q_1$ to represent the earth, P Q P_1 Q_1 the celestial concave, and m a heavenly body. Then a spectator at A, not being able to estimate the distance of m, would imagine it to be in the celestial concave at M.



This figure will enable us to explain the terms *true* and *apparent place* of a heavenly body. The body m viewed from the surface of the earth would appear to a spectator A to be at M in the celestial concave: but if it could be seen from the center of the earth C, the point occupied by m would be M_1, the extremity of a line drawn from the center C of the earth through the heavenly body to the celestial concave. M is called the *apparent* place, and M_1 the *true* place of the heavenly body m.

(30.) The *axis of the earth* is that diameter about which it revolves: the *poles* of the earth are the extremities of the axis.

(31.) The *terrestrial equator* is that great circle on the earth that is equidistant from each pole.

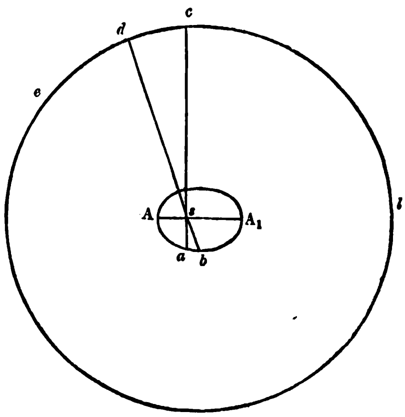
(32.) A spectator on the earth, not being sensible of the motion by which in fact he describes daily a circle from west to east with the spot on which he stands, views in appearance the heavens moving past him in the opposite direction, or from east to west. The sphere of the fixed stars, or, as it is more usually called, the *celestial concave*, thus appears to revolve from east to west round an imaginary line which is the axis of the earth produced to the celestial concave: this line is therefore called the *axis of the heavens.*

(33.) The *poles of the heavens* are the extremities of the axis of the heavens.

(34.) The *celestial equator* is that great circle in the celestial concave

which is perpendicular to the axis of the heavens; or it may be defined to be the terrestrial equator expanded or extended to the celestial concave. The poles of the celestial equator and the poles of the heavens are therefore identical.

(35.) While the earth thus performs its daily revolution, it is carried with great velocity from west to east round the sun, and describes an elliptic orbit once every year. This *annual* motion of the earth round the sun causes the latter body, to a spectator on the earth, insensible of his own change of place, to appear to describe a great circle in the celestial concave from west to east. This may be explained by a figure. Let $A b A_1$ be the earth's orbit, s the sun, and $c d e l$ the celestial concave; then, to a spectator at a the sun is seen at a point c in the celestial concave: but when the earth has arrived at b the spectator (not being sensible of his motion from a to b) imagines the sun to be at d, and thus it would seem to have described the arc $c\ d$ in the time the earth actually moved from a to b. It appears from this, that when the earth has arrived again at a, the sun will again be at c, having described one complete circle in the celestial concave among the fixed stars. The great circle thus described by the sun is called the *ecliptic.*



(36.) The axis of the earth as it is thus carried round the sun, continues always parallel to itself, and it may be assumed without any sensible error, on account of the smallness of the earth's orbit (small, when compared with the distance of the heavenly bodies), to be always directed to the same points in the celestial concave, namely, the *poles* of the heavens.

(37.) From observation, the celestial equator is found to be inclined to the ecliptic at an angle of about 23° 28′. This inclination of the equator to the ecliptic is called the *obliquity of the ecliptic.* The axis of the earth, therefore, which is perpendicular to the equator, is inclined to the ecliptic, or, as it is in the same plane, to the earth's orbit, at an angle of 66° 32′.

(38.) In consequence of the whirling motion of the earth about its axis, the parts near the equator, which have the greatest velocity, acquire thereby a greater distance from the center than the parts near the poles. By actual measurement of a degree of latitude in different parts of the earth, it has been computed that the equatorial diameter is longer than the axis or polar diameter by 26 miles: the former being about 7924 miles; the latter about 7898 miles, and that the form of the earth is that of an *oblate spheroid.* It is usual, however, in drawing the figure of the earth to exaggerate very

much its ellipticity; this is done for the sake of drawing the lines about the figure with greater clearness; for if it were constructed according to its true dimensions, the line pp_1 (fig. art. 29) (being only about the $\frac{1}{300}$th part of itself less than qq_1), would appear to the eye of the same length as qq_1, and we should see that the figure that more nearly resembles the earth would be a sphere.

(39.) If AG, a perpendicular to the earth's surface, be drawn passing through A, the angle A G q formed by the line A G with the plane of the equator is the *latitude*, or true latitude of the point A.

(40.) If A C be a line drawn from A to C, the center of the earth, then the angle A C q is called the reduced or central latitude of the point A. The difference between the true and reduced latitude is not great: it is, however, of importance in some of the problems in Nautical Astronomy. This correction has accordingly been calculated, and forms one of the Nautical Tables.

(41.) Sections of the earth passing through the poles, as p A p_1, are called *meridians* of the earth. If the earth is considered as a sphere (which it is very nearly), the meridians will be circles: on this supposition, moreover, the perpendicular A G would coincide with A C, and the latitude of a place on the surface of the earth may, on this supposition, be defined to be the arc of the meridian passing through the place, intercepted between the place and the equator. If G A be produced to meet the celestial concave at Z, the point Z is the zenith of the spectator at A. If C A be produced to the celestial concave at Z′, then Z′ is called the *reduced* zenith of the spectator at A. The point opposite to Z in the celestial concave is called the *Nadir*. In the figure the terrestrial equator qq_1 is extended to the celestial concave, and therefore Q c Q$_1$ is the plane of the *celestial* equator.

By means of this figure we may define the zenith, reduced zenith, latitude, and reduced latitude, as follows:

(42.) The *zenith* is that point in the celestial concave which is the extremity of the line drawn perpendicular to the place of the spectator, as Z.

(43.) The *reduced* zenith is that point in the celestial concave which is the extremity of a straight line drawn from the center of the earth, through the place of the spectator, as Z′.

(44.) The *latitude* of a place A on the surface of the earth, is the inclination of the perpendicular A G to the plane of the equator: thus the angle A G Q is the latitude of A. The arc Z Q in the celestial concave measures the angle A G Q; hence Z Q, *or the distance of the zenith from the celestial equator, is equal to the latitude of the spectator.*

(45.) The *reduced latitude* of the place A is the inclination of Z′C or A C to the plane of the equator: or it is the angle A C Q or arc Z′Q, which measures the angle. Since the curvature of the earth diminishes from the equator to the poles, the reduced latitude Z′Q must be always less than the true lati-

tude ZQ, and therefore the difference ZZ′ must be subtracted from the true latitude to get the reduced latitude.

The formula for computing the difference between the true and reduced latitude of any place is investigated in *Navigation*, Part II.

(46.) The *visible horizon* is that circle in the celestial concave which touches the earth where the spectator stands, as hAr; and a circle parallel to the *visible* horizon, and passing through the center of the earth, is called the rational horizon: thus HCR is the rational horizon. These two circles, however, form one and the same great circle in the celestial concave: thus R and r in the figure must be supposed to coincide. This may be readily conceived, when we consider that the distance of any two points on the surface of the earth will make no sensible angle at the celestial concave; therefore either of these two circles is to be understood by the word horizon. The *poles* of the horizon of any place are manifestly the zenith and nadir.

(47.) Great circles passing through the zenith are called *circles of altitude* or *vertical* circles. That circle of altitude which passes through the poles of the heavens is called the *celestial meridian*. The points of the horizon through which the celestial meridian passes are called the *north* and *south* points. A circle of altitude at right angles to the meridian is called the *prime vertical*. This last circle cuts the horizon in two points called the *east* and *west* points. The east and west points are manifestly the poles of the celestial meridian.

(48.) Since the horizon and celestial equator are both perpendicular to the celestial meridian, the points where the horizon and celestial equator intersect each other, must be 90° distant from every part of the meridian (Jeans' *Trig.* P. II. art. 65); that is, the celestial equator must cut the horizon in the east and west points.

(49.) The ecliptic (art. 35) is divided into twelve parts, called signs, which receive their names from constellations lying near them. These divisions or signs are supposed to begin at that intersection of the celestial equator and ecliptic which is called the *first point* of Aries.

(50.) Great circles passing through the poles of the heavens are called *circles of declination;* and great circles passing through the poles of the ecliptic are called *circles of latitude.*

(51.) *Parallels of declination* and of *latitude* are small circles parallel respectively to the celestial equator and ecliptic.

(52.) The *declination* of a heavenly body is the arc of a circle of declination passing through its place in the celestial concave, intercepted between that place and the celestial equator.

(53.) The *right ascension* of a heavenly body is the arc of the equator, intercepted between the first point of Aries and the circle of declination passing through the place of the heavenly body in the celestial concave, measuring from the first point of Aries, eastward, from 0° to 360°.

(54.) The *latitude* of a heavenly body is the arc of a circle of latitude

passing through its place in the celestial concave, intercepted between that place and the ecliptic.

(55.) The *longitude* of a heavenly body is the arc of the ecliptic intercepted between the first point of Aries and the circle of latitude passing through the place of the heavenly body in the celestial concave, measuring from the first point of Aries, eastward, from 0° to 360°.

(56.) The *true altitude* of a heavenly body is the arc of a circle of altitude passing through the true place of the body intercepted between the place and the horizon.

(57.) The *azimuth*, or bearing of a heavenly body, is the arc of the horizon intercepted between the north or south points and the circle of altitude passing through the place of the body; or it is the corresponding angles at the zenith between the celestial meridian and the circle of altitude passing through the body.

(58.) The *amplitude* of a heavenly body is the distance from the east point at which it rises, or the distance from the west point at which it sets, the arcs or distances being measured on the horizon.

(59.) The *hour angle* of a heavenly body is the angle at the pole between the celestial meridian and the circle of declination passing through the place of the body.

(60.) A *solar year* is the interval between the sun's leaving the first point of Aries and returning to it again.

(61.) A *sidereal year* is the interval between the sun's leaving a fixed point, as a star, and returning to that point again.

(62.) The length of the solar years is found to differ a little from each other, on account of certain irregularities in the sun's apparent motion, and that of the first point of Aries. The *mean length* of several solar years is therefore the one made use of in the common division of time, and called the *mean solar year*.

(63.) The *sidereal day* is the interval between two successive transits of the first point of Aries over the same meridian. It begins when the first point of Aries is on the meridian.

(64.) The *apparent solar day* is the interval between two successive transits of the sun's center over the same meridian. It begins when that point is on the meridian.

(65.) The length of an apparent solar day is variable chiefly from two causes:

1st. From the variable motion of the sun in the ecliptic.

2d. From the motion of the sun being in a circle inclined to the equator.

(66.) To obtain, therefore, a proper measure of time, we proceed as follows. An imaginary, or as it is called a *mean sun*, is supposed to *move uniformly in the celestial equator with the mean velocity of the true sun.* A *mean solar day* may therefore be defined to be the interval between two

successive transits of the mean sun over the same meridian. It begins when the mean sun is on the meridian.

(67.) *Sidereal* time is the angle at the pole of the heavens between the celestial meridian and a circle of declination passing through the first point of Aries, measuring from the meridian westward.

(68.) *Mean solar time* is the angle at the pole between the celestial meridian and a circle of declination passing through the mean sun, measuring from the meridian westward.

(69.) *Apparent solar time* is the angle at the pole between the celestial meridian and a circle of declination passing through the place of the sun's center, measuring from the meridian westward.

(70.) *The equation of time* is the difference in time between the places of the true and mean sun.

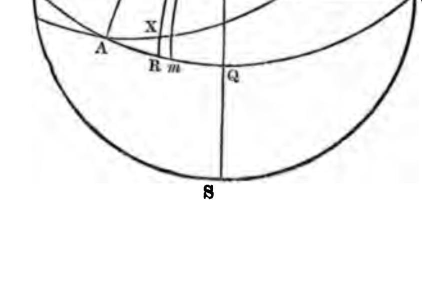
(71.) A *sidereal clock* is a clock adjusted so as to go 24 hours during one complete revolution of the earth; that is, during the interval of two successive transits of a fixed star: or supposing the first point of Aries to be invariable, between two successive transits of the first point of Aries.

(72.) A *mean solar clock* is a clock adjusted to go 24 hours during one complete revolution of the mean sun.

These definitions are more fully explained and illustrated in *Navigation*, Part II. pp. 8 to 11, by means of constructions or diagrams similar to the following one, which is the diagram for definitions 67, 68, 69, and 70. The student should endeavour to explain each of the other definitions by a similar construction.

Construct a figure, and show what is meant by sidereal time, apparent solar time, mean solar time, and the equation of time.

Let NWSE represent the horizon, P the pole, AE the equator, A the first point of Aries, and AC the ecliptic. Let X be the place of the sun in the ecliptic, and *m* the mean sun; through X and *m* draw the circles of declination PR and P*m*. Then *sidereal time* is the angle QPA, or arc QA; *apparent solar time* is the angle QPR, or arc QR; and *mean solar time* is the angle QP*m*, or arc Q*m*,—these angles or arcs being always measured from the meridian NZS westward. Also the angle *m*PR, or arc *m*R, is the *equation of time*.



INTRODUCTORY RULES IN NAUTICAL ASTRONOMY.

Nautical day and Astronomical day.

(73.) The nautical or civil day begins at midnight and ends the next midnight. The astronomical day begins at noon and ends at noon, and is later than the civil day by 12 hours. Again, in the astronomical day the hours are reckoned throughout from 0^h to 24^h; in the nautical day there are twice 12 hours, the first 12 hours being before noon, or before the commencement of the astronomical day (denoted by A.M., *ante meridiem*); the latter are afternoon, and distinguished by the letters P.M. (*post meridiem.*)

Rule 1. *Given civil or nautical time, to reduce it to astronomical time.*

1. If the given nautical time be P.M., it will be also astronomical time; P.M. being omitted.

2. If the given nautical time be A.M., add 12^h thereto, and put the day one back, omitting the letters A.M.; thus:

(1.) April 27, at $4^h\ 10^m$ P.M. (civil) is April 27, at $4^h\ 10^m$ (astro.).
(2.) April 27, at 4 10 A.M. (civil) is April 26, at 16 10 (astro.).

EXAMPLES.

Reduce the following civil or nautical times to astronomical times.

Civil times.	Astronomical times.
157. Sept. 10th, $4^h\ 10^m$ P.M.	*Ans.* Sept. 10th, $4^h\ 10^m$
158. June 3 2 42 A.M.	,, June 2 14 42
159. July 1 6 18 A.M.	,, June 30 18 18
160. Dec. 10 3 42 P.M.	,, Dec. 10 3 42

Rule 2. *Given astronomical time, to reduce it to civil or nautical time.*

1. If the astronomical time is less than 12 hours, it will also be nautical time P.M.

2. If the astronomical time be greater than 12 hours, reject 12 and put the day one forward; the result will be civil time A.M.; thus:

(1.) April 27, at $4^h\ 10^m$ (astro.) is April 27, at $4^h\ 10^m$ P.M. (civil).
(2.) April 27, at 16 10 (astro.) is April 28, at 4 10 A.M. (civil).

EXAMPLES.

Reduce the following astronomical times to nautical or civil times.

Astronomical times.				Civil times.					
161. Sept.	10th,	4^h	32^m	*Ans.*	Sept. 10th,	4^h	32^m	P.M.
162. July	5	16	32	,,	July 6	4	32	A.M.
163. July	10	18	42	,,	July 11	6	42	A.M.
164. Dec.	21	23	59	,,	Dec. 22	11	59	A.M.

Rule 3. *To reduce degrees into time.*

1. Divide the degrees by 15, the quotient is hours.

2. Multiply the remaining degrees, if any, by 4; the result is minutes in time.

3. Divide the minutes in arc by 15; the quotient is minutes in time.

4. Multiply the remaining minutes of arc, if any, by 4; the result is seconds of time.

5. Divide the seconds in arc by 15; the quotient is seconds in time, carried to decimals if necessary. The sum will be the arc in time.

EXAMPLE.

165. Reduce 34° 44′ 34″ into time.

$$
\begin{aligned}
34^\circ &= 2^h\ 16^m\ 0^s \\
44' &= \quad\ 2\ \ 56 \\
34'' &= \qquad\ \ 2{\cdot}26 \\
\therefore\ 34^\circ\ 44'\ 34'' &= 2\ \ 18\ \ 58{\cdot}26
\end{aligned}
$$

EXAMPLES.

Reduce the following arcs into time.

166.	84°	42′	30″	*Ans.*	5^h	38^m	50^s
167.	96	10	45	,,	6	24	43
168.	108	24	22	,,	7	13	37·4
169.	178	48	45	,,	11	55	15
170.	140	32	10	,,	9	22	8·66
171.	240	32	10	,,	16	2	8·66

Rule 4. *To reduce time into degrees.*

1. Multiply the hours by 15; the result is degrees.

2. Divide the minutes in time by 4; the quotient is degrees.

3. Multiply the minutes remaining, if any, by 15; the result is minutes of arc.

4. Divide the seconds of time by 4; the quotient is minutes of arc.

5. Multiply the seconds (and parts of seconds) remaining, if any, by 15 the result is seconds of arc.

The sum will be the arc in degrees.

EXAMPLE.

172. Reduce $2^h\ 18^m\ 58^s{\cdot}26$ into degrees.

$$
\begin{array}{lcrrr}
2^h & = & 30^\circ & 0' & 0'' \\
18^m & = & 4 & 30 & 0 \\
58^s{\cdot}26 & = & & 14 & 33{\cdot}9 \\
\hline
\therefore\ 2^h\ 18^m\ 58^s{\cdot}26 & = & 34 & 44 & 33{\cdot}9
\end{array}
$$

EXAMPLES.

Find the arcs corresponding to the following times,

173.	3^h	52^m	4^s	*Ans.*	58°	$1'$	$0''$
174.	17	8	22	„	257	5	30
175.	8	17	15·5	„	124	18	52·5
176.	12	14	16·75	„	183	34	11·25
177.	9	13	8	„	138	17	0
178.	15	17	18·4	„	229	19	36

By means of the following Table* arcs to the nearest minute are more readily expressed in time and the converse, than by the preceding rules.

TABLE

To reduce degrees into time, and the converse.

$1'= 0^m\ 4^s$	$21'= 1^m\ 24^s$	$41'= 2^m\ 44^s$	$1^\circ=0^h\ 4^m$	$10^\circ= 0^h\ 40^m$
$2'= 0\ \ 8$	$22'= 1\ \ 28$	$42'= 2\ \ 48$	$2^\circ=0\ \ 8$	$20^\circ= 1\ \ 20$
$3'= 0\ \ 12$	$23'= 1\ \ 32$	$43'= 2\ \ 52$	$3^\circ=0\ \ 12$	$30^\circ= 2\ \ 0$
$4'= 0\ \ 16$	$24'= 1\ \ 36$	$44'= 2\ \ 56$	$4^\circ=0\ \ 16$	$40^\circ= 2\ \ 40$
$5'= 0\ \ 20$	$25'= 1\ \ 40$	$45'= 3\ \ 0$	$5^\circ=0\ \ 20$	$50^\circ= 3\ \ 20$
$6'= 0\ \ 24$	$26'= 1\ \ 44$	$46'= 3\ \ 4$	$6^\circ=0\ \ 24$	$60^\circ= 4\ \ 0$
$7'= 0\ \ 28$	$27'= 1\ \ 48$	$47'= 3\ \ 8$	$7^\circ=0\ \ 28$	$70^\circ= 4\ \ 40$
$8'= 0\ \ 32$	$28'= 1\ \ 52$	$48'= 3\ \ 12$	$8^\circ=0\ \ 32$	$80^\circ= 5\ \ 20$
$9'= 0\ \ 36$	$29'= 1\ \ 56$	$49'= 3\ \ 16$	$9^\circ=0\ \ 36$	$90^\circ= 6\ \ 0$
$10'= 0\ \ 40$	$30'= 2\ \ 0$	$50'= 3\ \ 20$	$10^\circ=0\ \ 40$	$100^\circ= 6\ \ 40$
$11'= 0\ \ 44$	$31'= 2\ \ 4$	$51'= 3\ \ 24$	$11^\circ=0\ \ 44$	$110^\circ= 7\ \ 20$
$12'= 0\ \ 48$	$32'= 2\ \ 8$	$52'= 3\ \ 28$	$12^\circ=0\ \ 48$	$120^\circ= 8\ \ 0$
$13'= 0\ \ 52$	$33'= 2\ \ 12$	$53'= 3\ \ 32$	$13^\circ=0\ \ 52$	$130^\circ= 8\ \ 40$
$14'= 0\ \ 56$	$34'= 2\ \ 16$	$54'= 3\ \ 36$	$14^\circ=0\ \ 56$	$140^\circ= 9\ \ 20$
$15'= 1\ \ 0$	$35'= 2\ \ 20$	$55'= 3\ \ 40$	$15^\circ=1\ \ 0$	$150^\circ=10\ \ 0$
$16'= 1\ \ 4$	$36'= 2\ \ 24$	$56'= 3\ \ 44$	$16^\circ=1\ \ 4$	$160^\circ=10\ \ 40$
$17'= 1\ \ 8$	$37'= 2\ \ 28$	$57'= 3\ \ 48$	$17^\circ=1\ \ 8$	$170^\circ=11\ \ 20$
$18'= 1\ \ 12$	$38'= 2\ \ 32$	$58'= 3\ \ 52$	$18^\circ=1\ \ 12$	$180^\circ=12\ \ 0$
$19'= 1\ \ 16$	$39'= 2\ \ 36$	$59'= 3\ \ 56$	$19^\circ=1\ \ 16$	$190^\circ=12\ \ 40$
$20'= 1\ \ 20$	$40'= 2\ \ 40$	$60'= 4\ \ 0$	$20^\circ=1\ \ 20$	$200^\circ=13\ \ 20$

* The table is computed to the nearest minute of arc; when seconds are to be reduced (which is seldom required) the student must proceed as pointed out in the preceding rules.

Use of the Table.

179. Reduce $34° 44' 34''$ into time.

$34° 44' 34'' = 34° 45'$ nearly.
By Table...$30°$.......$2^h\ 0^m$
4 16
$45'$... 3
$\therefore 34° 44' 34'' = \overline{2\ 19}$ nearly

180. Reduce $2^h\ 18^m\ 58^s{\cdot}26$ into degrees.

$2^h\ 18^m\ 58^s{\cdot}26 = 2^h\ 19^m$ nearly.
By Table...2^h......$30°\ 0'$
16^m 4 0
3 45
$\therefore 2^h\ 18^m\ 58^s{\cdot}26 = \overline{34\ 45}$ nearly.

In some nautical tables, the angles in the log. sine table are given both in arc and in time. The reduction from degrees to hours, and the converse, is by means of such a table readily made.

Given the time at Greenwich, to find the time at the same instant at any other place, and the converse.

(74.) To find the time at any place, as Greenwich, corresponding to a given time at any other place, or the converse, we must remember that since the earth revolves through 360° in 24 hours, from west through south to east, or 15° in 1 hour; then at a place 15° to the eastward of a spectator the sun will be on the meridian 1 hour before, and at a place 15° to the westward, the sun will be on the meridian 1 hour later than at the place of the spectator: hence, when it is 10 o'clock at a given place, it will *at the same instant* be 11 o'clock at a place 15° to the eastward, and 9 o'clock at a place 15° to the westward. If, therefore, the longitude of a place is known, that is, the number of degrees it is to the east or west of Greenwich, we can readily tell what time it is at the place corresponding to a given time at Greenwich, and the converse. To find the time at Greenwich, corresponding to any given time at a place, is required in almost every nautical problem; and even if the longitude and time are only known nearly, the approximate true time at Greenwich, deduced from the estimated longitude and estimated time at the place, is an important element in nautical astronomy. The time at Greenwich, obtained in this manner, is called an approximate Greenwich time, or more frequently *the Greenwich date.*

TO FIND THE GREENWICH DATE.

Rule 5. *First method. By estimated ship time and longitude.*

1. Express the time at the ship astronomically to the nearest minute (p. 73).

2. Reduce the longitude into time to nearest minute, and put it under ship time (p. 72).

3. If west longitude, add longitude in time to ship time; the sum, if less than 24 hours, will be the time at Greenwich, or the Greenwich date on the same day as at the ship.

But if the sum be greater than 24 hours, reject 24 hours; the result will be the Greenwich date on the day following the ship date.

4. If east longitude, subtract longitude in time from ship time, the remainder will be the Greenwich date. If the longitude in time is greater than the ship time, 24 hours must be added to the ship time before subtraction is made, and the remainder will be the Greenwich date on the day preceding the ship date.

EXAMPLES.

181. June 10, at 6^h 10^m P.M. mean time nearly in longitude by account 32° 42′ W.; required the Greenwich date, or the Greenwich time to the nearest minute.

Ship, June 10......	6^h 10^m
long. in time........	2 11 W.
∴ Green. June 10.....	8 21

182. July 12, at 4^h 5^m A.M. in long. 63° 45′ W.; required the Greenwich date.

Ship, July 11.......	16^h 5^m
long. in time........	4 15 W.
∴ Green. July 11....	20 20

EXAMPLES.

Required the Greenwich date in each of the following examples.

	Ship times.		Answers, Greenwich dates.
183.	Mar. 7, at 3^h 15^m A.M.	Long. 15° 45′ E. .	Mar. 6, at 14^h 12^m
184.	Mar. 15 „ 10 35 P.M.	„ 43 5 E. .	Mar. 15 „ 7 43
185.	May 12 „ 4 30 A.M.	„ 45 50 W. .	May 11 „ 19 33
186.	May 9 „ 5 16 P.M.	„ 90 35 E. .	May 8 „ 23 14
187.	May 5 „ 11 30 P.M.	„ 55 47 W. .	May 5 „ 15 13
188.	May 20 „ 10 25 A.M.	„ 150 15 W. .	May 20 „ 8 26

The time at Greenwich, and therefore the Greenwich date, is more correctly found by means of a chronometer whose error on Greenwich mean time is known at the moment of observation as follows.

TO FIND THE GREENWICH DATE.

Rule 6. *Second method. By chronometer and its error on Greenwich mean time.*

To the time shown by chronometer, apply its error on Greenwich mean time; adding if error is slow, and subtracting if error is fast, on Greenwich mean time; the result is the Greenwich date in mean time. Sometimes 12 hours must be added to this result, and the day put one back. To determine when this must be done, get an approximate Greenwich date in the usual way by means of ship mean time and the estimated longitude; if the difference between the Greenwich dates found by the two methods is nearly 12 hours, then the Greenwich date *by chronometer* must be increased by 12 hours, and the day put one day back, if necessary, so as to make the two dates more nearly agree both in the day and hour.

The following examples will remove any doubt as to putting the day one back, or not.

EXAMPLE 1.

189. July 10th, at 6^h 34^m P.M. mean time nearly, in longitude 60° W., a chronometer showed 10^h 42^m 3^s, its error on Greenwich mean time being 2^m 10^s fast; required mean time at Greenwich, or the Greenwich date.

Greenwich time by chronometer.				Greenwich date.		
July 10th, chro.......	10^h	42^m	3^s	Ship, July 10th...	6^h	34^m
Error on G. M. T....		2	10	long. in time.......	4	0 W.
Gr. July 10th.........	10	39	53	Gr. July 10th......	10	34

As these two results come out near to each other, the correct Greenwich time is, July 10th, 10^h 39^m 53^s, and the Greenwich date is therefore July 10th, 10^h 40^m.

EXAMPLE 2.

190. Aug. 3d, at 5^h 42^m P.M. mean time nearly, in long. by account 150° 30′ W., a chronometer showed 3^h 23^m 15^s, and its error on Greenwich mean time was 10^m $10^s\cdot4$ slow; required the Greenwich date.

Greenwich time by chronometer.				Greenwich date.		
Aug. 3d, at......	3^h	23^m	15^s	Ship, Aug. 3d...	5^h	42^m
Error............		10	10·4	long. in time.....10		2 W.
Aug. 3d.........	3	33	25·4	Gr. Aug. 3d......15		44
Add..............12						
Gr. Aug. 3d.....	15	33	25·4			

In this example 12 hours must be added to the Greenwich date by chronometer, without putting the day one back.

EXAMPLE 3.

191. March 10th, at 2^h 10^m A.M. mean time nearly, in longitude 20° 42′ E., a chronometer showed 0^h 2^m 50^s, and its error on Greenwich mean time was 45^m 16^s slow; required the Greenwich date.

Greenwich time by chronometer.				Greenwich date.		
Mar. 10th......	0^h	2^m	50^s	Ship, Mar. 9th......	14^h	10^m
Error............		45	16	long. in time.........	1	23 E.
Mar. 10th......	0	48	6	Gr. Mar. 9th.........	12	47
Add.............	12					
Gr. Mar. 9th...	12	48	6			

In this example 12 hours must be added, and the day put one back, to bring the chronometer Greenwich time more nearly alike to the Greenwich date.

Find accurately the Greenwich times in the following examples.

M. T. nearly.	Long.	Chro. show.	Err. of Chro.	Answers.
192. Aug. 10 1^h 20^m P.M.	35° 42′ W.	4^h 2^m 10^s	18^m $45{\cdot}4^s$ fast	10 3^h 43^m $24{\cdot}6^s$
193. July 13 3 42 A.M.	150 50 W.	1 30 0	10 50·6 slow	13 1 40 50·6
194. June 10 10 42 P.M.	42 0 E.	7 44 10	8 12·0 slow	10 7 52 22
195. June 19 6 42 A.M.	50 50 W.	10 14 15	12 3·7 fast	18 22 2 11·3
196. Sept. 3 10 42 A.M.	19 15 E.	9 10 45	12 15·3 slow	2 21 23 0·3
197. Dec. 30 11 45 P.M.	110 35 W.	7 10 30	9 5·0 fast	30 19 1 25

CHAPTER III.

EXPLANATION AND USE OF THE NAUTICAL ALMANAC.

(75.) THE *Nautical Almanac* contains the declination, right ascension, &c., of the principal heavenly bodies, for certain fixed times at Greenwich. The declination and right ascension of the sun and planets are given for every day at $0^h\ 0^m\ 0^s$; for the moon, for every hour at Greenwich.

To obtain these quantities for any other time, we may either apply the common rules of proportion,* or—which is in most cases the simplest method—make use of certain tables computed for the purpose, called tables of *proportional logarithms.* The tables of proportional logarithms are the following:

1. The proportional logarithms (properly so called).
2. The Greenwich date proportional logarithm of the sun.
3. The Greenwich date proportional logarithm of the moon.
4. The logistic logarithms.

The explanation of these tables is given in *Nav.* Part II. p. 138.

TO TAKE OUT THE SUN'S DECLINATION.

Rule 7. *First method, by using proportional logarithms.*

1. Get a Greenwich date, thus:

(*a.*) Put down the ship mean time expressed astronomically.

(*b.*) Under which put the longitude in time: add if west, subtract if east (adding or subtracting 24 hours, according to Rule 5. p. 74).

2. Take out of the *Nautical Almanac* the sun's declination for the two consecutive noons between which the Greenwich date lies.

3. Take the difference of the declinations when their names are alike (that is, both north or both south); but when the names of the declinations are unlike, take their sum; the result will be the change of declination in 24 hours. Mark it + or −, or with the same or different letter, according as the declination is seen to be increasing or decreasing.

* The results obtained by the rules of proportion are only true when the daily or hourly change is *invariable*, and this is seldom the case in the motion or apparent motion of the heavenly bodies. When very great accuracy is required, we must apply a further correction, called *the equation of second difference.* For all the common purposes of navigation, however, a simple proportion, as directed in the following rules, will be sufficiently correct

4. Add together Greenwich date logarithm for the sun and proportional logarithm of the change in 24 hours; the result is the proportional logarithm of change of declination for the given time, which take from the table of proportional logarithms and apply to the declination at first noon, either by subtracting or adding it, according as the declination is seen to be decreasing or increasing.

Another method of taking out the sun's declination is to make use of the hourly changes of declination given in the *Nautical Almanac.*

TO TAKE OUT THE SUN'S DECLINATION.

Rule 8. *Second method, by using hourly differences.*

1. Find a Greenwich date, as in last rule.

2. Take out of the *Nautical Almanac* the declination at noon of the Greenwich date, and put down a little to the right thereof the difference for one hour found in the same page, and close to the declination taken out. Multiply this quantity by the hours in Greenwich date, and the fractional parts of the hour if necessary; the product will be the change of declination in the time from noon; apply this, reduced to minutes and seconds, to the declination taken out, adding it if the declination is seen to be increasing, and subtracting if decreasing. The result is the declination of the sun at the time required.*

EXAMPLES.

198. March 2, at $4^h\ 23^m$ P.M. mean time nearly, in long. 32° 42′ W.: required the sun's declination. (1.) By proportional logarithms. (2.) By hourly differences.

Ship, Mar. 2	4^h	23^m
long. in time	2	11 W.
Gr. Mar. 2	6	34

First method. By proportional logarithms.

Sun's decl. Mar. 2		7°	7′	0″ S.
„ 3		6	44	2 S.
			22	58 N.
Gr. date log. ☉ ...	56287			
pro. log. 22′ 58″.	89417			
	1·45704 ...		6	17 N.
∴ sun's decl. at $6^h\ 34^m$ ...		7	0	43 S.

* The corrections for Greenwich date of the quantities taken out of the *Nautical Almanac*, when small, are frequently made *by inspection;* the results thus obtained are generally found sufficiently correct.

Second method. By hourly differences.

Sun's decl.			
Mar. 2...7° 7′ 0″ S.... hourly diff. ... 57·4″ decreasing.			
cor. ... 6 16·9 −		6	
∴ decl. = 7 0 43·1 S.		344·4	change in 6^h
	30... $\frac{1}{2}$	28·7	
	3... $\frac{1}{10}$	2·9	" " 34^m
	1... $\frac{1}{3}$	0·9	
	60	376·9	
∴ cor. for 6^h 34^m......		6′ 16·9″ −	

199. March 21st, at 4^h 23^m A.M. in long. 100° 10′ E.: required the sun's declination. (1.) By proportional logarithms. (2.) By hourly differences.

Ship, Mar. 20	16^h 23^m
long. in time	6 41 E.
Gr. Mar. 20	9 42

(1.) By proportional logarithms.

Sun's decl. Mar. 20		0° 5′ 32·3″ S.
" 21		0 18 8·3 N.
		23 40·6 N.
Gr. date log. ⊙ ...	·39344	
pro. log. 23′ 41″ ...	·88083	
	1·27427...	9 33·5 N.
∴ sun's decl. at 9^h 42^m		4 1·2 N.

(2.) By hourly differences.

Sun's decl.		
Mar. 20...0° 5′ 32·2″ S....hourly diff. 59·2″ decreasing.		
cor. ... 9 33·2 N.		9
∴ decl. = 0 4 1·0 N.	30... $\frac{1}{2}$	532·8
	10... $\frac{1}{3}$	29·6
	2... $\frac{1}{10}$	9·8
		1·0
	60	573·2
∴ for 9^h 42^m......		9′ 33·2″ N.

200. July 30, 1845, at 3^h 20^m P.M. mean time, in long. 9° 0′ W.: required the sun's decl. *Ans.* 18° 28′ 14″ N.

201. Dec. 10, 1845, at 6^h 32^m A.M. mean time, in long. 32° 30′ W.: required the sun's decl. *Ans.* 22° 56′ 4″ S.

202. Aug. 1, 1845, at 4^h 52^m P.M. mean time, in long. 152° 33′ E.: required the sun's decl. *Ans.* 18° 4′ 16·5″ N.

Elements from Nautical Almanac.

		Hourly diff.
July 30......18°30′39″N.	July 31......18°15′57″N.	.......36·7″ dec.
Dec. 9........22 51 17 S.	Dec. 10......22 56 50 S.	......12·7 dec.
July 31......18 15 58 N.	Aug. 1 ......18 0 58 N.	.....37·5 dec.

Rule 9. *To take out the* EQUATION OF TIME.

First method, by using proportional logarithms.

1. Get a Greenwich date.
2. Take out the equation of time for two consecutive noons between which the Greenwich date lies, and take their difference.
3. Add together the Greenwich date logarithm for sun and proportional logarithm of difference: the sum is the proportional logarithm of correction, which find from the table, and apply it with its proper sign to the equation of time first taken out; the result is the equation of time required.

Second method, by using hourly differences.

1. Get a Greenwich date, as in first method.
2. Take out the equation of time for the noon of Greenwich date and the hourly difference opposite thereto.
3. Multiply hourly difference by the hours of the Greenwich date, and, if great accuracy is required, by the fractional parts of hour in the Greenwich date; the result will be the correction to be applied with its proper sign to the equation of time taken out.

EXAMPLE.

203. July 12, 1853, at 5^h 8^m A.M. mean time nearly, in long. 160° W.: required the equation of time.

Ship, July 11.........	17^h 8^m
long. in time..........	10 40 W.
Greenwich, July 11..	27 48
Greenwich, July 12..	3 48

Equation of time.

July 12	5^m	$15{\cdot}7^s$
„ 13	5	23·1
		7·4
Greenwich d. log. sun		0·80043
Prop. log. $7{\cdot}4^s$		3·16419
Prop. log. cor.		3·96462
Cor.		1·2
Equation required	5	16·9

Or thus, by hourly difference.

Diff. for 1 hour		$0{\cdot}308^s$ incr.
		3
		0·924
	30 ½	·154
	15 ½	·077
Cor.		1·155
Or,		$1{\cdot}2^s$
	5^m	15·7
Equation required	5	16·9

Find the equation of time in the following examples:

204.	March 2, 1853, at	$6^h 10^m$ P.M.	mean time, in long.	38°42′ W.
205.	„ 16 „ „	5 42 A.M.	„ „	152 45 W.
206.	„ 29 „ „	10 42 A.M.	„ „	87 8 E.

Elements from Nautical Almanac.

			Diff. 1 hour.
Eq. of time, March	2, 12^m $22{\cdot}1^s$	March 3, 12^m $9{\cdot}3^s$	$0{\cdot}53^s$
„ „	16 8 48·4	„ 17 8 30·8	0·72
„ „	28 5 9·0	„ 29 4 50·4	0·77

Ans. to 204, 205, 206: 12^m $17{\cdot}4^s$, 8^m $45{\cdot}5^s$, 4^m $56{\cdot}0^s$.

Rule 10. *To take out the* MOON'S SEMIDIAMETER AND HORIZONTAL PARALLAX.

The moon's semidiameter and horizontal parallax are put down in the *Nautical Almanac* for every mean noon and mean midnight at Greenwich; to find these quantities for any other time we may proceed as follows:

First. To find the moon's semidiameter.

1. Get a Greenwich date.

2. Take out of the *Nautical Almanac* the moon's semidiameter for the two times between which the Greenwich date lies, and take the difference. To the Greenwich date logarithm for moon add the proportional logarithm of the difference just found; the result will be the proportional logarithm of an arc, which being found and added to the semidiameter first taken out, or subtracted therefrom (according as the semidiameter is increasing or decreasing), will be the semidiameter at the given time.

Second. To find the moon's horizontal parallax.

Proceed in a similar manner to that pointed out above for finding the moon's semidiameter.

EXAMPLES.

207. Aug. 3, 1853, at 4^h 10^m P.M. mean time nearly, in long. 56° 15′ W.: required the moon's semidiameter and horizontal parallax.

Ship, Aug. 3	4^h	10^m
long. in time	3	45 W.
Greenwich, Aug. 3	7	55

Moon's semidiameter.		Moon's horizontal parallax.	
August 3, at noon......	15′ 6·6″	August 3, at noon......	55′20·6″
" mid.......	15 10·6	" mid.......	55 35·3
	4·0		14·7
Gr. d. log. for moon...	0·18064	Gr. d. log. for moon...	0·18064
Prop. log. for 4·0″......	3·43136	Prop. log. for 14·7″....	2·86611
Prop. log. cor...........	3·61200	Prop. log. cor...........	3·04675
Cor.......................	2·6	Cor.......................	9·7
Required semi. 15	9·2	Required hor. par......	55 30·3

NOTE. It is better in examples of this kind, where the differences taken out of the *Nautical Almanac* are only a few seconds, to learn to estimate the correction *at sight:* this, after a little practice, will not be difficult to do. The above method, however, by logarithms, should be adopted by beginners for the practice it gives in learning the use of the tables.

208. July 14, 1853, at 6^h 42^m A.M. mean time nearly, in long. 30° W.: required the moon's semidiameter and horizontal parallax.

Ship, July 13.........18^h 42^m
long. in time.......... 2 0 W.
Greenwich, July 13..20 42

Moon's semidiameter.		Moon's horizontal parallax.	
July 13, mid..................	16′ 2·7″	July 13, mid.	58′45·8″
" 14, noon................	16 7·5	" 14, noon................	59 3·5
	4·8		17·7
Gr. d. log. moon for $8^h 42^m$*...	0·13966	Gr. d. log. moon for $8^h 42^m$..	0·13966
Prop. log. 4·8″................	3·35218	Prop. log. 17·7″..............	2·78545
Prop. log. cor.................	3·49184	Prop. log. cor.................	2·92511
Cor.............................	3·5	Cor............................	12·8
Required semi.	16 6·2	Required hor. par.	58 58·6

Find the moon's semidiameter and horizontal parallax in the following examples:

209.	March 2, 1853......at	6^h 42^m P.M.......in long.	100° 0′W.
210.	" 14 " ......at	3 30 A.M....... "	120 0 W.
211.	" 24 " ......at	10 10 P.M....... "	60 42 E.

Elements from Nautical Almanac.

Moon's semidiameter.		Moon's hor. par.	Answers.
March 2, mid.	16′ 5·1″	Mid. 58′54·7″	Semi. 16′ 4·7″
" 3, noon	16 2·1	Noon 58 43·6	H. P. 58 53·4
March 13, mid.	14 48·9	Mid. 54 15·7	Semi. 14 47·9
" 14, noon	14 47·8	Noon 54 11·5	H. P. 54 11·7
March 24, noon	16 18·7	Noon 59 44·6	Semi. 16 21·2
" 24, mid.	16 23·7	Mid. 60 1·8	H. P. 59 53·3

*** When the Greenwich date exceeds 12 hours, as in this example, look out the Greenwich date logarithm moon for the excess of the Greenwich date above 12 hours.**

Rule 11. *To take out the* SUN'S RIGHT ASCENSION.

1. Get a Greenwich date.

2. Take out the right ascension for two consecutive noons between which the Greenwich date lies, and take their difference.

3. Add together the Greenwich date logarithm for sun and proportional logarithm of difference; the sum will be the proportional logarithm of correction to be added to the right ascension for noon of Greenwich date.

EXAMPLE.

212. July 13, 1853, at $6^h\ 31^m$ A.M. mean time nearly, in long. 172° 10′ W.: required the sun's right ascension.

		Sun's right ascension.
Ship, July 12......$18^h\ 31^m$		July 13......$7^h\ 30^m\ 30^s$
long. in time.......11 29 W.		„ 14......7 34 33
30 0		4 3
or ship, July 13... 6 0		0·60206
		1·64782
		2·24988...... 1 1
	∴	sun's R. A....7 31 31

This and the following examples may have been worked out by using the hourly difference, as in Rule 8. Sometimes the simplest method is to estimate the correction *by sight*, as in the above example, where we have to find the change of right ascension in 6 hours, the change in 24 hours being $4^m\ 3^s$, the correction is evidently $1^m\ 1^s$ nearly.

Find the sun's right ascension in the following examples:

213.	March 11, 1853...at	$6^h\ 42^m$ P.M.	mean time...	long.	42° 41′ W.
214.	„ 21 „ ...at	10 10 A.M.	„ ...	„	100 41 E.
215.	„ 21 „ ...at	0 0 A.M.	„ ...	„	142 14 W.

Elements from Nautical Almanac.

Sun's right asc., March 11,	$23^h\ 26^m\ 26{\cdot}3^s$	March 12, $23^h\ 30^m\ 6{\cdot}6^s$
„ „ 20	23 59 20·0	„ 21 0 2 58·2
„ „ 21	0 2 58·2	„ 22 0 6 36·4

Ans. to 213, 214, 215: $23^h\ 27^m\ 54{\cdot}3^s$, $0^h\ 1^m\ 40{\cdot}0^s$, $0^h\ 4^m\ 24{\cdot}2^s$.

Rule 12. *To take out the moon's* DECLINATION *and* RIGHT ASCENSION.

The moon's declination and right ascension are recorded in the *Nautical Almanac* for the beginning of every hour of mean time at Greenwich. To find them for any other time we may proceed as follows:

First. To find the moon's declination for any given time.

1. Get a Greenwich date.

2. Take out of the *Nautical Almanac* the moon's declination for two consecutive hours between which the Greenwich date lies, and take the difference.

First method. By logistic logarithms.

1. Add together the logistic logarithm of minutes in Greenwich date and proportional logarithm of difference, the sum will be the proportional logarithm of correction, which take from the table and apply it to the declination for the hour of Greenwich date, adding or subtracting according as the declination is seen to be increasing or decreasing. The result is the declination required.

Second method. By 10 minutes' differences.

1. Take out "Diff. Dec. for 10^m" opposite the first declination taken out.

2. Multiply the 10^m diff. by the number of minutes in Greenwich date, and remove the decimal point one place to the left: the result is the correction in decl. for Greenwich date in seconds, which turn into minutes and seconds if necessary, and apply to the decl. as in first method.

To take out the MOON'S RIGHT ASCENSION.

First method. By logistic logs.

Proceed as in the first method just given for finding the moon's declination.

Second method. By hourly difference, or by practice.

Multiply hourly difference, turned into seconds, by the number of minutes in Greenwich date, and divide by 60: the result will be the correction in right ascension for Greenwich date in seconds, which turn into minutes and seconds if necessary, and add to the right ascension at the hour of Greenwich date; or this correction may be obtained by the common rule of Practice.

EXAMPLES.

216. January 24, at 5^h 40^m P.M. mean time, in long. 60° 10′ W.: find the moon's declination and right ascension.

Ship, Jan. 24......5^h 40^m
long. in time.......4 1 W.

Gr. Jan. 24.........9 41

Moon's declination.

First method. By logistic logarithms.

24 at 9^h............19° 39′ 12″N.
10 ....19 34 21 N.
4 51 S.

0·16537
1·56953
1·73490...... 3 19 −

∴ moon's decl. at 9^h 41^m......19 35 53

Second method. By 10^m differences.

10^m diff............48·6″ dec.

```
              48·6
                41
              ----
               486
              1944
             -----
          60)199·26
             ------
             3′ 19·3″—
```

change in decl. as before.

Moon's right ascension.

First method. By logistic logarithms.

```
24 at  9h............6h 53m 24·1s
       10 ...........6  55  30·1
                     ------------
                         2   6·0
       0·16537
       1·93305
       -------
       2·09842......     1  26·0+
                     ------------
∴ moon R.A. at 9h 41m......6  54  50·1
```

Second method. By hourly differences.

```
diff. in 60m............126s
                         41
                        ---
                        126
                       504
                    60)516·6
                       -----
                        86·1
                    or 1m 26·1s
```

By Practice.

```
30...1/2  | 126·0s
10...1/3  |  63·0
 1...1/10 |  21·0
          |   2·1
          |  ----
          |  86·1
     or 1m 26·1s
```

The same result as before.

217. June 2, at 10^h 30^m A.M. mean time, in long. 53° 15′ W.: find the moon's right ascension and declination.

Ans. R. A. 2^h 47^m $29{\cdot}4^s$, decl. 17° 12′ 45″ N.

218. Sept. 7, at 4^h 15^m A.M. mean time, in long. 56° 30′ E.: find the moon's right ascension and declination.

Ans. R. A. 14^h 53^m $52{\cdot}7^s$, decl. 16° 51′ 39″ S.

219. July 10, at 9^h 30^m A.M. mean time, in long. 44° 20′ W.: find the moon's right ascension and declination.

Ans. R. A. 11^h 27 $48{\cdot}2^s$, decl. 1° 23′ 0″ S.

Elements from Nautical Almanac.

	Moon's right ascension.	Moon's decl.	10m diff.
June 2, at 2......	$2^h\ 47^m\ 23{\cdot}1^s$.........	17° 12′ 26″ N.........	63·4″ incr.
„ at 3......	2 49 28·1	17 18 46 N.	
Sept. 6, at 12......	14 52 44·7	16 48 19 S.........	68·9″ incr.
„ at 13......	14 55 3·7	16 55 13 S.	
July 10, at 0......	11 26 55·2	1 17 58 S.......	111·7 incr.
„ at 1......	11 28 52·9	1 29 9 S.	

Rule 13. *To take out the* RIGHT ASCENSION OF THE MEAN SUN (*called in the Nautical Almanac* SIDEREAL *time*).

The right ascension of the mean sun, or the sidereal time at mean noon, is given in the *Nautical Almanac* for every day at mean noon. To find it for any other time we may proceed as in the rule for finding the right ascension of the apparent or true sun; but as the motion of the mean sun is uniform throughout the year (the motion in every 24 hours being $3^m\ 56{\cdot}555^s$), the change in any given number of hours, minutes, and seconds is more easily found by means of a table. This table is given in the *Nautical Almanac*, under the title of "Time Equivalents;" it may be also found, arranged in a very convenient form, in Inman's *Nautical Tables*, p. 12*.

EXAMPLE.

220. July 23, 1853, at $2^h\ 32^m$ P.M., in long. 80° 42′ E.: required the right ascension of the mean sun.

	Right asc. mean sun.	Or thus, by table.
Ship, July 23... $2^h\ 32^m$	July 22...$8^h\ 0^m\ 35^s$	July 22....$8^h\ 0^m\ 35^s$
long. in time..... 5 23 E.	„ 23...8 4 32	cor. for 21^h 3 27
Green., July 22. 21 9	3 57	„ 9^m 1·5
	0·05490	R. A.......8 4 3·5
	1·65868	as before.
	1·71358 3 29	
	Right asc. mean sun...8 4 4	

Find the right ascension of mean sun (called in the *Nautical Almanac* sidereal time) in the following examples:

221.	March 2, 1853, at $10^h\ 42^m$ P.M. mean time,	in long.	48° 10′ W.
222.	„ 15 „ „ 6 6 A.M.	„	100 0 W.
223.	„ 21 „ „ 10 10 P.M.	„	100 0 E.

Elements from Nautical Almanac, and Answers.

Sidereal time March 2, at noon,	$22^h\ 40^m\ 44{\cdot}9^s$....	*Ans.*	$22^h\ 43^m\ 2{\cdot}0^s$
„ „ 15, „	23 32 0·1....	„	23 32 7·6
„ „ 21, „	23 55 39·4....	„	23 56 13·9

Rule 14. *To take out the* LUNAR DISTANCES *for any given time at Greenwich.*

1. Get a Greenwich date.

2. Find two consecutive distances in the *Nautical Almanac* at times between which the Greenwich date lies. Take the difference of the distances. To the proportional logarithm of the excess of the Greenwich date above the first of the times taken from the *Nautical Almanac* add proportional logarithm of difference of distances; the sum will be the proportional logarithm of an arc; which arc being applied to the distance at first time with its proper sign will be the distance required.

EXAMPLE.

224. September 24, at $6^h 10^m$ P.M. mean time nearly, in long. 60° 15′ W.: required the distance of Aldebaran from the moon.

		Dist. of Aldebaran.
Ship, Sept. 24... $6^h 10^m$		At 9^h...18° 57′ 35″
long. in time..... 4 1 W.		12 ...20 23′ 37
Green. Sept. 24...10 11	Proportional log...32061	... 1 26 2
	prop. log. $1^h 11^m$...40401	
	log. cor. 72462	... 0 33 56
	∴ dist. of Aldebaran at $6^h 10^m$	...19 31 31

Required the distance of the moon from certain stars in the following examples:

225. Jan. 24, at $4^h 30^m$ P.M. mean time nearly, in long. 30° 30′ E.: required the distance of Regulus from the moon. *Ans.* 69° 33′ 6″.

226. May 20, at $6^h 20^m$ A.M. mean time nearly, in long. 40° 0′ E.: required the distance of α Pegasi from the moon. *Ans.* 56° 59′ 7″.

227. June 10, at 9^h 40^m P.M. mean time nearly, in long. 32° 45′ W.: required the distance of α Aquilæ from the moon. *Ans.* 74° 32′ 35″.

228. July 2, at 7^h 20^m A.M. mean time nearly, in long. 30° 0′ E.: required the distance of Jupiter from the moon. *Ans.* 54° 16′ 52″.

229. Sept. 19, at $10^h 30^m$ A.M. mean time nearly, in long. 63° 15′ E.: required the distance of Aldebaran from the moon. *Ans.* 72° 0′ 51″.

230. Dec. 15, at 2^h 0^m P.M. mean time nearly, in long. 19° 40′ E.: required the distance of Pollux from the moon. *Ans.* 58° 56′ 47″.

Elements from Nautical Almanac.

Distance of	Regulus	at noon	68°11′ 7″......	at	3^h	69°50′50″
„	α Pegasi	„ 15^h	57 17 6	„	18	55 56 9
„	α Aquilæ	„ 9	75 56 46	„	12	74 28 9
„	Jupiter	„ 15	55 41 18	„	18	53 52 44
„	Aldebaran	„ 18	72 9 14	„	21	70 40 29
„	Pollux	„ noon	58 32 51	„	3	60 17 54

In rule (L), for finding the longitude by lunar observations, we have to calculate the *true* distance of the moon from some heavenly body at the time of observation. If the heavenly body is one whose distance is recorded in the *Nautical Almanac* for every three hours, we may find the mean time at Greenwich corresponding to the computed true distance for the time of observation as follows:

Rule 15. *To find the* TIME AT GREENWICH *corresponding to a* GIVEN DISTANCE *of a heavenly body from the moon.*

1. Under the given distance put down the two computed distances of the same heavenly body found in the *Nautical Almanac* between which the given true distance lies.

2. Take the difference between the first and second, and also between the second and the third.

3. From the proportional logarithm of the first difference subtract the proportional logarithm of the second difference, the remainder is the proportional logarithm of the additional time to be added to the hours of the distance first taken out of the *Nautical Almanac*; the result is the mean time at Greenwich corresponding to the given distance.

EXAMPLES.

231. November 22, 1853, the true distance of Saturn from the moon was found to be 77° 52′ 45″; required Greenwich mean time when the observation was taken.

True distance at observation	......	77°52′ 45″
in *Naut. Almanac* dist. at 3^h	.....	77 14 40
	6	78 47 24
prop. logarithm	·67454	38 5
	·28804	1 32 44
	·38650 Cor.	$1^h 13^m 55^s$
	Adding	3
Greenwich mean time Nov. 22...		4 13 55

Find mean time at Greenwich from each of the following observations:

232. November 24, 1853, when true distance of Aldebaran was 93° 38′ 45″. *Ans.* 3^h 57^m 18^s.

233. Sept. 24, 1853, when true distance of Regulus was 58° 45′ 8″. *Ans.* 16^h 3^m 6^s.

234. May 27, 1853, when true distance of the sun was 110° 8′ 50″. *Ans.* 14^h 2^m 22^s.

Elements from Nautical Almanac.

Dist. Aldebaran, Nov. 24, at	3^h...	93° 7′57″	......at 6^h...	94°44′42″
„ Regulus, Sept. 24 ...	„ 15 ...	59 16 16	...... „ 18 ...	57 47 27
„ Sun, May 27	„ mid...	111 12 57	...... „ 15 ...	109 38 38

To take out a PLANET'S RIGHT ASCENSION AND DECLINATION.

Proceed as in the similar rules for finding the sun's right ascension and declination (pp. 82, 78).

The rules above given are sufficient to enable the student to acquire a competent knowledge of the principal contents of the *Nautical Almanac*. They will be found of the greatest use in the subsequent rules for finding the latitude and longitude.

Equation of Second Differences.

We have supposed in the above examples the motion of the heavenly body to be *uniform* in the interval between the Greenwich times taken out of the *Nautical Almanac*. This is seldom the case, although in most of the questions in *Nautical Astronomy* it may be so assumed without any practical error. When, however, very accurate results are required, a correction must be used called the *equation of second differences*. The investigation of this equation belongs to the theoretical part of Navigation, and as the equation is seldom required in the common rules of Navigation, we may omit for the present examples of its application.

CHAPTER IV.

PRELIMINARY PROBLEMS AND RULES IN NAUTICAL ASTRONOMY.

CORRECTIONS FOR PARALLAX, REFRACTION, CONTRACTION OF THE MOON'S SEMI-DIAMETER, AND DIP.

GIVEN mean solar time and the equation of time: to find the apparent solar time; or

Given apparent solar time and the equation of time: to find mean solar time.

Rule 16. (1.) Get a Greenwich date (p. 74).

(2.) Correct for this date the equation of time (taken out of page i. of *Nautical Almanac*, when apparent solar time is given, and out of page ii. when mean time is given) (Rule 9).

(3.) Apply the equation of time, with its proper sign (as shown in the *Nautical Almanac*), to the given time.

(4.) The result is the time required.

EXAMPLES.

235. April 27, 1846, at 9^h 10^m P.M. mean time, in long. 16° W.: required apparent solar time.

Mean time.			Equation of time (page ii. *Naut. Alm.*).		Mean time.	
Ship, April 27 .	9^h	10^m	27 . .	2^h $26{\cdot}9^m$ add to	Ship, April 27.	9^h 10^m 0^s
long. in time . .	1	4 W.	28 . .	2 36·4 mean time.	eq. of time . . .	2 31+
Gr., April 27 . .	10	14		9·5	∴	9 12 31
			0·37020		apparent time required.	
			3·05570			
			3·42590 . .	4·1		
			∴ eq. of time . .	2 31·0+		

236. June 22, 1852, at 5^h 42^m P.M. apparent solar time, in long. 100° 30′ E.: required mean solar time.

Apparent time.			Equation of time (page i. *Naut. Alm.*). Diff. for 1 hour, $0{\cdot}54^s$.		Mean time.	
Ship, June 22 .	5^h	42^m P.M.	21 . .	1^m $27{\cdot}2^s$ add to	Ship, June 22	5^h 42^m 0^s
long. in time .	6	42 E.	22 . .	1 40·2 app. time.	eq. of time .	1 39·6+
Gr., June 21 .	23	0		13·0	∴	5 43 39·6
			23×·54	12·4	mean time required.	
				1 39·6+		

237. July 4, 1853, at $6^h 10^m$ P.M. mean time, in long. 100° W.: required apparent solar time. *Ans.* $6^h 5^m 53{\cdot}2^s$.

238. Dec. 10, 1853, at $4^h 42^m$ P.M. apparent solar time, in long. 80° 45′ W.: required mean solar time. *Ans.* $4^h 35^m 18{\cdot}1^s$.

239. Feb. 23, 1848, at $10^h 40^m$ A.M. apparent solar time, in long 1° 6′ W.: required mean solar time. *Ans.* $10^h 53^m 43{\cdot}5^s$ A.M.

Elements from Nautical Almanac.

Equation of time,	July 4	... 4^h $1{\cdot}1^s$ sub.	July 5	...	4^m $11{\cdot}7^s$ sub.
,,	Dec. 10	... 6 53·5 sub.	Dec. 11	...	6 25·8 sub.
,,	Feb. 22	... 13 51·0 add.	Feb. 23	...	13 43·1 add.

Rule 17. *Given mean solar time: to find sidereal time.*

1. Get a Greenwich date (p. 74).

2. Correct the right ascension of the mean sun by the table of Time Equivalents (p. 86), or by proportional logarithms, or otherwise, for the Greenwich date.

3. Add together the corrected right ascension of mean sun and mean time at the ship.

4. The sum (rejecting 24 hours if greater than 24 hours) will be sidereal time.

EXAMPLES.

240. Feb. 24, 1848, at $10^h 40^m 30^s$ A.M. mean time, in long. 1° 6′ W.: required sidereal time.

	h	m	s
Ship, Feb. 23	22	40	30
long. in time	0	4	24
Gr., Feb. 23	22	44	54

Right ascension mean sun.

	h	m	s	
Feb. 23 . .	22	10	3·52	
cor. for 22^h . .		3	36·84	by table.
,, 44^m . .			7·23	by table.
,, 54^s . .			·15	by table.
right asc. mean sun	22	13	47·74	
ship mean time . .	22	40	30·00	
sidereal time . . .	20	54	17·74	(rejecting 24 hours).

241. July 10, 1853, at $0^h 42^m 10^s$ P.M. mean time, in long. 84° 42′ W.: required sidereal time. *Ans.* $7^h 56^m 29{\cdot}7^s$.

242. Sept. 30, 1853, at $6^h 42^m 10^s$ A.M. mean time, in long. 100° 42′ W.: required sidereal time. *Ans.* $7^h 18^m 58{\cdot}59^s$.

243. Dec. 8, 1853, at $10^h 10^m 42^s$ P.M. mean time, in long. 18° 32′ E.: required sidereal time. *Ans.* $3^h 20^m 47^s$.

Elements from Nautical Almanac.

Right ascension mean sun, July 10		7^h	13^m	$17{\cdot}14^s$
,,	Sept. 30	12	36	34·64
,,	Dec. 8	17	8	36·96

Rule 18. *Given apparent solar time: to find sidereal time.*

1. Get a Greenwich date (p. 74).

2. Correct the equation of time and also the right ascension of the mean sun for Greenwich date (pp. 80, 86).

3. Apply corrected equation of time to ship apparent time, and thus get ship mean time. Then, as in the last rule,

4. Add together ship mean time and right ascension of mean sun.

5. The sum (rejecting 24 hours if greater than 24 hours) will be sidereal time.

EXAMPLES.

244. May 24, 1853, at $6^h\ 8^m\ 40^s$ A.M. apparent solar time, in long. 20° 20′ W.: required sidereal time.

				Equation of time.		Right ascension mean sun.			
Ship, May 23	18^h	8^m	40^s	23 . 3^m $33{\cdot}2^s$	sub. from	23d, at noon .	4^h	4^m	$2{\cdot}37^s$
long. in time .	1	21	20 W.	24 . 3 28·3	app. time.	19^h . . .		3	7·27
Gr., May 23 .	19	30	0	4·9		30^m . . .			4·93
						R. A. mean sun	4	7	14·57
						ship M. T. . .	18	5	10·80
						sidereal time .	22	12	25·37

Prop. logs.				
0·09018				
3·34323				
3·43341			4·0	
eq. of time .		3	29·2	sub.
app. time .	18	8	40·0	
May 23 . .	18	5	10·8	ship mean time.

245. July 4, 1853, at $3^h\ 42^m$ A.M. apparent solar time, in long. 84° 42′ W.: required sidereal time. *Ans.* $22^h\ 35^m\ 10{\cdot}53^s$.

246. Oct. 21, 1853, at $8^h\ 48^m$ P.M. apparent solar time, in long. 88° 8′ E.: required sidereal time. *Ans.* $22^h\ 32^m\ 30{\cdot}87^s$.

Elements from Nautical Almanac.

Equation of time.						Right ascen. mean sun.
July 3,	3^m $50{\cdot}1^s$ add	4,	4^m	$1{\cdot}1^s$, add	......	3, 6^h 45^m $41{\cdot}24^s$
Oct. 21,	15 19·1 sub.	22,	15	28·1, sub.	......	21, 13 59 22·26

Rule 19. *Given mean time, or apparent time at the ship: to find what heavenly body will pass the meridian the next after that time.*

1. Get a Greenwich date (p. 74).

2. Find the right ascension of the mean sun (and, if the Greenwich date is in apparent time, find also the equation of time (p. 80) for that date, so as to get mean time (p. 90).

3. Add together ship mean time and the right ascension of mean sun.

4. The sum (rejecting 24 hours if greater than 24 hours) will be sidereal time, or the right ascension of the meridian.

5. Then that star, found in some catalogue of fixed stars, whose right ascension is the *next greater* will be the star required.

EXAMPLE.

247. Feb. 24, 1853, at 4^h 42^m P.M. mean time nearly, in long. 100° E.; find what bright star will pass the meridian the next after that date.

			Right ascension mean sun.		
Ship, Feb. 24. .	4^h 42^m	23 . . .	22^h 13^m $9{\cdot}0^s$	Ship, Feb. 24 .	4^h 42^m 0^s
long. in time . .	6 40 E.	22^h . .	3 36·8	R. A. mean sun	22 16 46
Gr., Feb. 23 . .	22 2	2^m. . .	·3	R. A. merid. .	2 58 46
			22 16 46·1		

Looking into the "Catalogue of the mean places of 100 principal fixed stars" (see *Nautical Almanac*), we find the star whose right ascension is next greater than 2^h 58^m is α Persei; therefore α Persei is the bright star that will come to the meridian the next after 4^h 42^m P.M. on Feb. 24.

Sometimes it is required to find what principal stars will pass the meridian between certain convenient hours for observing their transits: as, for instance, between 8^h and 11^h P.M. To do this, we must find the right ascension of the meridian for these two times by the above rule; then the stars whose right ascensions lie between will be the stars required.

EXAMPLES.

248. Oct. 3, 1853, in long. 90° W., find what bright stars put down in the *Nautical Almanac* will pass the meridian between the hours of 9 and 12 P.M.

Ship, Oct. 3............	9^h 0^m	Ship, Oct. 3	12^h 0^m
long. in time	6 0 W.	long. in time.........	6 0 W.
Greenwich, Oct. 3 ...	15 0	Greenwich, Oct. 3...	18 0

Right ascension mean sun.				Right ascension mean sun.			
Oct. 3	12^h	48^m	24^s	Oct. 3	12^h	48^m	24^s
15^h		2	27	18^h		2	57
	12	50	51		12	51	21
ship, Oct. 3	9	0	0	ship, Oct. 3	12	0	0
R. A. meridian ...	21	50	51	R. A. meridian ...	0	51	21

In Catalogue p. 432, *Nautical Almanac*, the stars whose right ascensions

lie between $21^h\ 50^m\ 51^s$ and $0^h\ 51^m\ 21^s$ are from α Aquarii to β Ceti inclusive.*

249. What bright stars put down in the *Nautical Almanac* will pass the meridian of a ship in long. 40° E., between 8^h and 10^h P.M. mean time, on Nov. 20, 1853? *Ans.* From α Andromedæ to α Arietis.

250. What bright star will pass the meridian of a ship in long. 30° W. the first after $10^h\ 30^m$ P.M. on Oct. 10, 1853? *Ans.* α Andromedæ.

251. What bright stars will pass the meridian of a ship in long. 56° W., between the hours of 6 and 10 P.M. on March 10, 1853?

Ans. From β Tauri to ι Argûs.

252. What bright stars put down in the *Nautical Almanac* will pass the meridian of Greenwich, between the hours of 7 and 9 P.M. mean time, on August 20, 1853? *Ans.* From ε Ursæ Minoris to β Lyræ.

253. What stars named in the *Nautical Almanac* will pass the meridian of a ship in long. 86° E., on Oct. 20, 1853, between the hours of 10 P.M. and midnight? *Ans.* From α Andromedæ to α Eridani.

254. What bright star will pass the meridian of Greenwich the first after 9^h P.M. on Sept. 12, 1853? *Ans.* α Cygni.

Elements from Nautical Almanac.

	R. A. mean sun.		R. A. mean sun.
Nov. 20	$15^h\ 57^m\ 39^s$	Aug. 20	$9^h\ 54^m\ 56^s$
Oct. 10	13 16 0	Oct. 20	13 55 26
Mar. 10	23 12 17	Sep. 12	11 25 37

Rule 20. *Given sidereal time: to find mean time.*

1. Take out of the *Nautical Almanac* the right ascension of the mean sun (called in the *Nautical Almanac* sidereal time) for noon of the given day.

2. From sidereal time (increased if necessary by 24 hours) subtract the quantity just taken out; the remainder is mean time nearly.

3. Find in the table of the acceleration of sidereal on mean solar time the correction for this time, and subtract it from the mean time nearly.

4. The remainder is the mean time required.

NOTE. In strictness we ought to have entered the table with the correct mean time, instead of that used; but it is evident we may obtain a still closer approximation to the truth by repeating the work, using the last approximate value instead of the preceding one. For all practical purposes this repetition is seldom necessary.

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* In the *Handbook for the Stars*, published by the author, there is a table of the approximate times of the transits of the principal fixed stars. This table enables the observer to find the name of the bright star that is on the meridian at any given time, and at any place, by *inspection*, and without any calculation.

255. April 27, when a sidereal clock showed $3^h\ 40^m\ 45^s$: required mean time.

Sidereal time		3^h	40^m	$45·00^s$
R. A. mean sun at mean noon		2	20	21·58
mean time nearly ...		1	20	23·42
cor. ...1^h	9·86			
„ 20^m ...	3·28			
„ 23^s	·06			13·20
∴ required mean time		1	20	10·22

256. March 2, when a sidereal clock showed $3^h\ 40^m\ 45^s$: required mean time.

Sidereal time		3^h	40^m	$45·00^s$
R. A. mean sun at mean noon		22	41	35·94
mean time nearly ...		4	59	9·06
cor. ...4^h ...	39·43			
„ 59^m .	9·69			
„ 9^s	·02			49·14
∴ required mean time		4	58	19·92

(76.) The clock of an observatory used for noting the transit of a heavenly body is generally adjusted to *sidereal* time. By means of the above rule we can determine the error of a chronometer or solar clock regulated to *mean* time, by comparing the chronometer with the sidereal clock at some *coincident* beat, and then, correcting the sidereal clock for its error, we can find the corresponding mean time at the instant; the difference between which and the time shown by the chronometer will be the error of the chronometer on mean time at the place.

EXAMPLE.

257. Greenwich, March 3, 1853, when a sidereal clock showed $6^h\ 10^m\ 20^s$ a chronometer showed $7^h\ 32^m\ 10^s$: required the error of the chronometer on Greenwich mean time; the error of sidereal clock being $2^m\ 42·5^s$ slow.

Sidereal clock		6^h	10^m	20^s	
error			2	42·5	slow.
sidereal time		6	13	2·5	
R. A. mean sun at mean noon		22	44	41·48	
Cor. 7^h............	$1^m\ 8·99^s$	7	28	21·02	
„ 28^m	4·60				
„ 21^s	·05				
	1 13·64		1	13·64	
required mean time		7	27	7·38	
chronometer showed		7	32	10·0	
error of chro. on Gr. mean time			5	2·62	fast.

(77.) When the calculations are made for any other meridian than that

of Greenwich, for which the quantities in the *Nautical Almanac* are calculated, we must take into consideration the change of the mean sun's place arising from the difference of longitude. For example, the tables of the *Connaissance des Tems* are computed for Paris, the long. of which is $9^m\ 22^s$ to the east of Greenwich: as in that time the mean sun moves to the eastward through an arc of $1{\cdot}53^s$ in time (for $24^h : 9^m\ 22^s :: 3^m\ 56{\cdot}55^s : 1{\cdot}53^s$), it follows that we must add $1{\cdot}53^s$ to all the right ascensions of the mean sun in the French tables to obtain those of the mean sun at mean noon at Greenwich. (See *Nav.* Part II. chap. vii.) Or thus, by the rule;

EXAMPLE.

258. April 27, 1841, the right ascension of the mean sun at mean noon at Paris, by the *Connaissance des Tems*, was $2^h\ 21^m\ 10{\cdot}09^s$: required the same for Greenwich mean noon.

Greenwich, April 27		0^h	0^m	0^s
long. in time			9	22 W.
Paris date, April 27............			9	22
R. A. mean sun at Paris		2^h	21^m	$10{\cdot}09^s$
Cor. 9^m	$1{\cdot}48^s$			
,, 22^s......	$\cdot05$			
	$1{\cdot}53$			$1{\cdot}53$
R. A. mean sun at Greenwich...........		2	21	$11{\cdot}62$

(78.) The longitude is usually found at sea by means of a chronometer showing Greenwich mean time at the instant the mean time at the ship is known. The mean time at the ship is deduced from the hour-angle of a heavenly body, and this hour-angle is calculated by means of the altitude of the body observed with a sextant and certain elements found in the *Nautical Almanac.*

Rules for calculating the hour-angle of a heavenly body from an observed altitude will be given hereafter. We will therefore suppose the hour-angle known, and proceed to show how mean time might be found from it.

Rule 21. *To find mean time at the ship, having given the hour-angle of a heavenly body, as a star or the moon.*

It is proved in *Navigation*, Part II. p. 34, that

1. When the star is WEST of meridian,

Mean time = star's hour-angle + star's right ascension − right ascension of mean sun.

2. When the star is EAST of meridian:

Mean time = (24^h − star's hour-angle) + star's right ascension − right ascension of mean sun.

To find ship mean time, we must proceed therefore as follows:

1. Get a Greenwich date.
2. Take out the star's right ascension.
3. Take out also the right ascension of the mean sun (called in *Nautical Almanac* sidereal time), and correct it for Greenwich date.
4. When heavenly body is WEST of meridian:

To the star's hour-angle add star's right ascension, and from the sum subtract the right ascension of mean sun (adding or rejecting 24 hours if necessary); the result is ship mean time.

5. When heavenly body is EAST of meridian:

First subtract hour-angle from 24 hours, then to the remainder add star's right ascension, and from the sum subtract the right ascension of the mean sun; the result (rejecting 24 hours if necessary) is ship mean time required.

EXAMPLES.

259. Feb. 10, 1847, at $9^h\ 22^m$ P.M. mean time nearly, in long. 27° 15′ W., the hour-angle of Regulus (α Leonis) was $3^h\ 15^m\ 17^s$ EAST of meridian: required mean time at the ship.

Ship, Feb. 10 ...	9^h	22^m			24^h	0^m	0^s
long. in time ...	1	49 W.		Star's H. A.......	3	15	17
Gr. Feb. 10 ...	11	11			20	44	43·0
R. A. mean sun.				star's R. A.......	10	0	15·3
Feb. 10	21^h	19^m	$46{\cdot}0^s$				
cor...11^h.........		1	48·4		30	44	58·3
„ 11^m.........			1·8	R. A. mean sun	21	21	36·2
R. A. mean sun.	21	21	36·2	∴ ship mean time	9	23	22·1

260. Sept. 10, 1844, at $7^h\ 11^m$ P.M. mean time nearly, in long. 32° E., the hour-angle of Arcturus (α Bootis) was $4^h\ 22^m\ 15^s$ WEST of meridian: required mean time at the ship. *Ans.* $7^h\ 11^m\ 31{\cdot}7^s$ P.M.

261. Nov. 22, 1853, at $7^h\ 15^m$ P.M. mean time nearly, in long. 22° 0′ W., the hour-angle of Aldebaran (α Tauri) was $5^h\ 10^m\ 20^s$ EAST of meridian: required mean time at the place. *Ans.* $7^h\ 10^m\ 14^s$.

262. June 23, 1853, at $4^h\ 15^m$ A.M. mean time nearly, in long. 100° 40′ E., the hour-angle of α Lyræ was $3^h\ 42^m\ 40^s$ WEST of meridian: required mean time at the place. *Ans.* $16^h\ 10^m\ 56^s$.

NOTE.—If the estimated ship time used for getting the Greenwich date differs several minutes from the true ship time, the R. A. mean sun, and therefore ship mean time deduced from it, may be a few seconds incorrect. To get a correct result we must use the ship mean time, found as in the above examples, instead of that first used, and thus obtain a *corrected*

Greenwich date, and then recalculate the R. A. mean sun for that date. It will be rarely necessary to repeat this method of approximation more than once; but the necessity for this repetition should be borne in mind in many of the subsequent rules when a *wrong Greenwich date* has been found to have been used. The following examples will show the effect of an error in the Greenwich date on the resulting ship mean time.

EXAMPLES.

263. August 11, 1846, at $8^h\ 50^m$ P.M. mean time nearly, in long. 90° W., the hour-angle of Arcturus was $3^h\ 56^m\ 55^s$ west of meridian: required correct mean time at the place.

Ship, Aug. 11	8^h	50^m
long. in time	6	0 W.
Greenwich, Aug. 11	14	50

Right ascension mean sun.							
Aug. 11	9^h	18^m	$16{\cdot}51^s$	Star's hour-angle...	3^h	56^m	$55{\cdot}0^s$
cor. 14^h		2	17·99	„ right asc. ...	14	8	40·14
„ 50^m			8·21		18	5	35·14
				rt. asc. mean sun...	9	20	42·71
	9	20	42·71	ship mean time ...	8	44	52·43

This result is slightly incorrect, arising from the estimated mean time, $8^h\ 50^m$, being different from the true time. When great accuracy is required, the operation should be repeated, using mean time last found, namely $8^h\ 45^m$, instead of the one used before; thus,

The operation repeated.

Ship, Aug. 11	8^h	45^m
long. in time	6	0
Greenwich, Aug. 11	14	45

Right ascension mean sun.							
Aug. 11	9^h	18^m	$16{\cdot}51^s$	Star's hour angle...	3^h	56^m	$55{\cdot}0^s$
cor. 14^h		2	17·99	„ right asc......	14	8	40·12
„ 45^m			7·39		18	5	35·12
				rt. asc. mean sun...	9	20	41·89
	9	20	41·89	cor. ship mean time	8	44	53·23

264. June 15, 1853, at $10^h\ 10^m$ P.M., supposed mean time nearly, in long. 10° 42′ W., the hour-angle of Arcturus was $2^h\ 2^m\ 30^s$ EAST of meridian: required mean time at the place.

Ans. 1st approximation, $6^h\ 30^m$; 2d approx. $6^h\ 30^m\ 35^s$.

Elements from Nautical Almanac.

Right ascension mean sun.					Right ascension star.			
Sept. 10, 1853......	11^h	18^m	28^s	......	α Bootis	14^h	8^m	35^s
Nov. 22, „	16	5	32	......	Aldebaran ...	4	27	32
June 22, „	6	2	10	......	α Lyræ.........	18	32	0
June 15, „	5	34	43	......	Arcturus	14	8	59

TO FIND SHIP MEAN TIME FROM THE HOUR-ANGLE OF THE SUN.

If the heavenly body observed be the sun, its hour-angle will also be *apparent time* at the place if P.M. at the time of observation, and what it wants of 24 hours if A.M. Therefore, to find the corresponding *mean* time, we have only to apply apparent time thus found to the equation of time, with its proper sign, as pointed out in Rule 16, p. 90.

Rule 22. *To find at what time any heavenly body will pass the meridian.*

1. Take out of the *Nautical Almanac* the right ascension of the heavenly body, and also the right ascension of the mean sun for noon of the given day.

2. From the right ascension of the heavenly body (increased if necessary by 24 hours) subtract the right ascension of the mean sun; the remainder is mean time at the ship nearly.

3. Apply the longitude in time, and thus get a Greenwich date; with this Greenwich date correct the right ascension of mean sun, and the right ascension of the heavenly body if necessary.

4. Then from the right ascension of the star subtract the right ascension of the mean sun thus corrected; the remainder is the mean time when the heavenly body is on the meridian.

As in the last problem, the table of acceleration for correcting the R. A. of mean sun ought to have been entered with the correct mean time; but the error in this case is inappreciable.

EXAMPLE.

265. At what time will Sirius pass the meridian of a place in long. 68° 30′ W. on Nov. 20, 1845?

				R. A. mean sun.							
Star's R. A.+24	30^h	38^m	23^s	Nov. 20.....	15^h	57^m	26^s	Star's R. A. ..	30^h	38^m	23^s
R. A. mean sun	15	57	26	cor. 19^h.....		3	7·3	R.A. mean sun	16	0	36
ship M.T. nearly	14	40	57	„ 15^m.....			2·5	∴ ship M. T..	14	37	47
long. in time ..	4	34		R.A. mean sun	16	0	35·8				
Gr. Nov. 20 ...	19	15									

Therefore the transit of Sirius is at 14^h 37^m 47^s on Nov. 20, or at 2^h 37^m 47^s A.M. on Nov. 21.

To find at what time it will pass the meridian on the morning of Nov. 20, we must evidently begin one day back, and take out the right ascension of the mean sun for Nov. 19.

266. At what time will α Pegasi pass the meridian of Portsmouth, long. 1° 6′ W., on Nov. 25, 1853? *Ans.* Nov. 25, $6^h\ 38^m\ 58^s$.

267. At what time will the star Regulus (α Leonis) pass the meridian of Land's End, long. 5° 42′ W., on May 30, 1845? *Ans.* May 30, $5^h\ 27^m\ 45^s$ P.M.

268. At what time will Antares pass the meridian of Portsmouth, long. 1° 6′ W., on Aug. 20, 1845? *Ans.* Aug. 20, $6^h\ 24^m\ 11^s$.

269. At what time will α Leonis pass the meridian of Lisbon, long. 9° 8′ W., on June 4, 1846? *Ans.* June 4, $5^h\ 9^m\ 4^s$.

270. At what time will the star Antares pass the meridian of Copenhagen, long. 12° 35′ E., on Aug. 20, 1846? *Ans.* Aug. 20, $6^h\ 25^m\ 21^s$.

271. At what time will the star Fomalhaut pass the meridian of Calcutta, long. 88° 26′ E., on Nov. 20, 1846? *Ans.* Nov. 20, $6^h\ 52^m\ 34^s$.

Elements from Nautical Almanac.

	Right ascension mean sun.			Right asc. of star.		
Nov. 25, 1853	16^h	17^m	22^s	22^h	57^m	26^s
May 30, 1845	4	31	25	10	0	9
Aug. 20, „	9	54	43	16	19	58
June 4, 1846	4	50	11	10	0	11
Aug. 20, „	9	53	45	16	20	2
Nov. 20, „	15	56	28	22	49	11

To find the meridian zenith distance of a heavenly body, or how far it will pass north or south of zenith:

1. Take out the declination, and correct it, if necessary, for the Greenwich date.

2. Under the latitude of the place put the declination, with their proper names N. or S.

3. If the names are *alike* (both north or both south), take the difference and mark it with the common name of the latitude and declination, if the declination be greater than the latitude, otherwise on the contrary name.

4. If the names are *unlike* (one north and one south), take the sum and mark it with the name of the declination.

5. The result will be the meridian distance of the heavenly body from the zenith N. or S., according as the result was marked N. or S.

EXAMPLE.

272. In latitude 25° N. find how far north or south of the zenith the

following heavenly bodies will pass the meridian, their declinations being 10° N., 30° N., 10° S., and 50° S. respectively:

(1.)	(2.)	(3.)	(4.)
lat. ...25° N.	lat. ...25° N.	lat. ...25° N.	lat. ...25° N.
decl....10 N.	decl....30 N.	decl....10 S.	decl....50 S.
diff....15 S.	diff. ... 5 N.	sum ...35 S.	sum ...75 S.

273. At what time will α Columbæ pass the meridian of a place in lat. 42° 20′ S. and long. 54° 40′ W. on May 10, 1856, and at what distance N. or S. of the zenith? *Ans.* $2^h\ 19^m$; 8° 11′ N. of zenith.

274. At what time will Sirius pass the meridian of a place in latitude 61° N. and long. 10° W. on March 16, 1860, and at what distance N. or S. of the zenith? *Ans.* $7^h\ 0^m$; 77° 32′ S. of zenith.

Elements from Nautical Almanac.

	R. A. mean sun.	Star's R. A.	Star's decl.
May 10......	$3^h\ 13^m\ 52^s$	$5^h\ 34^m\ 25^s$	34° 9′ 12″ S.
Mar. 16......	23 37 10	6 39 0	16 31 46 S.

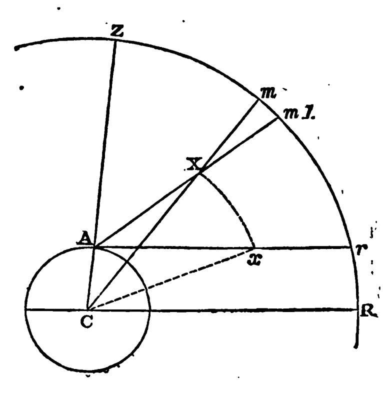
We will conclude this chapter by giving brief explanations of some of the principal corrections required for reducing the observations used for finding the latitude, longitude, time at the ship, and variation of the compass—the subjects of the next chapter.

CORRECTIONS OF THE OBSERVED ALTITUDE OF A HEAVENLY BODY.

(79.) The altitude observed at sea by means of the sextant is called the observed or apparent altitude. To obtain the *true* altitude, or that defined in p. 68, we must apply to the observed altitude (in addition to the index error of the instrument itself) several corrections, the principal of which are the *parallax in altitude*, *refraction*, and *dip*.

CORRECTION FOR PARALLAX IN ALTITUDE.

(80.) Let A be the place of the spectator on the surface of the earth, C the center, Z the zenith, X a heavenly body, and zmr the celestial concave.



Through X draw the two straight lines A X m_1 and C X m to the celestial concave. Then m_1 is the observed or apparent place, and m the true place of the heavenly body X.

Draw A r, a tangent to the earth's surface, at A; draw also C R through the center parallel to A r; then con-

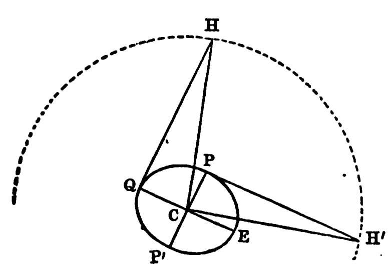
sidering the infinite distance of the points r and R from the earth, the earth's semidiameter AC will subtend no angle at r or R, and Ar may be conceived to coincide with CR, and therefore the arc rR=0. The *observed* altitude of X (without reckoning at present refraction) is measured by the arc $m_1 r$, and its true altitude by mR$=mr$. The difference mm_1 between the true and apparent altitudes, or the angle AXC, is called the *parallax in altitude.*

It appears from the figure that the effect of parallax is to *depress* bodies, so that the true altitude mR is greater on this account than the apparent altitude m_1R, and that the true altitude may be obtained by *adding* the parallax in altitude to the observed altitude.

If x be the same body when in the horizon, the angle AxC is called its *horizontal parallax.*

(81.) It is also evident from the figure that the parallax of a heavenly body is greatest when in the horizon, and that it diminishes to zero in the zenith; that the parallax for different bodies will differ, depending on their distance from the spectator; that the nearer the body is to the earth, the greater will be its parallax: thus the moon's parallax is the greatest of any of the heavenly bodies: the fixed stars, with perhaps a few exceptions, are at such an immense distance, that the earth dwindles to a point so indefinitely small that the radius of the earth AC subtends no measurable angle at a star; hence the fixed stars are considered to have no parallax.

Since the form of the earth is an oblate spheroid, the equatorial diameter being about 26 miles longer than the polar diameter or axis, the horizontal parallax of a heavenly body, as observed from some place on the equator, will be greater than the horizontal parallax of the same heavenly body if observed from the poles of the earth.



For let Q be a spectator at the equator, and H a heavenly body in his horizon, then the angle H is the equatorial horizontal parallax of the body at H. Similarly to a spectator at P, the pole of the earth, the horizontal parallax of the same body would be H′, which is evidently less than H, since it is subtended by a smaller radius of the earth; thus it appears from the figure that the horizontal parallax is greatest at the equator, and that it diminishes as the latitude increases. The moon's horizontal parallax put down in the *Nautical Almanac* is the *equatorial* horizontal parallax. To find the horizontal parallax for any other place a correction (see *Nav.* Part II. p. 123) must be applied, which is evidently subtractive: this correction is seldom made in the common problems of Navigation: in finding the longitude by occultations or solar eclipses, it ought not to be omitted. It is inserted in most collections of Nautical Tables.

(82.) The correction for parallax in altitude for the sun, moon, and planets has been calculated and formed into tables, so that this correction may be taken out by inspection. The tables are constructed from the following formula (*Nav.* Part II. p. 127):

Parallax in alt. = horizontal parallax × cos. altitude (corrected for refraction).

EXAMPLES.

275. Given the apparent altitude of the moon's center = 72° 42′ 15″ (cor. for ref. 18″ sub.), and horizontal parallax = 58′ 49″: find by table, and also by calculation, the parallax in altitude, and thence the true altitude.

(1.) *By Calculation.*

Obs. alt..	72° 42′ 15″	log. cos. alt.........	9·473304	hor. par..	58′ 49″
ref.	18−	„ hor. par. 3529″	3·547652		60
	72 41 57		3·020956		3529
par. in alt.	17 29+	∴ par. in alt.	1049″		
		or	17′ 29″		
true alt...	72 59 26				

(2.) *By Inman's Tables. Cor. of moon alt., Table* (*w*).

Entering with hor. par. 58′ ......cor. =		16′	57″
„ „ 49″ „ =			14
∴ cor. in alt., which includes refraction =		17	11+
obs. alt......	72	42	15
∴ true alt.......	72	59	26

276. Given the apparent altitude of the sun = 13° 14′ 30″ (cor. for ref. 4′ 3″ sub.), and horizontal parallax 8·8″: find by table, and also by calculation, the parallax in alt., and thence the true altitude.

(1.) *By Calculation.*

Obs. alt....	13° 14′ 30″	log. cos. alt...	9·988297
ref.........	4 3−	„ 8·8″......	0·944483
	13 10 27	„ par. in alt.	0·932780
par. in alt.	8·56+	∴ par. in alt....	8·56″
true alt. ...	13 10 35·6		

(2.) *By Table* (*b*).

Cor. by table ... 8·5″

277. Given the apparent altitude of Mars = 14° 6′ 50″ (cor. for ref. 3′ 50″

sub.), and horizontal parallax 22″: find by table, and also by calculation, the parallax in altitude, and thence the true altitude.

	(1.) *By Calculation.*			(2.) *By Table (b).*	
Obs. alt....	14° 6′ 50″	log. cos. alt...	9·986780	Cor. by table...	21·3″
ref.........	3 50−	„ 22″	1·342423		
	14 3 0	„ par. in alt.	1·329203		
par. in alt.	21·3+	∴ par. in alt....21·3″			
∴ true alt.	14 3 21·3				

CORRECTION FOR REFRACTION.

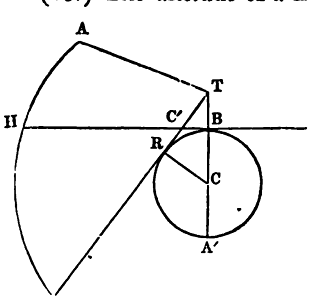
(83.) A ray of light passing obliquely from one medium to another of greater density, is found to deviate from its rectilineal course, and to bend towards a perpendicular to the surface of the denser medium. Hence to a spectator on the earth's surface, a heavenly body seen through the atmosphere appears to be raised, and its true place, on this account, is below its apparent place. Observations show that refraction is greatest when the body is in the horizon (about 34′), and that it diminishes to zero in the zenith. A table of refractions for every altitude has been computed and inserted in the Nautical Tables.

The corrections for parallax and refraction are frequently combined, so that they form one correction, called the *correction in altitude.* The two tables of the correction in altitude for the sun and moon may also be found in most collections of Nautical Tables.

(84.) The investigation of the formula for computing a table of refractions belongs more directly to a work on Optics. In any elementary book on that branch of mathematics the student will find this subject more satisfactorily explained than can be done in the brief space that could be assigned to it in the present work.

CORRECTION FOR DIP.

(85.) The altitude of a heavenly body, observed from a place above the surface of the earth, as on the deck of a ship, will evidently be greater than its altitude observed from the surface, since the observer brings the image of the body down to his horizon, which is lower than the horizon seen from the surface of the sea immediately below him. The difference of altitude from this cause, expressed in minutes and seconds, is called the *dip* of the sea horizon. Let a tangent at B, the point directly beneath the spectator at T, meet the celestial concave at H; from T draw the tangent T*h*,



touching the earth at R. Then, if A be the place of a heavenly body, AH is the altitude observed from B, the surface of the earth, and Ah is the altitude from T. The arc Hh is the dip (very much exaggerated in the figure) for the height TB of the spectator above the surface of the earth, and is evidently subtractive, to get the true altitude. This correction is found in all collections of Nautical Tables.

The table may be constructed from the following formula, *Nav.* Part. II, p. 132):

$$\text{dip} = \cdot 984\sqrt{\text{height of eye}}.$$

EXAMPLES.

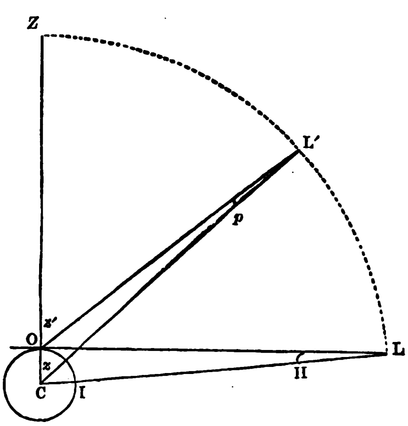
278. Calculate the dip for the height of the eye above the sea$=110$ feet.
Ans. Dip$=\cdot 984\sqrt{110}=\cdot 984\times 10\cdot 488=10'\ 19\cdot 2''$.

279. Find dip for 20 feet and for 30 feet. *Ans.* $4'\ 24''$; $5'\ 23\cdot 4''$.

(86.) The corrections just described are required in almost every example in Nautical Astronomy. Besides these, there are others of not so frequent occurrence, such as the corrections called "*The augmentation of the moon's semidiameter,*" "*The contraction of the moon's semidiameter,*" "*The correction of the moon's meridian passage;*" and in rare observations, such as *occultations*, &c., for determining the longitude, the oblate figure of the earth must be taken into consideration, and corrections called "*The correction of the moon's equatorial horizontal parallax,*" and "*Correction of the latitude for the spheroidal figure of the earth,*" must be applied to several of the terms used in the calculation. These corrections we will now very briefly describe, referring the student for fuller information to *Navigation*, Part II., where these corrections are investigated and useful practical formulæ obtained adapted to logarithms.

AUGMENTATION OF THE MOON'S HORIZONTAL SEMIDIAMETER.

(87.) When the moon is above the horizon, as at L′, its distance OL′ from a spectator at O is less than its distance OL when in the horizon at L. For the distance CL of the earth's center from the moon is about 60 times the earth's radius, therefore CL$=60\times$CI. But as the horizontal parallax is small, OL is nearly equal to CL, and therefore LI is less than LO by nearly the earth's radius. Hence if two observers were placed at O and I, one would see the moon when at L in his horizon, and the other in his zenith; but to the spectator at O the moon would be a little more, and to the spectator at I a little less, than 60 times its radius, and the diameter would



appear to the former about 30″ less than to the latter. It is evident that at any intermediate altitude, as at L′, the distance OL′ is less than OL, and therefore the moon's diameter at L′ would appear to be greater than the true or horizontal diameter at L; that is, the diameter at L′ would be *augmented.* The correction to be made to the moon's horizontal semidiameter on this account is called the *augmentation.* It has been computed for every degree of altitude, and may be found in the Nautical Tables.

In *Navigation*, Part II. p. 134, is given the following formula for calculating the augmentation of the moon's semidiameter.

$$\text{Aug.} = 2\text{R}.\ \text{cosec.}\ (z'-p)\ \cos.\ (z'-\tfrac{1}{2}p)\ \sin.\ \tfrac{1}{2}p$$

where R=horizontal semidiameter, z'=apparent zenith distance,

and p=parallax in altitude = hor. par. × cos. app. alt. (*Navigation*, Part II. p. 125.)

EXAMPLE.

280. Calculate the augmentation of the moon's horizontal semidiameter when the apparent altitude of the center is 32° 42′, the horizontal parallax being 54′ 42·5″, and horizontal semi. (in *Nautical Almanac*) 14′ 56″.

$$\begin{array}{rrrl}
\text{R}= & 14' & 56'' & =896'' \\
z'=57^\circ & 18 & 0 & \\
p=\text{par. in alt.} & & & \\
=\ 0^\circ & 46' & 3'' & \\
z'=57 & 18 & 0 & \\
\hline
\therefore z'-p=56 & 32 & 0 & \\
\tfrac{1}{2}p=\ 0 & 23 & 0 & \\
z'=57 & 18 & 0 & \\
\hline
\therefore z'-\tfrac{1}{2}p=56 & 55 & 0 &
\end{array}$$

1. To find par. in alt.

log. 3282·5	3·516205
„ cos. 32° 42′	9·925069
„ par. in alt.	3·441274

∴ par. in alt. = 2763″
= 46′ 3″

2. To find augmentation.

log. 2	0·301030
„ R	2·952308
„ cosec. $(z'-p)$	0·081170
„ cos. $(z'-\frac{1}{2}p)$	9·725219
„ sin. $\frac{1}{2}p$	7·825451
„ aug.	0·885178

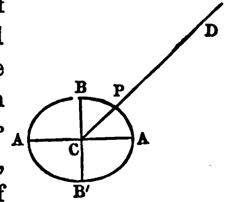
∴ augmentation = 7·68″

281. Calculate the augmentation of the moon's horizontal semidiameter when the apparent altitude of the center is 72° 0′, the horizontal parallax 58′ 43·4″, and horizontal semi. 16′ 0″.

Ans. By formula, 15·86″; by table, 15·8″.

CONTRACTION OF THE MOON'S SEMIDIAMETER ON ACCOUNT OF REFRACTION.

(88.) When the moon is near the horizon its disc assumes an elliptical form, as ABB′, in consequence of the unequal effect of refraction at low altitudes, the lower limb being raised more than the center, and the center more than the upper limb. If, therefore, a contact is made between a distant object in the direction D and some point P on the moon's limb, the contracted semidiameter CP, to be added to the distance to obtain the distance of the centers, must be less than CA the uncontracted semidiameter. This correction has been calculated, and may be found in the Nautical Tables.



The formula investigated in *Navigation*, Part II. p. 135, for computing the contraction, is the following:

$$\text{Contraction}=\text{C} \,.\, \sin.^2\theta,$$

where C = difference of refraction for center and vertex,

θ = inclination of line joining the centers of the two bodies of the horizon.

EXAMPLE.

282. Calculate the correction for contraction of the moon's semidiameter when the altitude = 4° 30′, and the line joining the centers is inclined at an angle of 40°, the moon's semidiameter being 15′ 30″.

Alt. of vertex...4° 45′ 30″...	Refraction 10′ 22″		log. C............1·447158
„ center...4 30 0	„ 10 50		„ sin. 40° ...9·808067
	∴ C= 28		„ sin. 40 ...9·808067
			„ contr. ......1·063292
			∴ contraction = 11·57″

283. Calculate the correction for contraction of the moon's semidiameter when the altitude = 30° 0′, and the line joining the centers is inclined at an angle of 36°: the moon's semi. being 16′ 5″.

Ans. By formula, 0·3459″; by table, 1·0″.

CORRECTION OF MOON'S MERIDIAN PASSAGE.

(89.) The time of the transit of any heavenly body can be found by means of Rule 22, p. 99; but in the case of the moon, the following approximate method of finding the time of her passage over a given meridian may be sometimes used with advantage.

The mean time of the moon's transit for every day at Greenwich is put down in the *Nautical Almanac*. At any place to the east of Greenwich, the time of the transit, owing to the moon's proper motion to the eastward, must take place sooner (independent of that due to the difference of longi-

tude), and to a place to the westward of Greenwich, later than the time recorded in the *Nautical Almanac.* Thus, if we suppose the moon's daily motion to be 60 minutes: to a place 90° to the east of Greenwich the transit will take place 6 hours earlier than that at Greenwich (on account of the difference of longitude) $+\frac{90}{360}$ of 60^{m}, or 15 minutes, due to the moon's motion, supposed equable, to the eastward in the 6 hours before she reaches the meridian of Greenwich. To a place west of the first meridian, a retardation will take place for the same reason.

The moon's daily motion in RA varies between 40^{m} and 60^{m}, so that it would not be difficult to construct a small table of the correction of the transits given in the *Nautical Almanac* for any given longitude: this has accordingly been done, and may be found in Inman's *Nautical Tables*, p. 5.

The moon's daily motion used should be that found by taking the difference between the two transits at Greenwich that happen before and after the one at the place: that is, if the place be in west longitude, the difference should be taken between the transit on the given day and the one following; if in east longitude, that on the given day and the one preceding. By observing this rule, the error arising from the unequal motion of the moon in RA is diminished.

An example or two of finding the time at Greenwich at the transit of the moon over a given meridian will show the use of the table.

284. April 27, required Greenwich mean time nearly at the transit of the moon over the meridian of a place in longitude 50° W.

By *Nautical Almanac*, mer. pass. on 27th...	11^{h}	$46{\cdot}3^{m}$
,, ,, on 28th...	12	32·0
moon's motion in 24 hours...		45·7
correction from table...		6·3 +
∴ time of transit at place...	11	52·6
long. in time...	3	20·0 W.
∴ Greenwich mean time of transit at ship...	15	12·6

285. April 27th, required Greenwich mean time nearly at the transit of the moon over the meridian of a place in longitude 50° E.

	By *Nautical Almanac*, mer. pass. on 27th...	11^{h}	$46{\cdot}3^{m}$
	,, ,, on 26th...	11	2·7
			43·6
Cor. $=\frac{50}{360}\times 43{\cdot}6$	correction...		6·0 −
$=6{\cdot}06^{m}$	transit at place...	11	40·3
or by table $=6{\cdot}0$	long. in time...	3	20·0 E.
	∴ time at Greenwich...	8	20·3

Required the mean time at the place of the moon's meridian passage on July 19 (astronomical day), in longitude 60° W., and on July 27 (astrono-

mical day), in longitude 175° E., having given the following quantities from the *Nautical Almanac:*

Gr. mer. pass. on July 19......	11^h $24{\cdot}3^m$	July 27......	17^h $30{\cdot}1^m$
" " 20......	12 19·2	" 26......	16 49·5

Ans. Mer. pass. at place on July 19 at 11^h $33{\cdot}3^m$

" " 27 at 17 11·1=July 28 at 5^h $11{\cdot}1^m$ A.M.

The corrections for moon's *equatorial horizontal parallax* and for the *figure of the earth* are fully investigated and explained in *Navigation,* Part II. pp. 129, 123.

THE SEXTANT.

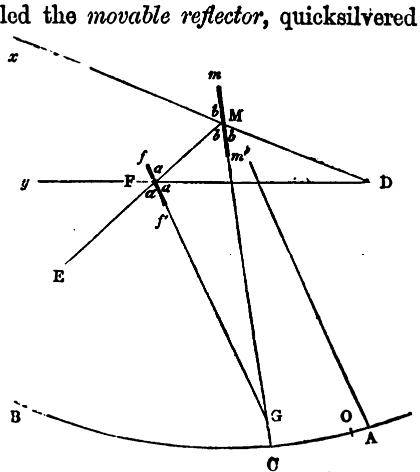
(90.) The student should begin as early as possible to learn to measure angles and take altitudes with the sextant. Before, therefore, we proceed to apply the preceding corrections to the observed altitude of a heavenly body, we will describe briefly the *construction, use,* and *principal adjustments* of this important instrument.

Construction and use of the Sextant.

(91.) The sextant is adapted for measuring angles in any plane whatever; differing in this respect from the theodolite, which is used for observing horizontal and vertical angles only.

The construction of the sextant may be explained by means of the annexed figure.

A small piece of glass, mm', called the *movable reflector,* quicksilvered at the back, is placed at M, the center of the arc AB. It is attached to a *movable radius,* MC, by moving which the plane of its surface produced (supposed perpendicular to the plane of AB) can be made to cut the arc at any required point, C. Another piece of glass, ff', called the *fixed reflector,* also perpendicular to the plane of AB, is placed at F, the lower half only of which is quicksilvered.



Now suppose a ray of light proceeding from the object x in the direction xD to impinge on the surface of the movable reflector M at the angle xMm; then, by a well-known optical law, the ray will be reflected back in the direction MF, making an angle FMm', with the movable reflector equal to the angle xMm. Again, at the fixed reflector F, the ray MF will suffer another

reflection in the direction FD, making, with the reflecting surface ff', the angle $\text{DF}f'$=angle $\text{MF}f$. If we suppose an observer's eye to be placed at D, and another ray of light to proceed from the object y along the same line yFD, the two objects x and y will thus appear to come to the spectator from the same point y; the image of the object x having been transmitted to him from the quicksilvered part of F, and the direct image of y through the upper part of F, which is left transparent for that purpose. The angular distance between x and y, which is the object required to be found, is the angle D, and this angle D will be proved (see below) to be double of the angle AMC, measured by the arc AC; AM being supposed to be drawn parallel to the surface ff' of the fixed reflector F. Hence, if the arc AB, which may be supposed to be the sixth part of a circle, or to contain 60°, be so graduated that it shall contain twice that number, or 120°, then the reading off on the arc AC will be the value of the angle at D: and this is the method adopted in dividing the arc of the sextant.

To observe, therefore, the angle between any two objects, x and y, the observer at D* looks directly at the left-hand object y through the fixed reflector F: he then moves the radius MC, attached to the movable reflector M, in the plane passing through D and the two objects, until he sees the ray proceeding from x in the same direction as the object y. Then the reading off on the arc AC measures the angle at D, the angular distance between the two objects x and y; this may be proved as follows:

Proof that the arc AC *measures the angle at* D *between the two objects* x *and* y.

Produce MF to E and ff' to cut the line MC in G; then the angles xMm and DMG are equal, being vertical angles; also the angle xMm is equal to the reflected angle FMG;† mark therefore these three angles at M with the same letter b; in the same manner the three angles, marked a, formed at F by the reflected ray, may be shown to be equal.

Now in the triangle MDF, the exterior angle EFD=FMD+D, or $2a=2b+\text{D}$.

$$\therefore\ a=b+\tfrac{1}{2}\text{D};$$

also in the triangle FGM, the exterior angle $a=b+\text{FGM}$.

$$\therefore\ b+\tfrac{1}{2}\text{D}=b+\text{FGM},\ \text{or}\ \tfrac{1}{2}\text{D}=\text{FGM}=\text{GMA},$$

since FG is parallel to MA (p. 108).

But the arc AC, which measures the angle GMA, is divided into double the number of degrees due to its length, the divisions commencing at the point A; therefore the reading off on the arc AC measures the angle D, the angular distance between the two objects.

* The observer's eye is seldom exactly at the point D, but in some other point in the line DF; this, however, will make no appreciable difference when the objects x and y are at a considerable distance from the spectator, as the sun or moon.

† See any work on Optics.

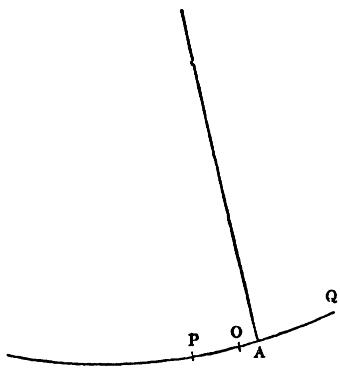
The index correction of the Sextant.

(92.) We have supposed above that the graduations of the arc commenced at A, the point in the arc cut by a line MA, parallel to the fixed reflector F; but this is seldom the case, the zero point of the arc being often a little to the right or left of A.

Let us suppose the graduation to commence at O, to the left of A, then the angle D would be equal to the reading off on OC + the small arc OA. The arc OA is called the *index correction*, the value of which is usually determined by measuring a small angle, as the sun's diameter, off and on the arc, that is, to the right and left of O; to enable us to do which, the divisions of the arc are continued a little to the right of the zero point O.

To find the index correction by measuring the sun's diameter.

Let A be the point on the arc of the sextant through which the movable radius MC (fig. p. 109) passes when its reflector M is parallel to the fixed reflector F; then if the graduation of the arc had commenced at A, it is evident that the reading off on any arc AC (p. 109) would have measured D, the angle between the two objects x and y. But let us suppose that the commencement of the graduation on the arc, or the zero point as it is called, to be at O. Then OA is the error of the instrument or index correction to be determined.



Let P be the point on the arc through which the movable radius MC passes when there is a contact of the direct and reflected limbs of the sun *on* the arc, and Q the point through which MC passes when there is a contact of the two limbs to the right of O, and therefore called *off* the arc; then, since the direct and reflected suns must coincide when the movable radius is at A, the arc AP = arc AQ.

Let a = OP, the measure of sun's diameter on the arc;
b = OQ, the measure of sun's diameter off the arc;
x = OA, the index correction required.

Then, since AP = AQ,

$\therefore \quad x + a = b - x;$

or, $\quad 2x = b - a,$

$\therefore \quad x = \frac{1}{2}(b - a);$

or the index correction is equal to half the difference of the measures of the sun's diameter off and on the arc.

In this case, the index correction OA, or x, must be added to the arc OC, to get the angle between the two bodies x and y; this is evident from the

figure: hence the index correction is said to be additive when the reading on the arc is less than the reading off the arc.

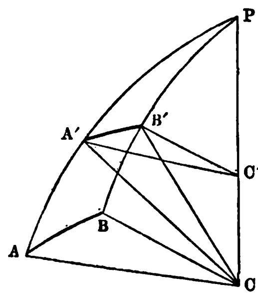
In the same manner it may be shown that if the zero point o is to the right of A, the index correction $x=\frac{1}{2}(a-b)$, and is subtractive; that is, when the reading on the arc is greater than the reading off the arc, when a contact of the true and reflected limbs of the sun is made.

Line of collimation.

(93.) *The line of collimation*, or optical axis of the telescope of the sextant, is the imaginary line joining the centers of the object and eye glass. This line should be parallel to the plane of the instrument.

The visual ray coming from any point of an object, viewed through the telescope of the sextant, passes through the center of the object-glass; and the instrument must be held so that it enters the eye through the center of the eye-glass, or the middle point between the two wires at the eye end of the tube. If this ray, or line of sight, is not parallel to the plane of the instrument, the angle read off on the arc will differ from the angle between the two objects. This will be proved hereafter.

To ascertain whether the line of collimation is parallel to the plane of the arc.



Let A and B be two luminous objects, the latter of which is viewed directly through the middle point between the two wires (supposed to be placed parallel to the plane of the instrument), and the reflected image of the former (A) is brought into contact with B by moving the index along the arc. Now we may ascertain if the tube is properly adjusted, by making a contact at the middle of the upper wire, and then (before any perceptible change, arising from the motion of the two bodies, takes place) bringing the same point of contact to the lower wire: if the two bodies still remain in contact, the instrument may be considered in adjustment; but if this is not the case, the difference must arise from the want of parallelism of the line of collimation. For let a contact be made between the two objects A and B, by bringing the object A, on the right hand, up to B, on the left. Then, if the instrument is properly adjusted, the angle ACB is in the plane of the instrument, and will be measured by its arc. Let now the two objects A and B be supposed to move through equal arcs, AA′ and BB′, in circles vertical to the plane of the instrument, and without moving the sextant, let the axis of the telescope be directed to B in its new position at B′, and thus inclined to the plane of the sextant. This being done, the object A will still be seen to coincide with B, while the angle they subtend at the eye, supposed to be at C, is changed; this may be shown as follows:

Let C be the eye, B and A the two objects, as the sun and moon, ACB the angle between them; the instrument being supposed to be perfectly adjusted, ACB will be in the plane of the instrument, and therefore will be measured by the division on the arc to which the index is set. Now the image of A being supposed in contact with B, let A and B be shifted up AP BP, through the equal arcs AA′ BB′, perpendicular to the plane ACB. In this position of the objects the image of A′ will still be seen in contact with B′. For let the eye be raised up CP, perpendicular to ACB, to C′, so that the plane A′C′B′ is parallel to the plane ACB; then the angle A′C′B′ is equal to ACB. Consequently, the reflectors remaining as before, and being perpendicular to A′C′B′, the image of A must be transferred to A′.

But the eye is actually at C, and views A′ and B′ under the angle A′CB′, which evidently differs from ACB or A′C′B′. It follows, that when the axis of the telescope or of vision is inclined to the plane of the instrument, the image of an object as A′ will be seen coincident with another object B′ when the division which the index is set to differs from the angle between the two objects; and it is manifest that the difference is the same, whether the axis of vision is inclined to the same degree from or towards the plane of the sextant.

Hence is deduced the practical rule for determining whether the line of collimation is parallel to the plane of the instrument, given under the head of the third adjustment of the sextant.

Investigation of a formula for determining the error in the observed angle arising from a given error in the line of collimation.

(94.) In fig. p. 112, let A′CB′ be the true angle between the objects A and B, subtended at the eye of the observer at C, and ACB the instrumental angle, or the one read off on the arc, and AA′ or BB′ the measure of the inclination of the line of collimation of the telescope to the plane of the instrument.

$$\text{Then } \cos. \text{AA}' = \frac{\text{arc A}'\text{B}'}{\text{arc AB}} \quad (\textit{Trig.}\ \text{Part II. art. 69})$$

$$= \frac{\text{chord A}'\text{B}'}{\text{chord AB}}$$

$$= \frac{2 \sin. \frac{1}{2}\text{A}'\text{CB}'}{2 \sin. \frac{1}{2}\text{ACB}}$$

$$\therefore \sin. \tfrac{1}{2}\text{A}'\text{CB}' = \sin. \tfrac{1}{2}\text{ACB} \,.\, \cos. \text{AA}'.$$

This formula determines the angle A′CB′: the difference between which and the angle ACB is the error in the angle observed.

EXAMPLES.

Required the error in the observed angles 90° and 150°, when the inclination of the line of collimation is 1°.

$$\sin. \tfrac{1}{2}\text{A}'\text{CB}' = \sin. \tfrac{1}{2}\text{ACB} \,.\, \cos. \text{AA}'.$$

$\text{ACB}=90^\circ$ $\quad\quad$ $\text{AA}'=1^\circ$ $\quad\quad$ $\text{ACB}=150^\circ$

(1.)		(2.)	
sin. 45°	9·849485	sin. 75°	9·984944
cos. 1°	9·999934	cos. 1°	9·999934
sin. $\frac{1}{2}$ A′C′B′	9·849419	sin. $\frac{1}{2}$ A′CB′	9·984878
∴ $\frac{1}{2}$ A′CB′=	44° 59′ 30″	∴ $\frac{1}{2}$ A′CB′=	74° 58′ 0″
and $\frac{1}{2}$ ACB=	45	and $\frac{1}{2}$ ACB=	75
$\frac{1}{2}$ error=	0 0 30	$\frac{1}{2}$ error=	0 2 0

∴ The error in one case is 1′ 0″; in the other 4′ 0″.

From this it appears that a slight inclination of the line of collimation to the plane of the instrument produces a considerable error in determining the true angle between the two objects; that this error increases as the angle increases; and that the observer should always take care, in nice observations, to make the contact as near the middle point of the field of view as possible.

Adjustments of the Sextant.

(95.) The principal adjustments are the following:

1. The movable reflector M (fig. p. 109) should be perpendicular to the plane of the instrument.

2. The fixed reflector F should be perpendicular to the plane of the instrument.

3. The line of collimation should be parallel to the plane of the instrument.

To examine the adjustments.

First adjustment.—To see if the movable reflector is perpendicular to the plane of the instrument.

Place the movable radius MC near the middle of the arc, as at C (fig. p. 109); turn the face of the instrument upwards, and look obliquely into the reflector M. Then the image of the arc BC will be seen in the reflector M; and if this image appears in one unbroken line with the arc BC itself, the reflector M is perpendicular to the plane of the instrument.

If the reflection of the line BC appears above or below the line BC itself, then the reflector M is out of adjustment, and must be adjusted by certain screws or studs at the back of the reflector. This adjustment, in good instruments, seldom requires to be made; and when it does happen, it is best to send it to the maker to be rectified.

Second adjustment.—To see if the fixed reflector is perpendicular to the plane of the instrument.

Look through the telescope and the fixed reflector F (fig. p. 109) at the

sun, or any other well-defined object; hold the instrument with its face in a horizontal position; bring the index towards the commencement of the divisions, move it gently backwards and forwards until the image of the object is placed as near as possible upon the object itself; then, if the image is found to obliterate or coincide exactly with the object itself, the fixed reflector F is perpendicular to the plane of the instrument.

If any portion of the direct object is seen not coinciding with the image, then the fixed reflector F is not perpendicular to the plane of the instrument; and the adjustment is made by means of a screw, which in some instruments is under the glass, in others behind it, and in others at the side. The screw must be turned gradually till the image is made to coincide with the object. This adjustment is frequently required to be made.

Third adjustment.—To see if the optical axis of the telescope (called the line of collimation) is parallel to the plane of the instrument.

(It is usual to examine this adjustment in practice by making a contact between the sun and moon.)

The telescope being placed in the collar of the sextant, turn the eye-piece round till the two wires are parallel to the plane of the instrument. Bring the darkened image of the sun (when at a considerable distance from the moon, *i. e.* from 90° to 120°) to touch the edge of the moon at the middle point of the upper wire, and then immediately, before any perceptible change in the distance of the two bodies can take place from their own proper motion, bring the point of contact of the two bodies to the lower wire, at which, if they appear in contact, the axis of the telescope may be considered to be parallel to the plane of the instrument; if otherwise, the adjustment is made by means of two screws in the collar—by slackening one and tightening the other. In some instruments, however, these screws are wanting, the adjustment of the parallelism of the tube being supposed to be carefully made before the instrument leaves the maker's hands.

Reading off on the Sextant.

(96.) The following brief directions for reading off will be more readily understood by the student if he place a sextant before him for reference and examination.

It will be seen that the arc is divided into degrees, and (in the best instruments) into the sixths of degrees, or 10 minutes. We will suppose it is an instrument of this kind before us. The *index* lines are cut on the plate at the end of the movable radius, and therefore called the *index plate.* The index itself is the commencement of the reading off on the index plate, and is generally distinguished from the other lines on the plate by a diamond-shaped mark, resembling a spear-head. First let us suppose this index line to coincide exactly with some line on the arc; for example, with the second line to the left of 50°; then the reading off will be 50° 20′, since

each line on the arc represents 10′. Next, let us suppose the index line not to stand exactly at any line on the arc, but somewhere between two, as, in the above example, between the second and third line from 50°; suppose it appeared to be about halfway between the second and third lines (the reader may place it in that position), then it is evident that the reading off would be about 50° 25′. But as this is a rough and imperfect way of estimating the additional minutes and seconds beyond the second division from 50°, the exact value is found by means of the ingenious arrangement of certain lines, called the *vernier*, cut on the index plate to the left of the index line. It will be seen that the divisions of the vernier are nearer to each other than the arc divisions; so that the line on the vernier immediately to the left of the index is somewhat nearer to the corresponding one on the arc than the small space the value of which is to be determined: and it is manifest that it must be nearer by the difference between the width of one division on the arc and one on the index plate. In like manner the second line on the vernier, reckoning from the index line, must be nearer to the corresponding line on the arc by two differences, the third by three, and so on. By carrying the eye along the vernier in this manner, it will be at length seen, by aid of the small reading-off glass, or microscope, attached to the movable radius, that a complete coincidence takes place between a line on the vernier and one on the arc.

Now it is evident, since the lines on the vernier have approached those on the arc through the small space the index is in advance of 20′, that this small space must be equal to as many times the difference of two divisions as there are lines reckoning from the index before the coincidence takes place. Hence, if we know the value of a difference, we shall know the value of the small arc to be measured; and this may be discovered in the following manner. It will be seen, by examining the arc of the sextant before us, that 60 divisions of the vernier just cover or coincide with 59 divisions on the arc; or the difference between a division on the arc and one on the vernier is $\frac{1}{60}$ of a division of the arc: if therefore a division on the arc is 10′, the difference in question will be $\frac{1}{60}$ of 10′, or 10″. Let us now suppose the index to stand between the second and third divisions from the 50°, and that, by carrying the eye along the vernier, we at length find the coincidence of the two lines to take place at the fourth line to the left of the line on the vernier marked 5; then the value of the space to be determined will be 5′ 40″, every sixth division on the vernier being distinguished by a figure indicating minutes. The magnitude of the whole angle is therefore 50° 20′+5′ 40″, or 50° 25′ 40″. The sextant supposed under examination is marked to read off to the nearest 10″; some instruments are graduated to 15″, or 30″, &c.; but the same method of reading off is to be followed as pointed out above.

The graduation of the arc of the sextant is usually continued to the right of 0°, or zero: this is done to enable the observer to take a small angle to

the right as well as to the left of the index line, or zero; as the measure of the sun's diameter off and on the arc to determine the index correction, &c. In this case we shall have to read off on an arc divided from left to right by means of an index, which we must suppose divided from right to left: this, however, is easily done, if we recollect that the line on the index plate marked 10′ must be considered as the commencement of the divisions; 9′ must be considered as 1′; 8′ as 2′; 7′ as 3′; &c.: thus, if the coincidence of the lines on the arc and index plate is at 6′ 40″, we must read this as 3′ 20″, and so on.

These few rules and brief observations on the adjustments and use of the sextant must be considered as introductory to other works written more expressly on the use of astronomical instruments.

Rule 23. *Given a* STAR'S *observed altitude: to find its true altitude.*

The stars are at such a distance from the spectator, that (excepting probably a few) the earth's orbit subtends no angle at the star: hence a star is considered to have no parallax (p. 101); and the only corrections used for reducing the observed altitude to the true are the *index correction*, the *dip*, and *refraction*. Hence this rule.

1. To the observed altitude apply the index correction with its proper sign.
2. Subtract the dip (taken from table of dip of horizon).
3. Subtract the refraction (taken from table of refraction).
4. The result is the true altitude of the star.

EXAMPLE.

286. The observed altitude of Arcturus (α Bootis) was 36° 10′ 20″, index correction +2′ 42″, and height of eye above the sea was 20 feet: required the true altitude.

Observed altitude	36°	10′	20″
index correction		2	42+
	36	13	2
dip		4	24−
star's apparent altitude ...	36	8	38
refraction		1	20−
star's true altitude	36	7	18

287. The observed altitude of Aldebaran (α Tauri) was 13° 4′ 30″, index correction −10′ 40″, and height of eye above the sea was 16 feet: required the true altitude. *Ans.* 12° 45′ 43″.

288. The observed altitude of γ Tauri was 62° 42′ 15″, index correction +0′ 40″, and height of eye above the sea was 20 feet: required the true altitude. *Ans.* 62° 38′ 1″.

289. The observed altitude of α Canis Majoris (Sirius) was 32° 42′ 30″,

index correction was $-3' \, 30''$, and height of eye above the sea was 12 feet: required true altitude. *Ans.* $32° \, 34' \, 4''$.

Rule 24. *Given a* PLANET'S *observed altitude: to find its true altitude.*

The effect of parallax on the true altitude of a heavenly body is to diminish it (p. 101): the correction of parallax in altitude must therefore be added to the observed, to get the true altitude. Hence this rule.

Correct the observed altitude for index correction, dip, and refraction, as in 1, 2, 3 (p. 117).

4. To the result add the parallax in altitude (taken out of the table of parallax in altitude of sun and planets).

5. The result is the true altitude of the planet.

EXAMPLE.

290. January 4th, 1848, the observed altitude of Mars was $21° \, 41' \, 10''$, index correction $+2' \, 42''$, and height of the eye above the sea 24 feet, horizontal parallax (in *Nautical Almanac*) being $10{\cdot}1''$: required the true altitude.

Observed altitude...........	21°	41′	10″
index correction		2	42+
	21	43	52
dip		4	49−
	21	39	3
refraction.....................		2	26−
	21	36	37
parallax in altitude			9+
true altitude..................	21	36	46

291. Jan 24, 1848, the observed altitude of Mars was $9° \, 8' \, 30''$, index correction $-3' \, 45''$, and height of eye above the sea 16 feet: required the true altitude. The horizontal parallax from *Nautical Almanac* was $8{\cdot}3''$. *Ans.* $8° \, 55' \, 3''$.

292. Feb. 3, 1848, the observed altitude of Venus was $25° \, 8' \, 30''$, index correction $-10' \, 50''$, and height of eye above the sea 12 feet: required the true altitude. The horizontal parallax from *Nautical Almanac* was $8{\cdot}1''$. *Ans.* $24° \, 52' \, 17''$.

293. Jan. 30, 1848, the observed altitude of Jupiter was $10° \, 20' \, 10''$, the index correction was $+0' \, 14''$, and height of eye above the sea 18 feet: required the true altitude, the horizontal parallax in *Nautical Almanac* being $2{\cdot}0''$. *Ans.* $10° \, 11' \, 3''$.

Rule 25. *Given the* SUN'S *observed altitude: to find the true altitude.*

The true altitude of the sun's center is found by observing the altitude

of either the upper or lower limb, and then subtracting or adding the semidiameter taken from the *Nautical Almanac;* the other corrections, namely, for index correction, dip, refraction, and parallax, being made as in the preceding rules. In some of the nautical tables, the two corrections for refraction and parallax of the sun are combined in one table, and called the "correction in altitude of the sun." Hence this rule.

1. Correct the observed altitude for index correction and dip, as in articles 1, 2 (p. 101).

2. To this add the sun's semidiameter, if the altitude of the lower limb is observed; but subtract if the upper limb is observed; the result is the apparent altitude of the sun's center.

3. Subtract the refraction and add the parallax taken from the proper tables; or rather take out the "correction in altitude of the sun," and subtract it.

4. The remainder is the sun's true altitude.

EXAMPLE.

294. The observed altitude of the sun's lower limb (L. L.) was 47° 32′ 15″, the index correction was +2′ 10″, and the height of the eye above the sea 15 feet: required the true altitude of the sun's center, the semidiameter in *Nautical Almanac* being 15′ 49″.

Observed altitude	47°	32′	15″
index correction		2	10+
	47	34	25
dip		3	49−
	47	30	36
semidiameter		15	49+
apparent altitude............	47	46	25
correction in altitude			47−
true altitude..................	47	45	38

295. The observed altitude of the sun's L. L. was 48° 30′ 15″, index correction —2′ 50″, and height of eye above the sea 15 feet: required the true altitude, the semidiameter being 15′ 55″. *Ans.* 48° 38′ 46″.

296. The observed altitude of the sun's L. L. was 40° 42′ 16″, index correction +5′ 10″, and height of eye above the sea 20 feet: required the true altitude, the semidiameter being 16′ 4″. *Ans.* 40° 58′ 6″.

297. The observed altitude of the sun's upper limb (U. L.) was 55° 57′ 42″ index correction −3′ 40″, height of eye above the sea 19 feet: required the true altitude, the semidiameter being 16′ 6″. *Ans.* 55° 33′ 4″.

298. The observed altitude of the sun's L. L. was 39° 25′ 15″, index correction −3′ 15″, height of eye above the sea was 15 feet: required the true altitude, the semidiameter being 16′ 3″. *Ans.* 39° 33′ 11″.

Rule 26. *Given the* MOON'S *observed altitude: to find the true altitude.*

The moon's horizontal parallax and semidiameter change so perceptibly, that they cannot be considered (as in the corresponding case of the sun) to be constant for 24 hours. The parallax and semidiameter taken out of the *Nautical Almanac* must therefore be corrected for the Greenwich date in order to find the horizontal parallax and horizontal semidiameter at the time of the observation. Moreover, since the moon is nearer the earth when observed than when it was in the horizon, the horizontal semidiameter must also be corrected for augmentation (p. 106). The correction of the moon's apparent altitude for parallax and refraction is found inserted in most of the nautical tables: it is entered with the corrected horizontal parallax at top, and the apparent altitude at the side. Hence this rule.

1. Get a Greenwich date.

2. Correct the moon's semidiameter and horizontal parallax, taken from the *Nautical Almanac*, for the Greenwich date (p. 74).

3. Add to the semidiameter the augmentation, taken from the table of augmentation.

4. Correct the observed altitude for index correction, dip, and semidiameter, as in the preceding rules (p. 117, 119).

5. Add the moon's correction in altitude, taken out of the proper table.

6. The result is the moon's true altitude.

Note. The moon's correction in altitude may be found by *calculation* by the following formula (*Nav.* Part. II. p. 127):

Parallax in altitude=horizontal parallax × cos. app. alt. (corrected for refraction).

EXAMPLE.

299. April 7, 1853, at $4^h\ 47^m$ P.M. mean time nearly, in long. 10° W., the observed altitude of the moon's L. L. was 72° 15′ 0″, the index correction was −4′ 20″, and height of eye above the sea 15 feet: required the true altitude.

			Moon's semi.			Moon's hor. par.	
Ship, April 7 . .	4^h 47^m	7th, at noon .	15′ 40·7″	noon		57′ 32·0″	
long. in time . .	0 40 W.	„ mid. .	15 45·8	mid.		57 50·8	
Gr., April 7 . .	5 27		5·1			18·8	
		0·34279			0·34279		
		3 32585			2·75927		
		3·66864	2·3		3·10206	8·5	
			15 43·0			57 40·5	
		aug. .	15·2				
			15 58·2				

True alt. By Inman's table (*w*).				True alt. By Calculation.					
Obs. alt.. .	72°	15′	0″	Obs. alt. .	72°	15′	0″		
in cor. . .		4	20−	in cor. . .		4	20−		
	72	10	40		72	10	40	57′ 40·5″	
dip. . . .		3	49−	dip. . . .		3	49	60	
	72	6	51		72	6	51	3460·5=hor. par. in seconds.	
semi.. . .		15	58	semi. . .		15	58		
	72	22	49		72	22	49		
cor. for 57′.		16	57	ref. . . .			19−		
„ 40″			12		72	22	30	cos. alt.. . . .	9·481135
∴ true alt. .	72	39	58	par. in alt.		17	28+	hor. par. . . .	3·539139
				∴ true alt.	72	39	58		3·020274
								par. in alt.. . .	1048″
									or 17′ 28″

300. July 12, 1848, at $9^h\ 18^m$ P.M. mean time nearly, in long. 44° 40′ W., the observed altitude of the moon's L. L. was 27° 56′ 40″, the index correction +2′ 20″, and height of eye above the sea 20 feet: required the true altitude. *Ans.* 28° 56′ 11″.

301. May 15, 1848, at $10^h\ 25^m$ P.M. mean time nearly, in long. 55° 40′ W., the observed altitude of the moon's L. L. was 21° 14′ 10″, the index correction +2′ 20″, and height of eye above the sea 15 feet: required the true altitude. *Ans.* 22° 15′ 15″.

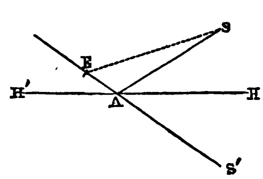
302. May 15, 1848, at $10^h\ 22^m$ P.M. mean time nearly, in long. 41° 30′ W., the observed altitude of the moon's U. L. was 45° 20′ 30″, the index correction +4′ 10″, and height of eye above the sea 20 feet: required the true altitude. *Ans.* 45° 42′ 32″.

Elements from Nautical Almanac.

	Moon's semidiameter.			Moon's horizontal parallax.		
July 12, mid......	14′	55·9″	....	mid......	54′	47·8″
„ 13, noon......	14	59·3	....	noon......	55	0·3
May 15, mid.......	14	41·1	....	mid......	53	57·0
„ 16, noon......	14	42·3	....	noon......	53	57·7

THE ARTIFICIAL HORIZON.

(97.) When the altitude of a heavenly body is observed by means of an artificial horizon, the reading off on the instrument will be the angular distance between the heavenly body and its image in the artificial horizon, and this will be double the altitude as observed from the true horizon. This will be easily seen by the following figure. Let SA, a ray of light proceeding from the body at S, be reflected by means of an artificial horizon placed at A, in the line AE. Then, if the spec-



tator's eye is in the line AE, as at E, the image of the body will appear in the direction EA coming from a point S′ below the horizon A. Now the observer is supposed to be placed so near A that the distance EA is inappreciable when compared with the distance AS of the heavenly body, that is, the angle observed between S and S′, namely, SES′, may be considered to be =SAS′, and this angle SAS′ is manifestly double SAH, the altitude above the horizontal plane HH. For by the principles of Optics it is proved that the angle SAH is equal to EAH′, which is equal to the vertical or opposite angle S′AH, that is, the horizontal line AH bisects the angle observed. Hence the following rule for finding the true altitude from an observed altitude in the artificial horizon.

Rule 27. *Given the observed altitude of a heavenly body in an artificial horizon: to find the true altitude.*

1. Correct the observed altitude for index correction.
2. Half of the result will be the apparent altitude of the point observed.
3. Then proceed as in the preceding rules to find the true altitude.

303. The observed altitude of the sun's lower limb in an artificial horizon was 98° 14′ 10″, index correction −4′ 10″: required apparent altitude of sun's lower limb.

Observed alt.	98°	14′	10″
in. cor.		4	10−
2)	98	10	0
∴ app. alt. sun's L. L.	49	5	0

304. The observed altitude of moon's L. L. in an artificial horizon was 112° 32′ 15″, index correction +3′ 25″: required apparent altitude of moon's lower limb.

Observed alt.	112°	32′	15″
in. cor.		3	25+
2)	112	35	40
∴ app. alt. moon's L. L.	56	17	50

The corrections for dip, semidiameter, and correction in altitude, are then applied as in the preceding rules to obtain the true altitude.

CHAPTER V.

THE LATITUDE.

(98.) In the following rules for finding the latitude and longitude the earth is considered *as a sphere*;* the meridians will therefore be great circles, and the latitude may be defined to be "the arc of the celestial meridian intercepted between the zenith and the celestial equator."

Rule 28. *The latitude by the meridian altitudes of a circumpolar star.*†

1. Correct the altitudes for index correction, height of eye, refraction, and parallax (or as many of these as are applicable to the case), and thus get the true meridian altitudes.

2. Add together the true meridian altitudes (reckoning from the same point of horizon), and half the result will be the latitude of the spectator.

3. If the heavenly body when passing the meridian above and below the pole is on different sides of the zenith, so that the altitudes are taken from opposite sides of the horizon, subtract the greater altitude from 180°, so as to reduce it to an altitude taken from the same point of the horizon as the other altitude; then proceed as directed by Rule (see Ex. 306).

EXAMPLES.

305. The meridian altitudes of α Ursæ Majoris were observed above and below the north pole to be 74° 10′ 10″ and 32° 42′ 15″ respectively (zenith south at both observations), index correction —2′ 10″, and height of eye above the sea 20 feet: required the latitude.

* The true figure of the earth is more nearly that of an *oblate spheroid*, that is, a solid generated by the rotation of an ellipse about its minor or shorter diameter; but as the major and minor diameters of the earth differ only about 26 miles, it is in Navigation considered, without any practical error, as a globe or sphere.

† A circumpolar star is one whose polar distance is less than the latitude of the spectator: it passes the meridian *above* the horizon, both at its superior and inferior transit.

Obs. alt. above pole	74°	10′	10″					
index cor.		2	10–	Obs. alt. below pole	32°	42′	15″
	74	8	0	index cor.		2	10–
dip		4	24–		32	40	5
	74	3	36	dip		4	24–
ref.			17		32	35	41
true alt. above	74	3	19	ref.		1	31
„ „ below.....	32	34	10	.... true alt. below......	32	34	10
2)	106	37	29				
latitude	53	18	44·5 N.				

306. The meridian altitudes of α Aurigæ (Capella) were observed above and below the north pole to be 81° 10′ 52″ (zenith north of star), and 3° 42′ 52″ (zenith south), index correction –3′ 10″, and height of eye above the sea 14 feet: required the latitude.

Obs. alt. from north point of horizon.	3°	42′	52″	Obs. alt. from south point of horizon.	81°	10′	52″
in. cor...............		3	10–	in. cor...............		3	10–
	3	39	42		81	7	42
dip..................		3	41–	dip.		3	41–
	3	36	1		81	4	1
ref.		12	53–	ref.			9–
true alt.	3	23	8	true alt.	81	3	52
true alt.	98	56	8		180		
2)	102	19	16	∴ tr. alt. from north point of horizon.	98	56	8
∴ latitude	51	9	38				

307. The meridian altitudes of a star were observed above and below the north pole to be 69° 20′ 45″ and 6° 14′ 30″ respectively (zenith south at both observations), index correction –1′ 45″, and height of eye 16 feet: required the latitude. *Ans.* Lat. 37° 37′ 35″ N.

308. The meridian altitudes of a star were observed above and below the north pole to be 85° 10′ 10″ and 10° 10′ 10″ respectively (zenith south at both observations), index correction –2′ 40″, and height of eye 20 feet: required the latitude. *Ans.* Lat. 47° 30′ 24″ N.

309. The meridian altitudes of a star were observed above and below the north pole to be 77° 8′ 10″ (zenith north of star) and 3° 40′ 45″ (zenith south), index correction +1′ 42″, and height of eye 12 feet: required the latitude. *Ans.* Lat. 53° 10′.3″ N.

310. August 12, 1850, the meridian altitudes of a star were observed above and below the south pole to be 85° 14′ 15″ (zenith south) and 4° 52′ 0″ (zenith north), index correction −8′ 14″, and height of eye above the sea was 30 feet: required the latitude. *Ans.* Lat. 49° 43′ 39″ S.

Rule 29 (*using sea horizon*). *The latitude by the meridian altitude of the* SUN, *and its declination.*

1. Find a Greenwich date in apparent time; namely, by adding the long. in time to $0^h\ 0^m$ when W., and subtracting it from 24^h (putting the day one back) when the long. is E.

2. By means of the *Nautical Almanac* find the sun's declination for this date (p. 77). Take out also the sun's semidiameter, which is to be added to the apparent altitude when the lower limb is observed, and subtracted when upper limb is observed.

3. Correct the observed altitude for index correction, dip, semidiameter, and correction in alt. (=refraction−parallax), and thus get the true altitude (p. 119), subtract the true altitude from 90°; the result will be the true zenith distance.

4. Mark the zenith distance N. or S. according as the zenith is north or south of the sun.

5. Add together the declination and zenith distance if they have the *same* names; but take the difference if their names be unlike; the result in each case will be the latitude, of the same name as the greater.

EXAMPLE.

311. April 27, 1853, in long. 87° 42′ W., the observed meridian altitude of the sun's lower limb was 48° 42′ 30″ (zenith north), the index correction was +1′ 42″, and the height of eye above the sea was 18 feet: required the latitude.

Ship, April 27	$0^h\ 0^m$	Sun's decl. (at app. noon).				Obs. alt. . .	48°	42′	30″
long. in time .	5 51 W.	27 . . .	13°	43′	53″N.	index cor. . .		1	42+
Gr., April 27 .	5 51	28 . . .	14	2	57 N.		48	44	12
				19	4	dip.		4	11−
		0·61306					48	40	1
		0·97500				semi.. . . .		15	54+
		1·58806		4	38	app. alt. center	48	55	55
		sun's decl.	13	48	31 N.	cor. in alt. . .			45−
						true alt. . .	48	55	10
							90		
						true zen. dist.	41	4	50 N.
						declination .	13	48	31 N.
						latitude . .	54	53	21 N.

312. January 14, 1853, in long. 72° 42′ W., the observed meridian alti-

titude of the sun's L. L. was 32° 42′ 10″ (Z. N.), the index correction +2′ 10″, and height of eye above the sea 14 feet: required the latitude.

Ans. Lat. 35° 50′ 34″ N.

313. March 20, 1853, in long. 72° 42′ E., the observed meridian altitude of the sun's L. L. was 45° 4′ 20″ (Z. S.), index correction −3′ 4″, and height of eye above the sea 20 feet: required the latitude.

Ans. Lat. 44° 56′ 54″ S.

314. July 4, 1853, in long. 100° 0′ W., the observed meridian altitude of the sun's L. L. was 62° 8′ 7″ (Z. N.), index correction −3′ 0″, and height of eye above the sea 15 feet: required the latitude.

Ans. Lat. 50° 34′ 59″ N.

315. March 21, 1853, in long. 62° 0′ W., the observed meridian altitude of the sun's U. L. was 50° 10′ 5″ (Z. N.), index correction +7′ 10″, and height of eye 14 feet: required the latitude. *Ans.* Lat. 40° 26′ 47″ N.

316. Sept. 24, 1853, in long. 33° 0′ E., the observed meridian altitude of the sun's U. L. was 42° 3′ 15″ (Z. N.), index correction −1′ 4″, and height of eye above the sea 18 feet: required the latitude.

Ans. Lat. 47° 49′ 39″ N.

317. June 3, 1853, in long. 178° 30′ W., the observed meridian altitude of the sun's U. L. was 16° 20′ 0″ (Z. S.), index correction +3′ 30″, and height of eye above the sea 20 feet: required the latitude.

Ans. Lat. 51° 35′ 39″ S.

Elements from Nautical Almanac.

	Sun's declination at apparent noon.		Sun's semidiameter.
Jan.	14...21° 16′ 4″ S.	15...21° 5′ 7″ S.	14...16′ 18″
March	19... 0 27 54 S.	20... 0 4 13 S.	19...16 5
July	4...22 53 8 N.	5...22 47 39 N.	4...15 46
March	21... 0 19 28 N.	22... 0 43 7 N.	21...16 5
Sept.	23... 0 8 3 S.	24... 0 31 28 S.	23...15 59
June	3...22 20 42 N.	4...22 27 50 N.	3...15 48

Meridian altitude by Artificial Horizon.

When the altitude is taken in artificial horizon, correct the observed altitude for index correction, and divide by 2. Then proceed as before.

EXAMPLES.

318. Oct. 21, 1853, in long. 1° 6′ W., observed the meridian altitude of

the sun's lower limb (in quicksilver horizon) to be 56° 14′ 0″ (Z. N.), index correction —0′ 10″: required the latitude.

Ship, Oct. 21 .	0h 0m	Sun's decl. (for app. noon).				Obs. alt. . . .		56°	14′	0″
long. in time .	4 W.	21 . . .	10°	35′	22″ S.	in. cor. . . .			0	10—
Gr., Oct. 21 .	0 4	22 . . .	10	56	45 S.		2)	56	13	50
		2·55630		21	23 S.			28	6	55
		0·92520				semi.			16	6+
		3·48150 .			3	app. alt. centre		28	23	1
		sun's decl..	10	35	25 S.	cor. in alt. . .			1	40—
						true alt. . . .		28	21	21
								90		
						true zen. dist. .		61	38	39 N.
						declination . .		10	35	25 S.
						latitude . . .		51	3	14 N.

319. Oct. 9, 1853, in long. 19° 20′ W., the observed meridian altitude of the sun's lower limb (in artificial horizon) was 44° 30′ 15″ (Z. S.), index correction —2′ 10″: required the latitude. *Ans.* Lat. 73° 53′ 28″ S.

320. June 10, 1853, in long. 23° 40′ E., the observed meridian altitude of the sun's lower limb (in quicksilver horizon) was 72° 15′ 20″ (Z. N.), index correction +4′ 5″: required the latitude. *Ans.* Lat. 76° 37′ 45″ N.

321. Aug. 7, 1853, in long. 62° 11′ E., the observed meridian altitude of sun's upper limb (in artificial horizon) was 83° 30′ 0″ (Z. N.), the index correction —3′ 15″: required the latitude. *Ans.* Lat. 65° 0′ 22″ N.

322. May 3, 1853, in long. 14° 20′ W., the observed meridian altitude of sun's upper limb (in artificial horizon) was 30° 2′ 30″ (Z. S.), index correction —1′ 15″: required the latitude. *Ans.* Lat. 59° 34′ 14″ S.

323. July 17, 1853, in long. 72° 30′ E., the observed meridian altitude of sun's upper limb (in artificial horizon) was 52° 30′ 0″ (Z. N.), index correction +2′ 10″: required the latitude. *Ans.* Lat. 85° 15′ 16″ N.

Elements from Nautical Almanac.

	Sun's declination.								Sun's semi.		
Oct. 9 ...	6°	20′	10″S.	10 ...	6°	42′	58″S.	9 ...	16′	4″
June 9 ...	22	57	36 N.	10 ...	23	2	20 N.	9 ...	15	46
Aug. 6 ...	16	40	46 N.	7 ...	16	24	4 N.	6 ...	15	48
May 3 ...	15	43	50 N.	4 ...	16	1	18 N.	3 ...	15	53
July 16 ...	21	21	46 N.	17 ...	21	11	43 N.	16 ...	15	46

Rule 30. *The latitude by the meridian altitude of the* MOON, *and its declination, &c.*

Since the moon's declination, &c., are given in the *Nautical Almanac* for Greenwich *mean* noon, we must get a Greenwich date in *mean* time.

1. Find a Greenwich date in mean time.*

2. By means of the *Nautical Almanac* find for this date the moon's declination, moon's semidiameter, and moon's horizontal parallax, augmenting the moon's semidiameter for altitude (p. 120).

3. Correct the observed altitude for index correction, dip, semidiameter, and parallax and refraction, and thus get the true altitude; subtract the true altitude from 90°, and thus get the true zenith distance.

4. Mark the zenith distance N. or S. according as the zenith is north or south of the moon.

5. Add together the declination and zenith distance if they have the same names, but take their difference if their names be unlike; the result in each case will be the latitude—in the former of the name of either, in the latter of the name of the greater.

EXAMPLES.

324. November 12, 1853, at $2^h\ 20^m$ P.M. mean time nearly, in longitude 60° 42′ W., observed the meridian altitude of the moon's lower limb to be 30° 30′ 40″ (Z. N.), the index correction +10′ 42″, and height of eye above the sea 16 feet: required the latitude.

Nov. 12, at.........................	$2^h\ 20^m$
long..................................	4 3+
Greenwich, Nov. 12	6 23

Moon's declination.				Moon's semi.			Hor. par.	
Nov. 12, at 6^h...2°	44′	20″ N.		Noon...15′	6·4″.........		55′	19·7″
„ at 7 ...2	57	38 N.		mid.....15	2·7		55	6·4
	13	18			3·7			13·3
0·41642				0·27413		0·27413		
1·13142				3·46522		2·90957		
prop. log. 1·54784	5	6		3·73935	2·0	3·18370		7·1
decl.................. 2	49	26 N.		15	4·4		55	12·6
				aug..........	7·4+			
				15	11·8			

* When the estimated time at ship is given, the Greenwich date is found in the usual way by applying the longitude in time (Rule 5), or the Greenwich date may be found by correcting the moon's transit (see p. 108).

Moon's alt.	30°	30′	40″
in cor.		10	42+
	30	41	22
dip		3	56−
	30	37	26
semi.		15	12+
	30	52	38
cor. in alt.		45	36
			11
true alt.	31	38	25
zenith dist.	58	21	35 N.
declin.	2	49	26 N.
latitude	61	11	1 N.

Or thus; true alt. by calculation (p. 121):

app. alt.	30°	52′	38″
ref.		1	37
	30	51	1
log. cos.		9·933747	
„ 3312·6″ ...		3·520169	
		3·453916	
∴ par in alt.			2844″
or		47′	24″
∴ true alt.	31	38	25

325. January 10, 1853, at 7^h 40^m P.M., mean time nearly, in longitude 5° 30′ E., the observed meridian altitude of the moon's lower limb was 10° 20′ 30″ (Z. N.), the index correction − 2′ 20″, and height of eye 14 feet: required the latitude. *Ans.* Lat. 56° 37′ 46″ N.

326. February 4, 1853, at 5^h 40^m A.M., mean time nearly, in longitude 72° 18′ W., the observed meridian altitude of the moon's lower limb was 40° 20′ 15″ (Z. N.), index correction + 3′ 40″, and height of eye 15 feet: required the latitude. *Ans.* Lat. 25° 17′ 10″ N.

327. March 7, 1853, at 3^h 20^m P.M., mean time nearly, in long. 19° 20′ W., the observed meridian altitude of the moon's lower limb was 19° 17′ 18″ (Z. S.), index correction − 1′ 15″, and height of eye 16 feet: required the latitude. *Ans.* Lat. 88° 0′ 44″ S.

328. July 5, 1853, at 1^h 7^m P.M., mean time nearly, in long. 33° 30″ E., the observed meridian altitude of the moon's upper limb was 25° 42′ 30″ (Z. N.), the index correction + 2′ 15″, and height of eye 20 feet: required the latitude. *Ans.* Lat. 88° 22′ 37″ N.

329. August 12, 1853, at 5^h 4^m A.M., mean time nearly, in longitude 94° 40′ E., the observed meridian altitude of the moon's upper limb was 72° 20′ 0″ (Z. S.), the index correction + 3′ 40″, and height of eye 22 feet: required the latitude. *Ans.* Lat. 31° 53′ 3″ S.

330. December 27, 1853, at 9^h 12^m A.M., mean time nearly, in longitude 15° 20′ W., the observed meridian altitude of the moon's upper limb was 19° 50′ 4″ (Z. S.), the index correction − 0′ 30″, and height of eye above the sea was 24 feet: required the latitude. *Ans.* Lat. 87° 35′ 20″ S.

Elements from Nautical Almanac.

	Moon's declination.				Moon's semi.			Hor. par.	
Jan. 10, at 7^h......	22°	1′	16″S.	noon	16′	0·1″......	58′	36·4″
,, ,, 8	21	55	8 S.	mid.	15	54·2	58	14·9
Feb. 3, at 22	23	20	51 S.	mid.	16	8·2	59	6·1
,, ,, 23	23	24	43 S.	noon	16	6·6	59	0·3
Mar. 7, at 4	18	25	4 S.	noon	15	33·4	56	58·5
,, ,, 5	18	15	51 S.	mid.	15	29·3	56	43·7
July 4, at 22	24	33	11 N.......	mid.	14	50·7	54	22·0
,, ,, 23	24	35	27 N.......	noon	14	52·9	54	30·1
Aug. 11, at 10	14	4	13 S.	noon	16	6·9	59	1·1
,, ,, 11	14	16	46 S.	mid.	16	9·2	59	9·7
Dec. 26, at 22	17	55	16 S.	mid.	16	27·4	60	16·7
,, ,, 23	18	7	6 S.	noon	16	33·2	60	37·6

Rule 31. *The latitude by the meridian altitude of* A FIXED STAR, *and its declination.*

The declination of a fixed star changes so slowly, that we may, without any practical error, take it out of the *Nautical Almanac* by *inspection;* a Greenwich date will therefore be unnecessary.

1. Correct the observed altitude for index correction, dip, and refraction, and thus get the true meridian altitude; subtract this from 90° to obtain the true zenith distance.

2. Mark the same N. or S. according as the zenith is north or south of the star.

3. Take out the star's declination by inspection from the *Nautical Almanac,* and apply it to the true zenith distance in the manner pointed out in Rule 28, and thus get the latitude.

EXAMPLES.

331. Feb. 10, 1853, the observed meridian altitude of α Hydræ was 35° 50′ 40″ (zenith north of star), the index correction was +2′ 10″, and height of eye 0 feet: required the latitude.

Observed altitude.......	35°	50′	40″	
index correction		2	10+	
	35	52	50	
refraction		1	21−	
true altitude	35	51	29	
	90			
true zenith distance	54	8	31 N.	
declination	8	1	29 S.	(*Naut. Alm.*)
latitude	46	7	2 N.	

332. May 21, 1853, the observed meridian altitude of α Bootis was 62° 42′ 10″ (Z. N.), the index correction −4′ 4″, and height of eye 18 feet: required the latitude. *Ans.* Lat. 47° 23′ 32″ N.

333. June 16, 1853, the observed meridian altitude of α Lyræ was 77° 1′ 50″ (Z. N.), index correction +2′ 10″, and height of eye 16 feet: required the latitude. *Ans.* Lat. 51° 39′ 4″ N.

334. May 6, 1853, the observed meridian altitude of α Virginis was 16° 52′ 5″ (Z. N.), index correction +1′ 45″, and height of eye 20 feet: required the latitude. *Ans.* Lat. 62° 50′ 4″ N.

335. Oct. 26, 1853, the observed meridian altitude of α Piscis Australis was 70° 10′ 0″ (Z. S.), the index correction −4′ 5″, and height of eye 10 feet: required the latitude. *Ans.* Lat. 50° 21′ 26″ S.

336. May 10, 1853, the observed meridian altitude of α² Centauri was 10° 4′ 15″ (Z. N.), index correction −2′ 10″, and height of eye 20 feet: required the latitude. *Ans.* Lat. 19° 54′ 9″ N.

337. August 1, 1853, the observed altitude of α Aquilæ was 50° 4′ 15″ (Z. N.), index correction −4′ 10″, and height of eye 14 feet: required the latitude. *Ans.* Lat. 48° 33′ 32″ N.

Elements from Nautical Almanac.

May 21 ...	α Bootis................	Decl.	19°	56′	57″	N.
June 16...	α Lyræ..................	,,	38	38	55	N.
May 6.....	α Virginis.............	,,	10	23	40	S.
Oct. 26....	α Piscis Australis......	,,	30	23	53	S.
May 10...	α² Centauri.............	,,	60	13	31	S.
Aug. 1.....	α Aquilæ	,,	8	29	7	N.

Rule 32. *The latitude by the meridian altitude of a* PLANET, *and its declination.*

1. Find a Greenwich date in mean time.

2. By means of the *Nautical Almanac* find the planet's declination for this date; and when great accuracy is required, take out the planet's semidiameter and horizontal parallax.

3. Correct the observed altitude for index correction, dip, refraction (and if necessary for semidiameter and parallax in altitude), and thus get the true altitude. Subtract the true altitude from 90° to get the true zenith distance.

4. Mark the zenith distance north or south according as the zenith is north or south of the planet.

5. Proceed then as in Rule 28.

EXAMPLE.

338. November 20, 1853, at 6^h 18^m A.M., mean time nearly, in long. 62° 42′ E., observed the meridian altitude of Mars' lower limb to be 52° 10′ 45″ (Z. N.), the index correction + 4′ 0″, and height of eye above the sea 16 feet: required the latitude.

Ship, Nov. 19	18^h	18^m	Planet's semi.......	3″
long. in time	4	11 E.	,, H. P......	6
Greenwich, Nov. 19	14	·7		

Planet's declination.

19	12°	55′	36″ N.
20	12	37	1 N.
		18	35
·23048			
·98615			
1·21663......		10	56
Planet's decl....	12	44	40 N.

Obs. alt..........	52°	10′	45″
in. cor.		4	0 +
	52	14	45
dip.		3	56
	52	10	49
semi.			3
	52	10	52
ref.			45 −
	52	10	7
par. in alt......			4 +
true alt.........	52	10	11
	90		
true zen. dist...	37	49	49 N.
planet's decl....	12	44	40 N.
latitude	50	34	29 N.

If the small corrections of the planet's semidiameter and parallax in altitude are neglected, the above example will be worked thus:

Ship, Nov. 19	18^h	18^m
long in time..................	4	11 E.
Greenwich, Nov. 19	14	7

Planet's declination.

19	12°	55′	36″ N.
20	12	37	1 N.
		18	35
·23048			
·98615			
1·21663		10	56
Planet's decl......	12	44	40 N.

Obs. alt............	52°	10′	45″
in. cor.		4	0 +
	52	14	45
dip.		3	56
	52	10	49
ref.			45 −
true alt............	52	10	4
	90		
true zen. dist. ...	37	49	56 N.
decl.	12	44	40 N.
latitude	50	34	36 N.

EXAMPLES.

339. May 4, 1853, at 2^h 45^m A.M. mean time nearly, in long. 42° 10′ W., the observed meridian altitude of Jupiter's center was 16° 42′ 10″ (Z. N.), index correction + 11′ 42″, and height of eye above the sea 20 feet: required the latitude. *Ans.* Lat. 50° 30′ 38″ N.

340. July 12, 1853, at 9^h 36^m P.M. mean time nearly, in long. 30° 30′ E., the observed meridian altitude of Jupiter's center was 10° 10′ 50″ (Z. N.), the index correction −4′ 4″, and height of eye above the sea 10 feet: required the latitude. *Ans.* Lat. 57° 45′ 37″ N.

341. November 27, 1853, at 6^h 3^m A.M. mean time nearly, in long. 100° 0′ W., the observed meridian altitude of Mars' center was 32° 40′ 10″ (Z. S.), index correction −8′ 10″, and height of eye 16 feet: required the latitude. *Ans.* Lat. 45° 45′ 0″ S.

342. Sept. 15, 1853, at 4^h 20^m A.M. mean time nearly, in long. 10° 6′ W., the observed meridian altitude of Saturn's center was 19° 42′ 10″ (Z. N.), index correction −6′ 45″, and height of eye 12 feet: required the latitude. *Ans.* Lat. 88° 55′ 24″ N.

343. Jan. 12, 1853, at 7^h 9^m P.M. mean time nearly, in long. 32° 0′ W., the observed meridian altitude of Saturn's center was 62° 42′ 10″ (Z. S.), index correction −8′ 10″, and height of eye 20 feet: required the latitude. *Ans.* Lat. 14° 36′ 41″ S.

344. June 7, 1853, at 5^h 40^m P.M. mean time nearly, in long. 72° 30′ E., the observed meridian altitude of Venus was 30° 40′ 10″ (Z. S.), index correction + 4′ 20″, and height of eye 24 feet: required the latitude. *Ans.* Lat. 35° 39′ 30″ S.

Elements from Nautical Almanac.

Jupiter,	decl.	May 3...	22°	43′	11″ S.	May 4...	22°	43′	1″ S.
Jupiter,	„	July 12...	22	16	10 S.	July 13...	22	15	49 S.
Mars,	„	Nov. 27...	11	48	44 N.	Nov. 28...	11	40	44 N.
Saturn,	„	Sept. 14...	18	24	50 N.	Sept. 15...	18	24	38 N.
Saturn,	„	Jan. 12...	12	54	5 N.	Jan. 13...	12	54	24 N.
Venus,	„	June 7...	23	42	15 N.	June 8...	23	48	1 N.

Rule 33. *The latitude by the meridian altitude of a heavenly body* BELOW *the pole, and its declination.*

1. Find the declination at the time of observation.
2. From the observed altitude get the true altitude; then
3. Add 90° to the true altitude, and from the sum subtract the declination; the remainder will be the latitude of the same name as the declination.

345. April 27, 1853, the meridian altitude of *α* Crucis below the south

pole was observed to be 14° 10′ 30″, the index correction was +4′ 4″, and the height of eye 20 feet: required the latitude.

Obs. alt.	14°	10′	30″
in. cor.		4	4+
	14	14	34
dip		4	24−
	14	10	10
ref.		3	47−
true alt.	14	6	23
	90		
	104	6	23
star's decl.	62	17	10 S.
∴ lat.	41	49	13 S.

346. June 18, 1853, at apparent midnight, in long. 100° W., the observed meridian altitude of the sun's lower limb below the north pole was 8° 42′ 10″, the index correction −3′, and height of eye above the sea 14 feet: required the latitude.

Ship, June 18	$12^h\ 0^m$	Sun's decl. (app. noon).				Obs. alt. . .	8°	42′	10″
long. in time	6 40 W.	18 . . .	23°	25′	36″ N.	in. cor. . . .		3	0−
Gr., June 18	18 40	19 . . .	23	26	39 N.		8	39	10
				1	3	dip.		3	41−
		·10915					8	35	29
		2·23408				semi. . . .		15	46+
		2·34323		0	49		8	51	15
		decl. . .	23	26	25 N.	cor. in alt. . .		5	51−
							8	45	24
							90		
							98	45	24
						sun's decl. . .	23	26	25 N.
						∴ lat. . . .	75	18	59 N.

347. Feb. 10, 1853, the meridian altitude of α Argûs below the pole was observed to be 6° 41′ 15″, index correction −2′ 10″, and height of eye above the sea 14 feet: required the latitude. *Ans.* Lat. 43° 50′ 18″ S.

348. January 11, 1853, the observed meridian altitude of α Ursæ Majoris below the pole was 14° 14′ 30″, the index correction −4′ 5″, and height of eye 20 feet: required the latitude. *Ans.* Lat. 41° 29′ 47″ N.

349. April 20, 1853, the observed meridian altitude of η Argûs below the pole was 20° 14′ 15″, the index correction −4′ 5″, and height of eye 10 feet: required the latitude. *Ans.* Lat. 51° 9′ 27″ S.

350. June 1, 1853, in long. 30° 52′ W., the observed meridian altitude

of the sun's lower limb below the pole was 10° 42′ 0″, the index correction +2′ 10″, and height of eye 20 feet: required the latitude.

Ans. Lat. 78° 41′ 0″ N.

351. June 10, 1853, at 2^h 40^m A.M. mean time nearly, in long. 30° W., observed the meridian altitude of the moon's lower limb below the pole to be 14° 30′ 10″, index correction +2′ 45″, and height of eye 14 feet: required the latitude. *Ans.* Lat. 81° 32′ 31″ N.

352. July 1, 1853, at 9^h 30^m P.M. mean time nearly, in long. 62° W., the observed meridian altitude of Mars below the pole was 10° 32′ 30″, index correction −3′ 0″, and height of eye 18 feet: required the latitude.

Ans. Lat. 79° 8′ 32″ N.

Elements from Nautical Almanac.

α Argûs . . Feb. 10, decl. .	52°	37′	14″ S.	Sun's decl., June 1 .	22°	5′	15″ N.
α Ursæ Majoris, Jan. 11, „ . .	62	32	26 N.	„ „ June 2 .	22	13	10 N.
η Argûs . . Apr. 20, „ . .	58	54	59 S.	„ semi. . . .		15	48

	Moon's decl.	Moon's semi.	Moon's h. par.		Planet's decl.
June 9 at 16^h	24° 3′ 51″ N.	mid. 15′ 0·4″	54′ 57·6″	July 1 .	21° 7′ 5″ N.
„ 17	24 0 11 N.	noon 15 4·0	55 11·0	„ 2 .	21 15 9 N.

LATITUDE BY OBSERVATIONS OFF THE MERIDIAN.

In the volume of astronomical problems* by the author will be found several methods for finding the latitude depending on some particular bearing or hour-angle of the heavenly body: as when it bears due east, or when it is in the horizon, or when the hour-angle is 6 hours, &c.; but since it is difficult to determine the precise moment when the heavenly body is in either of these positions, the methods referred to are of little use in practice. Problem 131 in that volume, however, is one from which a useful rule may be derived (*Nav.* Part II. p. 54), as it depends on the declination, altitude, and hour-angle of the heavenly body; and as it requires only the common table of sines, &c., we have selected it as the second method about to be given for finding the latitude near the meridian. The first method is deserving attention, being free from any distinction of cases; it requires, however, the tables of haversines and versines, and that the latitude should be known within a quarter of a degree of the truth, otherwise it may be necessary to repeat a part of the work perhaps more than once: but it is a useful method, and gives very accurate results. The altitude and declination are easily obtained at sea; the hour-angle is only known accurately when the ship time is given, and this is a quantity somewhat difficult to discover independently of an observation: the ship time, however, may always be consi-

* *Problems in Astronomy, &c., and Solutions*, pp. 33, 34, &c. These solutions of nearly 200 astronomical and nautical problems form a useful and interesting introduction to the theory of nautical astronomy.

dered to be known nearly. To render, therefore, a rule for finding the latitude, depending on the declination, altitude, and ship time, of practical value, we must ascertain in what position of a heavenly body an error of a few minutes in the ship time will produce the smallest error in the latitude deduced from it; and this we find will be the case if the observed altitude is taken when the body is *near the meridian* (see *Nav.* Part. II. p. 57). It is for this reason that single altitude observations taken off the meridian for finding the latitude are confined to bodies within half an hour of the meridian, when the time at the ship is uncertain to 3 or 4 minutes.

Another practical rule of more general application is deduced from problems 143 and 144. Two altitudes are taken of the same or different heavenly bodies at the same or at different times, from whence the latitude may be found. This is called the rule by DOUBLE ALTITUDE. In this method of finding the latitude the heavenly bodies need not be close to the meridian, but the effect of any error in the observations will be diminished if, in selecting the bodies to be observed, the difference of their bearings be always greater than the less bearing.

Rule 34. First method (using haversines). *Latitude from an altitude of the sun* NEAR THE MERIDIAN.

1. Find the Greenwich date in mean time.

2. Take out the declination and equation of time for this date, and sun's semidiameter.

3. *To find the sun's hour-angle.* To the Greenwich mean time found as accurately as possible apply the longitude in time, subtracting if west, and adding if east; the result will be ship mean time: to this apply the equation of time with its proper sign to reduce mean time into apparent time; the result will be the sun's hour-angle.

4. Add together the following logarithms:

Constant log. 6·301030.
Log. cosine declination.
Log. cosine estimated latitude.
Log. haversine hour-angle.*

reject 30 in the index, and look for the result as a logarithm, and take out its natural number.

5. Correct the observed altitude for index correction, dip, semidiameter, correction in altitude, and thus get a zenith distance.

6. From the versine of zenith distance subtract the natural number found as above. The remainder will be the versine of a meridian zenith distance, which find from the tables.

* Or, instead of log. haversine, take out twice the log. sine of half the hour-angle (rejecting in this case 40 from the index).

7. Under the meridian zenith distance put the declination, and proceed to find the latitude by one of the preceding rules for finding the latitude by a meridian altitude.

NOTE. If the latitude thus found differ much from the estimated latitude used in the question, the work should be corrected by using the last latitude found, in place of the former one.

EXAMPLES.

353. August 22, 1853, A.M., in latitude by account 50° 48′ N., and long. 1° 6′ W., a chronometer showed $11^h\ 50^m\ 22^s$, error on Greenwich mean time being $40{\cdot}2^s$ fast, when the observed altitude of the sun's lower limb (in artificial horizon) was 101° 14′ 10″ (Z. N.), index correction +30″: required the latitude.

Greenwich date and hour-angle.

Chr. showed	11^h	50^m	22^s A.M.
error, fast..........			40·2
	11	49	41·8
	12		
Gr. date, Aug. 21...	23	49	41·8
long. in time		4	24·0 −
ship mean time	23	45	17·8
equation of time...		2	39·1 +
	23	47	56·9
	24		
∴ hour-angle......	0	12	3·1

Obs. alt. in hor ...	101°	14′	10″
in cor.			30 +
	2)101	14	40
	50	37	20
semi.		15	51
	50	53	11
cor. in alt.			42 −
sun's true alt.	50	52	29
	90		
sun's zen. dist. ...	39	7	31

Sun's decl.

Aug. 21	12°	4′	57″ N.
„ 22	11	44	50 N.
		20	7
·00303			
·95172			
·95475		19	59
∴ sun's decl.	11	44	58 N.

Equation of time.

Aug. 21 ...	2^m	54^s add.
„ 22 ...	2	39
		15
·00303		
2·85733		
2·86036 ...		14·9
	2	39·1 add.

Semi. 15′ 51″

By First Method.

Const. log.	6·301030
log. cos. decl.	9·990803
„ cos. est. lat.	9·800737
„ hav. hour-angle..	6·839449
	2·932019
∴ natural number...	855

vers. ...39° 7′	0224137
31″	95
vers. zen. dist.	0224232
natural number	855
vers. mer. Z. D. ...	0223377
39° 2′	220
51″	157

mer. Z. D. ...	39	2	51 N.
decl.	11	44	58 N.
∴ LATITUDE..	50	47	49 N.

Rule 35. Second method (using sines, &c.). *Latitude by altitude of sun* NEAR THE MERIDIAN.

In *Navigation*, Part II., p. 54, it is shown that if h=hour-angle, p= polar distance, and a=altitude of a heavenly body, then the colatitude $=y\pm x$, where tan. x=cos. h. tan. p and cos. y=sec. p. cos. x, sin. a. From which formulæ the colatitude, and thence the latitude, is easily found, if we attend to the proper algebraic sign of each quantity, as pointed out in *Trigonometry*, Part I., art. 31. We may, however, deduce from these trigonometrical expressions a direct rule, and free from the distinction of cases arising from the use of signs, by modifying the above formulæ as follows:

$$\text{Let } z=90-x, \text{ and the decl.}=90\sim p=d,$$

Then the above formulæ become

$$\cot. z=\cot. d. \cos. h,$$
$$\cos. y=\text{cosec}. d. \sin. z . \sin. a,$$

where the arcs z and y may be looked upon as the approximate declination, and mer. zen. distance respectively, and marked N. or S., as in the rule 29 for latitude by meridian altitude. Hence this Rule.

1. Find Greenwich date, declination, equation of time, hour-angle, and true altitude, as in last Rule.

2. Add together log. cos. hour-angle, and log. cotangent of declination (taking out at the same opening of the tables, and putting a little to the right, the log. cosecant of declination).

3. The sum (rejecting 10 in index) of the two logarithms just added together will be log. cotangent of arc z, which find from the tables, and mark it N. or S., according as the declination is north or south.

4. Under log. cosecant of declination (already taken out) put log. sine of arc z, and log. sine of altitude: the sum of these three logarithms (rejecting 20 in index) will be the log. cosine of arc y, which take out, and mark N. or Z., according as the zenith is north or south of the heavenly body.

5. Under arc z put arc y, and take their sum or difference, according as they have the same or different names: the result will be the latitude required, to be marked north or south, as in the rule for latitude by meridian altitude.

354. May 10, 1853, A.M., in latitude by account 50° 50′ N., and long. 2° 10′ W., a chronometer showed $11^h\ 51^m\ 58^s$, error on Greenwich mean time being $11^m\ 31^s$ fast, when the observed altitude of the sun's lower limb was 56° 19′ 30″ (Z. N.), index correction — 3′ 20″, and height of eye 18 feet: required the latitude.

Greenwich date and hour-angle.

Chr. showed	11ʰ	51ᵐ	58ˢ A.M.	Obs. alt............	56°	19′	30″
error, fast		11	31	in cor.		3	20—
	11	40	27		56	16	10
	12			dep.		4	11—
Gr. date, May 9 ..	23	40	27		56	11	59
long. in time......		8	40—	semi.		15	52+
ship mean time ...	23	31	47		56	27	51
equation of time ..		3	49+	cor. in alt..........			33—
ship app. time	23	35	36	∴ true alt.........	56	27	18
	24						
hour-angle	0	24	24				

Sun's declination.				Equation of time.			Semi.
May 9	17°	24′	37″ N.	May 9 ...	3ᵐ	46·0ˢ add.	15′ 52″
„ 10	17	40	25 N.	„ 10 ...	3	49·0	
·00608		15	48			3·0	
1·05662				cor. ...		3·0	
1·06270..		15	35		3	49·0	
∴ sun's decl.....	17	40	12 N.				

The Second or Direct Method.

cot. z=cot. decl. cos. hour-angle.

cos. y=cosec. decl. sin. z . sin. alt.

log. cot. decl.	0·496782	log. cosec. decl....	0·517773
„ cos. hour-angle...	9·997534	„ sin. z	9·484402
„ cot. z..............	10·494316	„ sin. alt.	9·920876
∴ z	17° 45′ 45″ N.	„ cos. y	9·923051
y.............	33 6 30 N.	∴ y	33° 6′ 30″ N.
∴ latitude......	50 52 15 N.		

355. Nov. 14, 1853, P.M., in lat. by account 87° 41′ S., and long. 1° 0′ W., a chronometer showed 0ʰ 25ᵐ 27ˢ, error on Greenwich mean time being fast 5ᵐ 56·7ˢ, when the observed altitude of the sun's lower limb was 20° 26′ 20″ (Z. S.), index correction −2′ 20″, and height of eye 10 feet: required the latitude. *Ans.* Lat. 87° 42′ 15″ S.

356. June 30, 1853, A.M., in lat. by account 63° 20′ N., and longitude 23° 30′ W., a chronometer showed 11ʰ 30ᵐ 15ˢ, error on Greenwich mean time being 7ᵐ 32ˢ fast, when the observed altitude of the sun's upper limb was 44° 20′ 22″ (Z. N.), index correction +2′ 20″, and height of eye 14 feet: required the latitude. *Ans.* Lat. 63° 21′ N.

357. July 10, 1853, A.M., in lat. by account 57° 24′ N., and longitude 3° 40′ W., a chronometer showed $11^h\ 20^m\ 15^s$, error on Greenwich mean time being $30^m\ 30^s$ slow, when the observed altitude of the sun's lower limb was 54° 17′ 19″ (Z. N.), index correction −2′ 40″, and height of eye 20 feet: required the latitude. *Ans.* Lat. 57° 25′ 25″ N.

358. May 20, 1853, A.M., in lat. by account 79° 48′ N., and longitude 44° 30′ E., a chronometer showed $11^h\ 30^m\ 0^s$, error on Greenwich mean time being $15^m\ 20^s$ slow, when the observed altitude of the sun's lower limb (in artificial horizon) was 54° 30′ 20″ (Z. N.), index correction −4′ 30″: required the latitude. *Ans.* Lat. 79° 48′ 30″ N.

359. June 16, 1853, P.M., in lat. by account 52° 25′ N., and longitude 1° 6′ W., a chronometer showed $1^h\ 2^m\ 9^s$ error on Greenwich mean time being $40^m\ 30^s$ fast, when the observed altitude of the sun's lower limb was 60° 37′ 50″ (Z. N.), index correction −2′ 10″, and height of eye 17 feet: required the latitude. *Ans.* Lat. 52° 24′ 15″ N.

Elements from Nautical Almanac.

	Sun's declination.					Equation of time.			Sun's semi.	
Nov. 14,	18°	18′	5″	S	……	15^m	$22{\cdot}9^s$	to be added …	16′	13″
„ 15,	18	33	30	S	……	15	12·8			
June 29,	23	14	31	N	……	3	3·7	„ subtracted	15	46
„ 30,	23	11	3	N	……	3	15·6			
July 9,	22	21	48	N	……	4	50·8	„ subtracted	15	46
„ 10,	22	14	22	N	……	4	59·5			
May 19,	19	48	45	N	……	3	47·7	„ added …	15	50
„ 20,	20	1	23	N	……	3	44·9			
June 16,	23	22	15	N	……	0	18·8	„ subtracted	15	46
„ 17,	23	24	8	N	……	0	31·6			

Latitude by POLE-STAR *(using Inman's Table).*

The table for correcting the altitude of the pole-star, contained in Inman's *Nautical Tables*, has recently been recalculated, and adapted to the present and several subsequent years. As this table enables us to find the latitude sufficiently near for all ordinary purposes, the practical rule (a proof of which is given in Part II.) is now inserted.

Rule 36. 1. Get a Greenwich date.

2. Take out from the *Nautical Almanac* the right ascension of the mean sun (called there sidereal time), and correct it for the Greenwich date (p. 86).

3. Add together the right ascension of mean sun so corrected to the nearest minute and ship mean time (expressed astronomically).

The result, rejecting 24^h if greater than 24^h, is the meridian right ascension, the argument of the table.

4. Correct the observed altitude of the star for index correction, dip, and refraction, and thus get the true altitude.

5. Enter the table, called the "correction of pole-star," with the meridian right ascension at the side, and with the nearest latitude to that by account at the top, and take out the correction as near as can be estimated, with its proper sign.

6. Apply the correction to the true altitude, and the result will be the latitude required.

EXAMPLE.

360. April 10, 1860, at 1^h 30^m A.M., mean time nearly, in longitude 64° 30′ E., the observed altitude of the pole-star was 52° 30′ 40″, index correction — 1′ 20″, and height of eye above the sea 15 feet: required the latitude.

	h	m
Ship, April 9	13^h	30^m
long. in time	4	18 E.
Green. April 9.........	9	12

	R. A. mean sun.			Obs. alt.		
9th	1^h	11^m	47^s	52°	30′	40″
cor. for 9^h 12^m		1	30		1	20 —
	1	13	17	52	29	20
ship time	13	30			3	49 —
mer. R. A.	14	43		52	25	31
						45 —
				52	24	46
cor. from table				1	19	15 +
latitude				53	44	1 N.

EXAMPLES.

361. June 15, 1860, at 2^h 20^m A.M., mean time nearly, in longitude 10° 20′ W., the observed altitude of α Polaris was 46° 10′ 30″, index correction + 3′ 10″, and height of the eye 19 feet: required the latitude.

Ans. Lat. 45° 52′ N.

362. July 20, 1860, at 11^h 40^m P.M., mean time nearly, in longitude 42° E., the observed altitude of α Polaris was 35° 30′ 40″, index correction — 4′ 10″, and height of eye above the sea 15 feet: required the latitude.

Ans. Lat. 35° 12′ N.

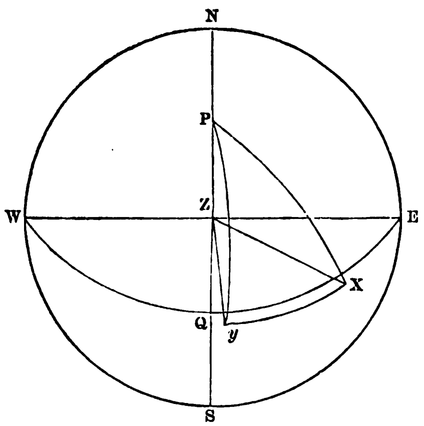
Elements from Nautical Almanac.

R. A. mean sun (called in *N. A.* sidereal time).

June 14th, at noon........................	5^h	32^m
July 20th, „	7	54

LATITUDE BY DOUBLE ALTITUDE.

The most general rule for finding the latitude by a double altitude of a heavenly body is the one selected as the first method; but the labour of reducing the observations is somewhat greater than in the second method,

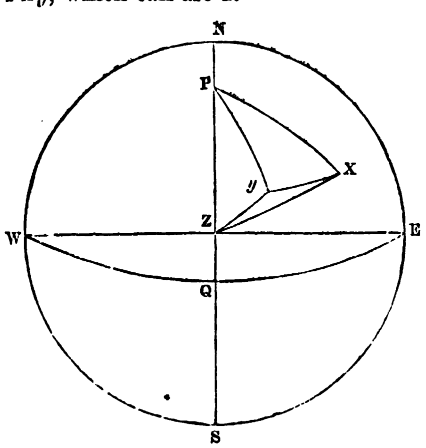


known as Ivory's Rule. The great advantage of the first method is that it may be applied to the same or different heavenly bodies, observed at the same instant or at different times, and that it is the simple application of two rules in Spherical Trigonometry.

Let P be the pole, Z the zenith, X and y the same heavenly body observed at different times; or different heavenly bodies observed at the same instant, or different heavenly bodies observed at different times. Let ZX Zy be their zenith distances. Then in the figure we know by observation ZX and Zy, and from the *Nautical Almanac* we can find the polar distances PX and Py; also by means of the elapsed time as measured by a watch, or from the right ascension of the bodies, or from both, we can compute the polar angle XPy; the colatitude PZ may then by computed in the following manner by the application of the common rules of Spherical Trigonometry.

1. In triangle PyX are given two sides PX, Py and the included angle XPy, to find Xy, which call arc 1.

2. In triangle PXy are given three sides PX, Py and arc 1, to find angle PXy, which call arc 2.



3. In triangle Z X y are given three sides ZX, Zy and arc 1, to find angle ZXy, which call arc 3.

4. Arc 2—arc 3=angle PXZ= arc 4. But if the arc X y drawn through X and y pass when produced between P and Z the pole and the zenith, then it is evident by the annexed figure that the arc 2+arc 3=PXZ or arc 4. If the arc Xy produced pass near Z, the bodies X and y in such a position should not be observed.

Lastly. In triangle P X Z are

given the two sides PX and ZX and arc 4 (namely, the included angle PXZ), to find PZ the colatitude, and thence the latitude.

Correction for run of ship.

If the ship have moved in the interval between the observations, the second altitude will in general differ from what it would have been if both observations had been taken at the same place. On this account it is usual to apply to the first altitude a correction so as to reduce it to what it would have been if taken at the place of the second observation; this quantity is called "the correction for run of the ship," and may be calculated as follows.

When a ship describes an arc on the surface of the sea, the zenith describes a similar arc in the celestial concave: let, therefore, Z be the zenith of the ship at the first observation, Z′ its zenith at the second observation; then arc ZZ′ measures the distance run in the interval. Let S be the place of the heavenly body at the first observation: with center S at distance SZ′, describe an arc cutting SZ, fig. 1, or SZ produced in D, fig. 2; then the triangle ZZ′D being small, may be considered as a right-angled plane triangle, and ZD is the correction to be applied to ZS in order to get Z′S the distance of S from the zenith at the second observation.

Fig. 1.

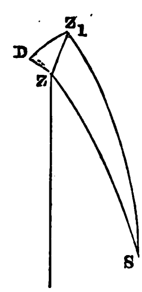


Fig. 2.

Now ZD=ZZ′ cos. Z′ZD=distance run × cos. angle between the direction of the ship's run and the bearing of the sun at the first observation.

This correction ZD may be readily found by means of the traverse table, for since (p. 43),

Diff. lat. = dist. cos. course; if therefore in triangle ZZ′D the angle ZZ′D be considered as the course, and ZZ′ the distance, the correction ZD for run will correspond in the traverse table to the difference of latitude.

The angle Z′ZD is the difference between the course of the ship in the interval and the true bearing of the body, when the run of the ship has been towards the place of the body, as in fig. 1; and what this angle wants of 180° or 16 points when the direction of the ship's run has been from the place of the body, as in fig. 2. In the former case it is manifest that the correction ZD for run must be added to the first observed altitude, and in the second subtracted, in order to get the altitude of the body, the same as it would have been if it had been also observed at the place of the ship at the second observation.

Correction for run. Rule.

1. Enter the traverse table with the distance run as a distance, and the angle (supposed less than 8 points) between the true bearing of the heavenly

body at the first observation and course of the ship, as a course, and take out the corresponding *diff. lat.*, which *add* to the first taken true altitude (the tenths in the diff. lat. being turned into seconds, by multiplying them by 60); the result will be the altitude corrected for run.

2. But if the above angle be greater than 8 points, subtract the same from 16 points, and look out the remainder as a course, and *subtract* the diff. lat. corresponding thereto from the first true altitude; the result will be the altitude corrected for run.

EXAMPLES.

363. The course of the ship was N.W.½W. 10 miles, and bearing of the sun E. by S.: required the correction for the first altitude for run.

The angle between N.W.½W. and E. by S. is 13½ points, subtracting 13½ from 16 points: enter traverse table with the remainder, namely, 2½ as a course and 10 miles as a distance: the corresponding diff. lat. is 8·8′= 8′ 48″ to be *subtracted* from the true altitude.

364. The course of the ship was E.N.E. 25 miles, and bearing of the sun E. by S.: required the correction of the first altitude for run.

The angle between E.N.E. and E. by S. is 3 points; entering traverse table with 3 points as a course, and 25 miles as a distance, the corresponding diff. lat. =20·8′=20′ 48″ to be *added* to the true altitude.

365. The true course of the ship was S.W.½W. 15 miles, and the true bearing of the sun S. by E.½E.: required the correction of the first altitude for run. *Ans.* + 5′ 42″.

366. The true course of the ship was W.½N. 19 miles, and the true bearing of the sun was S. by E.½E.: required the correction for run.

Ans. —7′ 18.″

Rule 37. Double Altitude. First method. (1.) *The latitude by two altitudes of the* sun.

1. From the estimated mean time at the ship at each observation, and the longitude, get two Greenwich dates.

2. By means of the *Nautical Almanac* find the declination for each Greenwich date. Take out also from the *Almanac* the sun's semidiameter.

3. Find the polar distance at each observation by subtracting the declination from 90°, if the estimated latitude and declination are of the same name; or by adding 90° to the declination, if the estimated latitude and declination are of different names.

4. Correct the two observed altitudes for index correction, dip, semidiameter, and correction in altitude.

5. Correct also the first altitude observed for the run of the ship (p. 143).

6. Subtract the true altitudes thus obtained from 90° and thus get the zenith distances.

7. Find the polar angle or elapsed time between the observations, by subtracting the time shown by chronometer at the first observation from the time shown by chronometer (increased if necessary by 12 hours) at second observation.

NOTE. When great accuracy is required, this elapsed time should be corrected for rate of chronometer, and also for the change in the equation of time in the interval; but these corrections are seldom made.

8. *To find arc* 1 (using Inman's Tables). Add together log. sin. polar distance at greater bearing, log. sin. polar distance at lesser bearing, and log. haversine of polar angle (rejecting the tens in the index); and look out the result as a log. haversine; the arc corresponding thereto is arc 1 nearly.*

9. *To find arc* 2. Under arc 1 put polar distance at greater bearing, and take the difference, under which put polar distance at lesser bearing; take the sum and difference of the two last quantities. Add together the log. cosecants of the two first arcs put down, and halves of the log. haversines of the two last arcs put down; the sum, rejecting 10 in index, is the log. haversine of arc 2, which take from the Tables.

10. *To find arc* 3. Under arc 1 put zenith distance at greater bearing, and take the difference, under which put zenith distance at lesser bearing; take the sum and difference of the last two quantities.

Add together the log. cosecants of the two first arcs put down, and halve the log. haversines of the two last arcs put down; the sum, rejecting the tens in index, is the log. haversine of arc 3, which take from the Tables.

11. *To find arc* 4. The difference between arc 2 and arc 3 is arc 4.

NOTE. When the arc joining the places of the sun at the two observations passes, when produced, between the zenith and pole (which the observer may easily discover at the time the observation is taken), then the sum of arcs 2 and 3 is arc 4.

12. *To find arc* 5. Add together log. sin. polar dist. at greater bearing, log. sin. zenith distance at greater bearing, and log. haversine of arc 4; the sum, rejecting 10 in the index, is log. haversine of arc, which take from the Tables, and call arc 5.

Take the difference between the polar distances at the greater bearing, and the zenith distance at greater bearing.

Add together versine of arc 5 and versine of the difference just found; the sum is the versine of the colatitude, which take from the Tables, and subtract from 90°; the result is the latitude required.

NOTE. If the student have only the tables of sines, &c. he may use Rule 41, p. 156, or the Trigonometrical method given in p. 160.

* To find arc 1 correctly. To the versine of arc found as above add the versine of the difference of polar distances: the sum will be the versine of arc 1. But this is rarely necessary to be done.

EXAMPLE.

367. Oct. 11, 1845, in latitude by account 54° N. and long. 83° 15′ W., the following double altitude of the sun was observed:

Mean time nearly.	Chro.	Alt. sun's L. L.	Bearing.
7^h 45^m A.M.	11^h 40^m 15^s	9° 0′ 20″	E.S.E. ¼ E.
10 35 A.M.	2 13 20	25 3 30	S.S.E.

The run of the ship in the interval was S. by W. 15 miles, index correction + 5′ 10″, and the height of eye above the sea was 18 feet: required the true latitude at the second observation.

At greater bearing.

Ship, Oct. 10	19^h	45^m	
long. in time	5	33	W.
Oct. 10	25	18	
Gr. Oct. 11	1	18	

At less bearing.

Ship, Oct. 10	22^h	35	
	5	33	W.
Oct. 10	28	8	
Gr. Oct. 11	4	8	

Decl. at greater bearing.

11	7°	4′	38″	S.
12	7	27	15	S.
		22	37	
1·26627				
·90084				
2·16711 cor.		1	14	
	7	5	52	S.
	90			
N. Pol. dist.	97	5	52	

At greater bearing.

Decl. at less bearing.

11	7°	4′	38″	S.
12	7	27	15	S.
		22	37	
·76391				
·90084				
1·66475 cor.		3	54	
	7	8	32	S.
	90			
N. Pol. dist.	97	8	32	

At less bearing.

Sun's altitude at greater bearing.

Obs. alt.	9°	0′	20″
in cor.		5	10+
	9	5	30
dip		4	11−
	9	1	19
semi		16	3+
	9	17	22
cor. in alt.		5	33−
	9	11	49
cor. for run		2	12+
true alt.	9	14	1
	90		
Z. D.	80	45	59

At greater bearing.

Sun's altitude at less bearing.

Obs. alt.	25°	3′	30″
in cor.		5	10+
	25	8	40
dip		4	11−
	25	4	29
semi		16	3+
	25	20	32
cor. in alt.		1	54−
true alt.	25	18	38
	90		
Z. D.	64	41	22

At less bearing.

To find arc 1.

Chro. times.					
	11h	40m	15s	Sin. P. D. at G. B. ...	9·996661
	14	13	20	sin. P. D. at L. B. ...	9·996617
				hav. pol. angle	9·031223
Pol. angle.........	2	33	5	hav. arc 1	9·024501
				arc 1	37° 58′ 0″

To find arc 2.

Arc 1...............	37°	58′	0″	.... cosec.	0·210982
pol. dist. at G.B. .	97	5	52	.... cosec.	0·003339
diff.	59	7	52		
pol. dist. at L. B.	97	8	32		
sum	156	16	24	.... ½ hav. ...	4·990618
diff.	38	0	40	.... ½ hav.....	4·512779
				Hav. arc 2	9·717718
				arc 2	92° 31′ 45″

To find arc 3.

Arc 1..............	37°	58′	0″	.... cosec.	0·210982
Z. D. at G. B......	80	45	59	.... cosec.	0·005664
diff.	42	47	59		
Z. D. at L. B......	64	41	22		
sum	107	29	21	.... ½ hav.....	4·906540
diff.	21	53	23	.... ½ hav.....	4·278481
				Hav. arc 3	9·401667
				arc 3	60° 17′ 0″

To find arc 4.

Arc 2	92°	31′	45″
arc 3	60	17	0
arc 4	32	14	45
Pol. dist. at G. B..	97°	5′	52″
zen. dist. at G. B..	80	45	59
diff.	16	19	53

To find arc 5.

Sin. pol. dist. at G. B.	9·996661
sin. zen. dist. at G. B.	9·994336
hav. arc 4	8·887148
hav. arc 4	8·878145
arc 5	31° 54′ 15″

Vers. 31° 54′ 15″	0151066			
vers. 16 19 53	0040347			
vers. colat.	0191413			
∴ colat.	36°	2′	31″	
	90			
latitude	53	57	29	N.

368. June 3, 1847, in latitude by account 52° N., and long. 72° E., the following double altitude of the sun was observed :

Mean time nearly.	Chro.	Alt. sun's L. L.	Bearing.
9^h 50^m A.M.	9^h 52^m 28^s	51° 17′ 45″	S.E.b.S.
11 15 A.M.	11 14 29	59 32 15	S.b.E.

The run of the ship in the interval was W. by S. 10 miles, index correction −0′ 40″, and height of eye 12 feet: required the true latitude at the second observation. *Ans.* Arc 1, 18° 57′ 45″; arc 2, 86° 3′ 30″; arc 3, 52° 9′ 15″; lat. 50° 48′ N.

369. April 11, 1847, in latitude by account 56° 20′ N., long. 10° 30′ E., the following double altitude of the sun was observed :

Mean time nearly.	Chro.	Alt. sun's L. L.	Bearing.
11^h 0^m A.M.	7^h 5^m 10^s	40° 10′ 15″	S.S.E.
2 0 P.M.	10 6 10	35 15 40	S.W.b.W.

The run of the ship in the interval was N.N.E. 29 miles, index correction +2′ 10″, and height of eye 18 feet: required the true latitude at the second observation. *Ans.* Arc 1, 44° 45′ 45″; arc 2, 86° 38′ 0″; arc 3, 66° 12′ 30″; lat. 56° 57′ N.

370. April 13, 1847, in latitude by account 41° 20′ N., long. 156° 15′ E., the following double altitude of the sun was observed :

Mean time nearly.	Chro.	Alt. sun's L. L.	Bearing.
10^h 45^m A.M.	7^h 30^m 20^s	53° 0′ 20″	S.E.b.S.
2 45 P.M.	11 29 40	40 59 10	S.W.b.W.

The run of the ship in the interval was S.S.E. 25 miles, index correction −5′ 20″, and height of eye 14 feet: required the true latitude at second observation. *Ans.* Arc 1, 59° 3′ 45″; arc 2, 85° 2′ 30″; arc 3, 43° 51′ 30″; lat. 41° 23′ 15″ N.

371. April 22, 1847, in latitude by account 50° 48′ N., and longitude 148° 30′ E., the following double altitude of the sun was observed :

Mean time nearly.	Chro.	Alt. sun's L. L.	Bearing.
10^h 0^m A.M.	10^h 2^m 25^s	44° 20′ 0″	S.E.b.S.
11 24 A.M.	11 24 34	50 20 0	S.b.E.

The run of the ship in the interval was 0, index correction +40″, and height of eye 0 : required the true latitude at second observation.
Ans. Arc 1, 20° 5′ 30″; arc 2, 87° 48′ 0″; arc 3, 62° 23′ 30″; lat. 50° 44′ 30″ N.

372. Oct 15, 1848, in latitude by account 53° N., and long. 54° E., the following double altitude of the sun was observed:

Mean time nearly.	Chro.	Alt. sun's L. L.	Bearing.
11h 20m A.M.	11h 15m 50s	27° 31′ 50″	S.b.E.
1 20 P.M.	0 50 32	25 45 5	S.S.W.

The run of the ship in the interval was S. by W. 14 miles, index correction +2′ 55″, and height of eye above the sea 15 feet: required the true latitude at second observation.

Ans. Arc 1, 23° 24′ 15″; arc 2, 91° 43′ 15″; arc 3, 79° 10′ 30″; lat. 53° 17′ 45″ N.

373. Oct. 24, 1849, in latitude by account 50° 40′ S., and long. 142° W., the following double altitude of the sun was observed:

Mean time nearly.	Chro.	Alt. sun's L. L.	Bearing,
10h 0m A.M.	10h 12m 34s	44° 20′ 0″	S.E.b.S.
11 24 A.M.	11 34 34	50 20 0	S.b.E.

The run of the ship in the interval was 0, index correction 0, and height of eye above the sea 0: required the true latitude at second observation.

Ans. Arc 1, 20° 3′ 0″; arc 2, 87° 48′ 0″; arc 3, 62° 23′ 0″; lat. 50° 45′ 30″ S.

Elements from Nautical Almanac.

Sun's declination.			Sun's semi.
June 2, 22° 8′ 50″ N.	......	June 3, 22° 16′ 34″ N.	... 15′ 47″
April 10, 7 49 12 N.	......	April 11, 8 11 21 N.	... 15 58
April 12, 8 33 22 N.	......	April 13, 8 55 14 N.	... 15 57
April 21, 11 44 26 N.	......	April 22, 12 4 46 N.	... 15 56
Oct. 14, 8 18 13 S.	......	Oct. 15, 8 40 28 S.	... 16 4
Oct. 24, 11 49 32 S.	......	Oct. 25, 12 10 19 S.	... 16 7

Rule 38. DOUBLE ALTITUDE. First method. (2.) *The latitude by altitudes of* TWO STARS *taken at the* SAME INSTANT.

1. Correct the observed altitudes for index correction, dip, and refraction, and thus find the true altitudes, which subtract from 90° for the true zenith distances.

2. Take out of the *Nautical Almanac* the right ascension and declination of the two stars, and get their polar distances as in (3) p. 144.

3. *To find the polar angle.* The difference between the right ascensions of the two stars is the polar angle.

4. *To find arc* 1 (using Inman's Tables). Put down the two polar distances under each other, and take their difference. Add together the log. sin. of the polar distance at greater bearing, the log. sin. of polar distance at

less bearing, and the log. haversine of polar angle; the result, rejecting the tens in the index, is the log. haversine of an arc, which take from the tables and call arc A.

Add together versine of arc A and versine of the difference of polar distances; the sum will be the versine of arc 1, which find in the tables. Then proceed to find arc 2, &c., as in Rule 37, p. 145.

EXAMPLE.

374. January 1, 1846, in latitude by account 38° 10′ N., the following altitudes of the stars α Pegasi and α Aquilæ were taken at the same instant:

Obs. alt. α Pegasi.	Bearing.	Obs. alt. α Aquilæ.	Bearing.
29° 49′ 27″	E.b.S.	57° 29′ 50″	S.S.E.
In. cor. −15″		In. cor. −15″	

The height of the eye was 41 feet: required the latitude.

At greater bearing. α Pegasi.				At less bearing. α Aquilæ.			
Observed alt. ...	29°	49′	27″	Observed alt. ...	57°	29′	50″
index correction.			15—	index correction.			15—
	29	49	12		57	29	35
dip		6	18	dip		6	18—
	29	42	54		57	23	17
refraction		1	42—	refraction		0	37—
true alt.	29	41	12	true alt.	57	22	40
	90				90		
zenith distance...	60	18	48	zenith distance...	32	37	20
Star's declination	14°	22′	50″ N.	Star's declination	8°	28′	2″ N.
	90				90		
P. D. at G. B. ...	75	37	10	pol. dist. at L. B.	81	31	58
R. A. α Pegasi .	22h	57m	6s	pol. dist. at G. B.	75°	37′	10″
R. A. α Aquilæ .	19	43	15	pol. dist. at L. B.	81	31	58
polar angle	3	13	51	diff. pol. dists. ...	5	54	48

To find arc 1.*

Sin. polar distance at greater bearing	9·986177
sin. polar distance at lesser bearing	9·995241
haversine polar angle	9·226458
haversine arc A	9·207876
arc A	47° 22′ 30″

* Arcs 1 and 2 may be readily found by using Shadwell's Tables, from which these two arcs may be taken out by inspection, and thus materially shortening this method of finding the latitude.

vers. arc A	0322696
	107
vers. difference polar distances	5319
vers. arc 1	0328122
arc 1	47° 47′ 16″

*To find arc 2.**

Arc 1	47°	47′	16″	.... cosec.	0·130382
P. D. at G. B.	75	37	10	.... cosec.	0·013823
difference...........	27	49	54	½ hav. S. ...	4·911662
P. D. at L. B.	81	31	58	½ hav. D....	4·654808
sum (S)	109	21	52	Haversine, arc 2	9·710675
difference (D)......	53	42	4	∴ arc 2	91° 34′ 0″

To find arc 3.

Arc 1	47°	47′	16″	.... cosec.	0·130382
zen. dist. at G. B..	60	18	48	.... cosec.	0·061110
difference	12	31	32	½ hav. S. ...	4·584171
zen. dist. at L. B..	32	37	20	½ hav. D. ...	4·241725
sum (S)	45	8	52	Haversine, arc 3	9·017388
difference (D)	20	5	48	∴ arc 3	37° 38′ 30″

To find arc 4.				*To find arc 5.*	
Arc 2	91°	34′	0″	Sin. pol. dist. at G. B.	9·986177
arc 3	37	38	30	sin. Z. D. at G. B.	9·938890
∴ arc 4	53	55	30	haversine, arc 4	9·312977
				haversine, arc	9·238044
				arc	49° 9′ 15″
				vers. arc	0345919
P. D. at G. B.	75°	37′	10″		55
Z. D. at G. B.	60	18	48	... vers. difference	0035443
difference	15	18	22		28
				vers. colat.	0381445
					363
					82
				colat.	51° 47′ 22″
					90
				∴ latitude ...	38 12 38 N.

* See note on previous page.

375. Sept. 17, 1844, in latitude by account 36° 45′ N., the following altitudes were observed at the same time :

Obs. alt. α Orionis.	Bearing.	Obs. alt. α Leonis.	Bearing.
55° 1′ 30″	S.E.b.E.	45° 13′ 30″	S.S.W.

The index correction was +55″, and height of eye 8 feet: required the true latitude. *Ans.* Arc 1, 62° 27′ 10″; arc 2, 88° 14′ 45″; arc 3, 38° 14′ 0″; lat. 36° 44′ N.

376. Feb. 20, 1846, in latitude by account 36° 40′ N., the following altitudes were observed at the same time :

Obs. alt. Sirius.	Bearing.	Obs. alt. Spica.	Bearing.
27° 50′	S.W.	12° 56′	E.S.E.

The index correction was +1′, and height of eye above the sea 10 feet: required the true latitude.

Ans. Arc 1, 96° 10′ 30″; arc 2, 108° 5′ 15″; arc 3, 59° 39′ 15″; lat. 36° 36′ 45″ N.

377. May 1, 1845, in latitude by account 41° 20′ N., the following altitudes of stars were taken at the same instant: required the true latitude.

True alt. α Pegasi.	Bearing.	True alt. α Tauri.	Bearing.
62° 44′	S.b.E.	19° 26′ 20″	E.

Ans. Arc 1, 79° 0′ 15″; arc 2, 78° 3′ 30″; arc 3, 26° 55′ 0″; lat. 41° 22′ 30″ N.

378. March 2, 1845, in latitude by account 41° 20′ N., long. 60° E., the altitudes of the two following stars were observed at the same time: required the true latitude.

True alt. α Andromedæ.	Bearing.	True alt. α Tauri.	Bearing.
73° 14′	S.b.E.	18° 27′ 30″	E.

Ans. Arc 1, 62° 9′ 30″; arc 2, 66° 11′ 0″; arc 3, 15° 8′ 45″; lat. 41° 23′ N.

379. January 2, 1847, in latitude by account 32° 10′ N., the following altitudes of the stars α Pegasi and α Aquilæ were observed at the same instant :

Obs. alt. α Pegasi.	Bearing.	Obs. alt. α Aquilæ.	Bearing.
22° 49′ 27″	E.b.S.	57° 29′ 50″	S.S.E.

The index correction −15″, and height of eye above the sea 41 feet: required the true latitude.

Ans. Arc 1, 47° 46′ 45″; arc 2, 91° 34′ 0″; arc 3, 31° 24′ 30″; lat. 32° 43′ N.

380. Dec. 27, 1847, the following altitudes were observed at the same instant, in latitude by account 37° 10′ N.: required the true latitude.

True alt. β Orionis.	Bearing.	True alt. α Hydræ.	Bearing.
31° 5′ 11″	S.W.b.W.	39° 47′ 33″	S.E.½S.

Ans. Arc 1, 62° 30′ 0″; arc 2, 94° 42′ 0″; arc 3, 58° 5′ 0″; lat. 37° 13′ N.

Elements from Nautical Almanac.

	Star's right ascension.				Star's declination.			
α Leonis	10h	0m	5·8s	....	12°	43′	24″	N.
α Orionis	5	46	47·9	....	7	22	23	N.
Spica.........	13	17	7·4	....	10	21	30	S.
Sirius	6	38	23·8	....	16	30	52	S.
α Tauri.........	4	27	3·0	....	16	11	30	N.
α Pegasi	22	57	4·0	....	14	22	24	N.
α Andromedæ	0	0	23·4	....	28	14	11	N.
α Pegasi	22	57	8·5	....	14	23	8	N.
α Aquilæ	19	43	18·2	....	8	28	12	N.
β Orionis	5	7	15·5	....	8	23	5	S.
α Hydræ.......	9	20	8·2	....	8	0	15	S.

When two heavenly bodies are observed at different times, the polar angle is to be found by the following rule:

Rule 39. 1. Subtract the time shown by the chronometer at the first observation (increased if necessary by 12 hours) from the time shown at the second observation, and thus find the elapsed time.

2. * Correct the elapsed time for rate of chronometer, if any; either by proportional logs. or by the common rule of proportion.

3. Add to the elapsed time so corrected the acceleration of sidereal on mean solar time (taken from table in *Nautical Almanac* or elsewhere). The result is the elapsed time expressed in sidereal time.

4. Add this elapsed time to the right ascension of the heavenly body first observed, and take the difference between the sum and the right ascension of the second heavenly body; the remainder (subtracted from 24 hours if greater than 12 hours) will be the polar angle required.

EXAMPLES.

381. The altitude of α Pegasi was observed when the chronometer showed $6^h\ 42^m\ 10^s$, and the altitude of α Aquilæ was observed when the chronometer showed $8^h\ 32^m\ 5^s$: required the polar angle between the two places observed, the rate of the chronometer being $12{\cdot}5^s$ gaining.

* When great accuracy is not required, and the elapsed time is small, these two corrections in 2 and 3 for rate of chronometer and acceleration may be omitted.

Times by chronometer.				
At second observation		8^h	32^m	5^s
at first observation		6	42	10
		1	49	55
Gr. date log. sun for 1^h 49^m ...	1·11697			
prop. log. for $12·5^s$	2·93651			
	4·05348			1—
		1	49	54
1^h	$9·86^s$			
49^m	8·05			
54^s	·15			
	18·06			18+
elapsed time in sidereal time		1	50	12
right ascension α Pegasi		22	57	14
		24	47	26
α Aquilæ		19	43	25
polar angle required		5	4	1

382. The altitude of Sirius was observed when the chronometer showed 2^h 10^m 20^s, and the altitude of Spica was observed when the chronometer showed 3^h 20^m 15^s: required the polar angle between the two places observed, the rate of chronometer being $2·5^s$ losing.

Times by chronometer.					
At second observation			3^h	20^m	15^s
at first observation............................			2	10	20
			1	9	55
rate of chronometer...........................					0
			1	9	55
acceleration	1^h	$9·8^s$			
	9^m	1·5			
	55^s	·1			
		11·4			11·4+
			1	10	6·4
right ascension Sirius			6	38	25·4
			7	48	31·8
Spica			13	17	10·9
polar angle			5	28	39·1

383. The altitude of β Orionis was observed when the chronometer showed 6^h 10^m 25^s, and the altitude of α Hydræ was observed when the chronometer showed 7^h 17^m 35^s: required the polar angle between the two places observed, the rate of chronometer being $6·3^s$ losing, and the right ascension of β Orionis 5^h 7^m 15^s, and of α Hydræ 9^h 20^m $8·2^s$.

Ans. 3^h 5^m 32^s.

Rule 40. DOUBLE ALTITUDE. First method. (3.) *The latitude by altitudes of two heavenly bodies observed at* DIFFERENT *times.*

1. Proceed as in (1) and (2), p 149, to get the zenith distances, the right ascension, and polar distances.
2. Find the polar angle, as in Rule 39, p. 153.
3. Find arc 1, as in (4), p. 149.
4. Then proceed to find arcs (2), (3), (4), &c., as in Rule 37, p. 144.

EXAMPLE.

384. Sept. 27, 1846, in latitude by account 43° 30′ N., the following altitudes of the stars α Pegasi and α Aquilæ were observed at different times.

	Observed altitude.	Time by chron.	Bearing.
α Pegasi	29° 49′ 30″	7^h 35^m 10^s	S.E.
α Aquilæ	54 29 0	8 2 10	S.¼W.

The run of the ship in the interval was S. 10 miles, the index correction +1′ 10″, and height of eye above the sea 20 feet: required the true latitude at the second observation.

At greater bearing. α Pegasi.

Observed alt.	29°	49′	30″
		1	10+
	29	50	40
		4	24−
	29	46	16
		1	41−
	29	44	35
		7	6+
	29	51	41
zenith distance	60	8	19

At less bearing. α Aquilæ.

Observed alt.	54°	29′	0″
		1	10+
	54	30	10
		4	24−
	54	25	46
		0	41−
	54	25	5
zenith distance	35	34	55

Right asc. α Pegasi, 22^h 57^m 9^s.
Declination, 14° 23′ 9″ N.

Right asc. α Aquilæ, 19^h 43^m 19^s.
Declination, 8° 28′ 19″ N.

To find the polar angle.

Chronom. at first observation	7^h	35^m	10^s
„ second „	8	2	10
elapsed time	0	27	0
right ascension α Pegasi	22	57	9
	23	24	9
right ascension α Aquilæ	19	43	19
polar angle	3	40	50

To find arc 1.

Pol. dist. at G. B.	75°	36′	51″
pol. dist. at L. B.	81	31	41
diff. pol. distances......	5	54	50
sin. pol. dist. at G. B.	9·986161		
sin. pol. dist. at L. B.	9·995236		
hav. pol. angle	9·331838		
hav. arc A	9·313235		
arc A	53°	56′	30″
vers. arc A	0411274		
	117		
vers. diff. pol. dists....	0005297		
	23		
vers. arc 1	0416711		
	695		
arc 1...... 54° 19′ 4″	16		

To find arc 2.

Arc 1	54°	19′	4″
pol. dist. at G. B.	75	36	51
diff.	21	17	47
pol. dist. at L. B.	81	31	41
sum	102	49	28
diff.	60	13	54
cosec. arc 1	090309		
cosec. pol. dis. at G.B.	013839		
½ hav. sum	4·893016		
½ hav. diff.	4·700498		
hav. arc 2	9·697662		
arc 2	89°	49′	45″

To find arc 3.

Arc 1	54°	19′	4″
zenith dist. at G. B. ..	60	8	19
diff.	5	49	15
zenith dist. at L. B. ..	35	34	55
sum	41	24	10
diff.	29	45	40
cosec. arc 1	·090309		
cosec. zen. dis. at G.B.	·061869		
½ hav. sum	4·548400		
½ hav. diff.	4·409623		
hav. arc 3	9·110201		
arc 3.....................	42°	4′	45″
pol. dist. at G. B.	75	36	51
zen. dist. at G. B.	60	8	19
difference...............	15	28	32

To find arc 4.

Arc 2	89°	49′	45″
arc 3	42	4	45
arc 4	47	45	0

To find the latitude.

Log.sin.pol. dis. atG.B.	9·986161		
log. sin. zen. dis. „	9·938131		
log. hav. arc. 4	9·214358		
hav. arc 5	9·138650		
arc 5......................	43°	33′	0″
vers. arc 5	0275227		
vers. diff. pol. dist. and zen. dist.	36214		
	41		
	0311482		
	34		
	48		
	46°	29′	14″
	90		
latitude	43	30	46N.

Rule 41. Double Altitude. Second method. *The latitude by two altitudes of the* sun. (This method requires only the common tables of sines, &c.)

1. From the time shown by the chronometer or watch at the second observation (increased if necessary by 12 hours) subtract the time shown at the first observation, divide by 2; the result is the half polar angle in time.

2. To the estimated mean time at the ship at the first observation add

the half polar angle; the sum will be the ship mean time at the middle time between the observations.

3. Apply the longitude in time, and thus get a Greenwich date.

4. Take out from the *Nautical Almanac* the declination for this date, and also the sun's semidiameter in the adjacent column.

5. Correct the observed altitudes for index correction, dip, semidiameter, and parallax and refraction.

6. Correct also the first true altitude for run of ship in the interval, and thus get the true altitudes for the same place.

7. Put the first true altitude under the second true altitude, take their sum and difference, and also the half-sum and half-difference; call the half-sum S, and the half-difference D.

8. Under the log. sin. half polar angle put log. cos. declination; at the same time take out and put a little to the right the log. sin. declination.

9. Add together the two logs. first taken out, and call the sum sin. arc 1.

10. At the same opening take out sec. arc 1, and put it under the log. sin. declination; take out also and put down in the same horizontal line the log. cosec. arc 1, and also log. sec. arc 1.

11. Add together log. sin. declination and log. sec. arc 1; the sum will be log. cos. arc 2; the arc corresponding thereto found in the Tables will be arc 2, if the latitude and declination are of the same name; but if the latitude and declination are of different names, subtract the arc taken out from 180°: the remainder is arc 2.

12. Under log. cosec. 1, and log. sec. 1, just taken out, put the following quantities:

Under log. cosec. 1 put log. cos. S.	Under log. cosec. 1 put log. sin. D.
„ sec. 1 „ sin. S.	„ sec. 1 „ cos. D.

Add together log. cosec. 1 and the two logs. placed beneath it; the sum will be the log. sin. arc 3.

13. Take out the log. sec. arc 3, and put it down twice; once under log. cos. D, and again a little to the right.

14. Add together the log. sec. 1, and the three logarithms beneath it; the result is log. cos. arc 4, which find in the Tables.

15. Under arc 4 put arc 2, and take the difference in all cases when the line drawn through the places of the sun at the two observations will when produced *not* pass through the zenith and pole (that is, the difference must be taken, if it is seen that their sum would exceed 90°), otherwise take their sum; the result is arc 5.

Lastly. Under log. sec. arc 3, already taken out, put log. sec. arc 5: the sum will be the log. cosec. of the required latitude.

The arrangement on the paper of the logarithms to be taken out, as directed by the rule, will be better seen in the following blank form; and it would also facilitate the working out questions in other rules of Navigation if blank forms, similar to the one now given, were constructed on thick drawing-paper by the student for each rule.

LATITUDE BY DOUBLE ALTITUDE OF SUN. (BLANK FORM.)

Times by Chronometer.

1st obs.
Sub. from
* 2d Obs.
Diff. or pol. A. .
$\frac{1}{2}$ pol. A.........

* Add 12 to hours at 2d observation if necessary.

Sin. $\frac{1}{2}$ pol. A.
+
Cos. decl.
Sin. (1) ....

Mean T. at 1st observation.

Ship
+
$\frac{1}{2}$ pol. A.
Ship ....
Long. in T.......
Gr. date ....
On ....

& Sin. decl.
+
Sec. (1)
Cos. (2) ....
* Arc (2) ......
180
Arc (2) ....

* If latitude and declination of *different* names, subtract this arc from 180° for arc (2).

Sun's declination.

At
At
Diff. ....

Sun's decl. ......

& Cosec. (1) ...
Cos. (S) ....
Sin. (D) ....
Sin. (3) ....

† If the sum of arc (4) and (2) equal or exceed 90°, then the *difference* is arc (5).

1st Observed altitude.

Obs. alt..........
Index cor. ......
Dip (−).........
Semidiameter...
Cor. in alt. (−)
Run ....
1st true alt. ...

& Sec. (1) ......
& Sin. (S) ......
& Cos. (D)......
& Sec. (3) ......
Cos. (4) ....
Arc (4)
− or + †
Arc (2)
Arc (5) ....

2d Observed altitude.

Obs. alt. ....
Index cor. ......
Dip (−).........
Semidiameter...
Cor. in alt. (−)
(2d) tr. alt.......
(1st) tr. alt. ...
Sum ....
Difference ......
(S) $\frac{1}{2}$ Sum ......
(D) $\frac{1}{2}$ Difference

& Sec. (3)
+
Sec. (5)
Cosec. ....
Lat. ....

EXAMPLES.

385. Oct. 11, 1845, in latitude by account 54° N., and long. 83° 15′ W., the following double altitude of the sun was observed :

Mean time nearly.	Chronometer.	Alt. sun's L. L.	Bearing.
7^h 45^m A.M.	11^h 40^m 15^s	9° 0′ 20″	E.S.E.
10 35 A.M.	2 13 20	25 3 30	S.S.E.

The run of the ship in the interval was S. by W. 15 miles, index correction +5′ 10″, and height of eye above the sea 18 feet : required the latitude at the second observation. *Ans.* 53° 59′ N.

386. March 20, 1845, in latitude by account 52° 10′ N., and longitude 55° 15′ W., the following double altitude of the sun was taken :

Mean time nearly.	Chronometer.	Alt. sun's L. L.	Bearing.
8^h 35^m A.M.	9^h 36^m	20° 0′ 30″	S.E.b.E.
1 45 P.M.	2 49	34 5 30	S.W.b.S.

The run of the ship in the interval was N.W. by W. 10 miles, index correction 0, and height of eye 20 feet : required the latitude at the second observation. *Ans.* 52° 27′ N.

387. Dec. 11, 1845, the following double altitude of the sun was observed :

Mean time nearly.	Chronometer.	Alt. sun's L. L.	Bearing.
6^h 0^m A.M.	6^h 3^m 30^s	19° 40′ 25″	E.b.S.
10 0 A.M.	10 4 25	50 20 40	N.E.

The run of the ship in the interval was E.N.E. 25 miles, index correction −1′ 50″, and height of eye 16 feet : required the latitude at second observation, the latitude by account being 60° S., and long. 79° 15′ W. *Ans.* 56° 57′ S.

388. Nov. 10, 1846, in latitude by account 35° 30′ N., long. 94° 30′ E., the following double altitude of the sun was observed :

Mean time nearly.	Chronometer.	Alt. sun's L. L.	Bearing.
1^h 15^m P.M.	1^h 45^m 15^s	33° 5′ 40″	S.S.W.
3 45 P.M.	4 15 17	12 55 10	S.W.b.W.

The run in the interval was S.S.E. 15 miles, index correction +4′ 10″, and height of eye 18 feet : required the true latitude at the second observation. *Ans.* 35° 31′ N.

389. Oct. 30, 1846, in latitude by account 52° 10′ N., and longitude 159° 45′ E., the following double altitude of the sun was observed ;

Mean time nearly.	Chronometer.	Alt. sun's L. L.	Bearing.
11^h 15^m A.M.	11^h 21^m 15^s	25° 26′ 20″	S.¾E.
11 30 A.M.	11 37 55	25 55 0	S.¼E.

The run of the ship in the interval was S. by W. 1 mile, index correction $+3' 50''$, and height of eye above the sea 20 feet: required the true latitude at second observation. *Ans.* 49° 56′ N.

Elements from Nautical Almanac.

	Sun's declination.								Sun's semi.	
Oct. 11 ...	7°	4′	38″S.	12 ...	7°	27′	15″S.		16′	3″
Mar. 20 ...	0	5	40 S.	21 ...	0	18	1 N.		16	4
Dec. 11 ...	23	1	55 S.	12 ...	23	6	54 S.		16	16
Nov. 9 ...	16	51	6 S.	10 ...	17	8	9 S.		16	11
Oct. 29 ...	13	26	6 S.	30 ...	13	45	56 S.		16	8

In the above rule, called Ivory's method, it is assumed that the declinations of the sun at the times of observation are the same as at the middle time between the two observations; but this is not so. The latitude deduced will therefore only be approximately true, differing perhaps 2′ or 3′ from the truth. Its principal recommendation is that it requires only the common table of log. sines, and is shorter than the first or Inman's method.

Inman's Rule for Double Altitude.

Adapted to the Tables of Sines, &c.

In p. 142 it is shown that the latitude by double altitude may be found by the simple application of two rules in spherical trigonometry. These are

1. Two sides and included angle given, to find the third side.
2. Three sides given, to find an angle.

These practical rules, *adapted to the tables of sines*, are given in the author's *Trigonometry*, pp. 63, 65.

Or the several parts of the problem may be calculated by the following two formulæ, *Trig.* Part II. pp. 65, 95.

$$(\alpha)\ \sin.\frac{a}{2}=\frac{\sin.\frac{1}{2}\text{A}\sqrt{\sin.b\ \sin.c}}{\sin.\theta}\ \text{where}\ \tan\theta=\frac{\sin.\frac{1}{2}\text{A}\sqrt{\sin.b\ \sin.c}}{\sin.\frac{1}{2}(b\sim c)}.$$

$$(\beta)\ \sin.^2\frac{\text{A}}{2}=\text{cosec.}\,b.\ \text{cosec.}\,c.\ \sin.\tfrac{1}{2}(a+\overline{b\sim c}).\sin.\tfrac{1}{2}(a-\overline{b\sim c}).$$

Arc 1, or Xy (fig. p. 142), is found by (α), where b and c represent the two polar distances, and A the polar angle. Arc 2, or angle PXy, is found by (β), where b and c represent respectively arc 1 and polar distance at greater bearing, and a the polar distance at lesser bearing. Arc 3, or angle ZXy, is found by (β), where b and c represent respectively arc 1 and zenith distance at greater bearing, and a the zenith distance at lesser bearing.

Then arc 4=arc 2$\mp$arc 3 (p. 145).

Lastly, the co-latitude PZ is found by (α), where b and c represent respectively the pol. dist. and zen. dist. at greater bearing, and A the arc 4.

These formulæ may be applied to all the cases of the double altitude; and it will be found, after a little practice, that the trigonometrical method is much easier than following the formal rules.

CHAPTER VI.

THE ERROR AND RATE OF CHRONOMETERS.

THERE are two methods of determining the *error of a chronometer on mean time*, the one by a single altitude of a heavenly body observed at some distance from the meridian; the other by means of equal altitudes of a heavenly body observed on both sides of the meridian.

Before going to sea, the *error* of the chronometer on Greenwich mean time at mean noon, and its *daily rate*, are supposed to have been accurately determined, either at an observatory by means of daily comparisons with an astronomical clock, or by observations taken with a sextant at a place whose longitude is known.

The *mean daily rate* of a chronometer can then be found by dividing the increase or decrease in its error by the number of days elapsed between the times when the observations were taken to determine its error; thus, suppose on April 27, at $9^h\ 30^m$ A.M., the error of a chronometer was found to be fast $10^m\ 10{\cdot}5^s$ on Greenwich mean time, and that on April 30th about the same hour its error was found to be $10^m\ 40{\cdot}5^s$ fast: then it appears that in the three days elapsed between the observations the chronometer has gained 30^s, hence its mean daily rate is 10^s gaining.

When the error and rate of a chronometer are given, we may determine what its error will be at some future time, provided the rate of the chronometer continues uniform in the interval, by the following rule.

Rule 42. *Given the* ERROR OF A CHRONOMETER *on Greenwich mean time at some given day and hour, and also its* MEAN DAILY RATE, *to find Greenwich mean time at some other instant.*

1. Get a Greenwich date.

2. Find the number of days and part of a day that have elapsed from the time when the error and rate were determined by the hour of the Greenwich date.

3. Multiply the rate of the chronometer by the number of days elapsed, and add thereto the proportionate part for the fraction of a day, found by

proportion or otherwise. The result is the accumulated error in the interval.

4. If the chronometer is gaining, subtract the accumulated error from the time shown by the chronometer; if losing, add.

5. To the result apply the original error of chronometer, adding if slow, subtracting if fast (increasing the time shown by chronometer by 24^h if necessary, and putting the day one back). The result (rejecting 24^h if greater than 24^h, and putting the day one forward) will be mean time at Greenwich at the instant of the observation.

6. If this time differs from the Greenwich date by 12 hours nearly, in that case 12 hours must be added to the Greenwich time, determined as above, and the day put one back, to get the astronomical Greenwich mean time.

EXAMPLES.

390. June 13, 1851, at $10^h\ 52^m$ P.M. mean time nearly, in long. 60° W., an observation was taken when a chronometer showed $2^h\ 50^m\ 42^s$. On June 1, at noon, its error was known to be $3^m\ 10{\cdot}2^s$ fast on Greenwich mean time, and its mean daily rate was $3{\cdot}5^s$ gaining: required mean time at Greenwich when the observation was taken.

Ship, June 13	10^h	52^m
long. in time .	4	0 W.
Gr., June 13 .	14	52

Interval from June 1 to June 13, at $14^h\ 52^m$ is $12^d\ 14^h\ 52^m = 12^d\ 15^h$ nearly

Daily rate .	$3{\cdot}5^s$
	12
	42·0
12^h is $\frac{1}{2}$	1·75
3 „ $\frac{1}{4}$	·44

Accumulated rate . . .			44·2 gaining
chronometer showed . .	2^h	50^m	$42{\cdot}0^s$
	2	49	57·8
original error		3	10·2 fast
Greenwich, June 14 . .	2	46	47·6 A.M.
∴ Greenwich, June 13 .	14	46	47·6

(12 hours are added and the day put back one, to make the time thus found agree more nearly with the Greenwich date.)

391. August 10, 1853, at $3^h\ 42^m$ A.M. mean time nearly, in long. 100° 30′ W., an observation was taken when a chronometer showed $10^h\ 30^m\ 45{\cdot}5^s$.

On August 1, its error was known to be $12^m\ 10{\cdot}5^s$ slow on Greenwich mean time, and its rate $11{\cdot}2^s$ gaining: required mean time at Greenwich when the observation was taken.

Ship, Aug. 9 .	15h 42m		Daily rate . . .		11·2s
long. in time .	6 42 W.				8
		Interval from			
Gr., Aug. 9 .	22 24	Aug. 1 to Aug. 9,	12h is ½		89·6
		at 22h 24m is	8 „ ⅓		5·6
		8d 22h 24m	2 „ ⅓		3·7
			24m „ ⅓ nearly		·9
					·2
					100·0

Accumulated error . .			1m 40·0	gaining
chronometer showed .	10h	30	45·5	
	10	29	5·5	
original error		12	10·5	slow
Greenwich, August 10 .	10	41	16·0	A.M.
∴ Greenwich, August 9	22	41	16·0	

(adding 12 hours, as directed by rule.)

Note. If the Greenwich time thus determined differs considerably from the Greenwich date used, the work should be repeated, using for the Greenwich date the approximate Greenwich time first found.

EXAMPLES.

392. Nov. 20, 1851, at $6^h\ 42^m$ P.M. mean time nearly, in long. 32° 0′ E., an observation was taken when a chronometer showed $4^h\ 30^m\ 6^s$.

On Oct. 9 its error was known to be $5^m\ 52{\cdot}4^s$ slow on Greenwich mean time, and its rate $2{\cdot}7^s$ losing: required mean time at Greenwich when the observation was taken. *Ans.* $4^h\ 37^m\ 52{\cdot}3^s$.

393. Dec. 31, 1851, at $10^h\ 10^m$ A.M. mean time nearly, in long. 150° E., an observation was taken when a chronometer showed $0^h\ 0^m\ 22{\cdot}3^s$.

On Nov. 20 its error was known to be $3^m\ 52{\cdot}4^s$ slow on Greenwich mean time, and its rate $2{\cdot}7^s$ losing: required mean time at Greenwich when the observation was taken. *Ans.* $12^h\ 6^m\ 4{\cdot}0^s$.

394. April 11, 1851, at $3^h\ 14^m$ P.M. mean time nearly, in long. 56° 42′ W., an observation was taken, when a chronometer showed $7^h\ 2^m\ 10{\cdot}5^s$.

On March 15 its error was known to be $1^m\ 32{\cdot}7^s$ fast on Greenwich mean time, and its daily rate $6{\cdot}3^s$ losing: required mean time at Greenwich when the observation was taken. *Ans.* $7^h\ 3^m\ 29{\cdot}7^s$.

THE HOUR-ANGLE AND APPARENT TIME.

Apparent time, and thence mean time, may be found by observing the altitude of a heavenly body, and calculating its corresponding hour-angle. If the time is noted by a chronometer when the observation is taken, the

the error of the chronometer will be found on mean time at the place by the following rules.

Rule 43. *To find the* ERROR OF A CHRONOMETER *on mean time at the ship by a single altitude of a heavenly body.*

First. *By sun's altitude.*

1. Get a Greenwich date.

2. Correct the sun's declination and equation of time for this date. Take out of the *Nautical Almanac* the sun's semidiameter, at the same time the declination and equation of time are taken out.

3. Correct the observed altitude for index correction, dip, semidiameter, and correction in altitude, and thus get the true altitude; subtract the true altitude from 90° to obtain the zenith distance.

NOTE. The HOUR-ANGLE, and thence apparent time, may be computed either by using the table of log. haversines (by far the most simple method), or by the common table of sines, &c.

4. *To find* SHIP APPARENT TIME. First method, using log. haversines, &c.

Under the latitude put the sun's declination, and, if the names be alike, take the difference; but if unlike, take their sum. Under the result put the zenith distance, and find their sum and difference.

Add together the log. secants of the two first terms in this form, and the halves of the log. haversines of the two last; and (rejecting the tens in the index) look out the sum as a log. haversine, to be taken out at the top of the page if the sun is west of the meridian, but at the bottom of the page if the sun is east of meridian. The result is apparent solar time at the instant of observation.

To find SHIP APPARENT TIME. Second method, using log. sines, &c.

Under the latitude put the sun's declination, and, if the names be alike, take the difference; but if unlike, take their sum. Under the result put the zenith distance, and find their sum and difference, and half-sum and half-difference.

Add together the log. secants of the two first terms in this form (rejecting the tens in index) and the log. sines of the two last, and divide the sum by 2; look out the result as a log. sine, and multiply the angle taken out by 2.

Reduce the angle thus found into time, and if the sun is west of meridian, the same will be apparent time; but if east of meridian, subtract the angle from 24 hours; the remainder will then be apparent solar time at the instant of observation.

5. *To find* SHIP MEAN TIME. To apparent solar time apply the equation

of time with its proper sign, as directed in the *Nautical Almanac;* the result is mean time at the place.

6. *To find error of chronometer on* SHIP MEAN TIME. The difference between mean time thus found and the time shown by chronometer at the observation will be the error of the chronometer on mean time at the place.

Rule 44. *To find the error of a chronometer on* MEAN TIME AT GREENWICH *by a single altitude of the sun.*

Find mean time at the place of observation as directed in preceding Rule. See 1, 2, 3, 4, and 5.

6. To the mean time at the place thus found apply the longitude in time, adding if west, and subtracting if east (rejecting or adding 24 hours if necessary): the result will be mean time at Greenwich at the time of the observation.

7. The difference between which and the time shown by chronometer will be the error of the chronometer on Greenwich mean time.

EXAMPLE.

395. May 10, 1842, at $8^h\ 44^m$ A.M. mean time nearly, in latitude 50° 48′ N., and long. 1° 6′ W., when a chronometer showed $8^h\ 26^m\ 59{\cdot}7^s$, the observed altitude of the sun's lower limb was 39° 14′ 30″, index correction +4′ 24″, and height of eye above the sea 20 feet: required the error of the chronometer on mean time at the ship, and also its error on Greenwich mean time.

Ship, May 9 . . .	$20^h\ 44^m$	Sun's semi.
Long. in time . . .	4 W.	15′ 51″
Greenwich, May 9 .	20 48	

Sun's declination.			Equation of time.					
9th . .	17° 19′ 24″ N.	9th . . .	$3^m\ 46{\cdot}2^s$ sub.	Obs. alt. . .	39°	14′	30″	
10th . .	17 35 18 N.	10th . . .	3 49·1	in. cor. . .		4	24	+
	15 54		2·9		39	18	54	
·06215		·06215		dip. . . .		4	24	−
1·05388		3·57103			39	14	30	
1·11603	13 46	3·63318	2·5	semi. . . .		15	51	+
Declin. .	17 33 10 N.		3 48·7		39	30	21	
				cor. in alt. .		1	3	−
					39	29	18	
					90			
				zen. dist. .	50	30	42	

First Method.
Apparent time by haversines.

Lat.	50°48′ 0″N.	sec. lat.	0·199263
decl.	17 33 10 N.	sec. decl.	0·020710
diff.	33 14 50	½ hav. S.	4·824491
zen.dis.	50 30 42	½ hav. D.	4·176307
sum	83 45 32 (S)	hav.	9·220771
diff.	17 15 52 (D)		
∴ apparent time			20^h 47^m 30^s
equation of time			3 48·7 −
∴ ship mean time May 9			20 43 41·3
chro. showed (adding 12^h)			20 26 59·7
∴ error on ship M. T.			16 41·6 slow

Second Method.
Apparent time by sines, &c.

Lat.	50°48′ 0″N.	sec.	0·199263
decl.	17 33 10 N.	sec.	0·020710
	33 14 50	sin. S_1.	9·824491
zen. dis.	50 30 42	sin. D_1	9·176300
sum	83 45 32		2) 19·220764
diff.	17 15 52		9·610382
½ sum	41 52 46 (S_1)		24° 3′ 45″
or ½ diff.	8 37 56 (D_1)		1^h 36^m 15^s
			2
			3 12 30
			24
		∴ ship ap. T.	20 47 30
			as before.

To find error of chronometer on Greenwich mean time.

Ship mean time, May 9	20^h	43^m	$41{\cdot}3^s$
longitude in time		4	24·0 W.
∴ Greenwich mean time, May 9	20	48	5·3
chronometer showed (adding 12^h)	20	26	59·7
∴ error on Greenwich mean time......		21	5·6 slow.

If the computed ship mean time differ several minutes from the estimated ship mean time, it will be advisable, when great accuracy is required, to recalculate the sun's declination and the hour-angle, using the approximate ship time just found to determine the Greenwich date; the following example will illustrate the mode of proceeding:

396. March 16, 1844, at 10^h 10^m A.M., mean time nearly, in lat. 50° 48′ N., and long. 1° 6′ W., when a chronometer showed 10^h 15^m $47{\cdot}2^s$, the observed altitude of the sun's lower limb was 58° 46′ 30″ (in artificial horizon), the index correction + 1′ 20″; required the error of chronometer on Greenwich mean time.

			Sun's declination.		Equation of time.	
March 15	22^h 10^m	15th	1° 58′ 28″ S.	15th		9^m $1{\cdot}7^s$ add
long. in time	4 W.	16th	1 34 45 S.	16th		8 44·4
Gr., March 15	22 14		23 43			17·3
		·03321			·03321	
Sun's semi.		·88022			2·79538	
16′ 5″		·91343	21 58		2·82859	16·0
		Declination	1 36 30 S.			8 45·7

Obs. alt. . .	58°	46′	30″
in. cor. . .		1	20+
2)	58	47	50
	29	23	55
semi. . . .		16	5
	29	40	0
cor. in alt. .		1	34−
	29	38	26
	90		
zen. dist. .	60	21	34

Lat. . .	50°	48′	0″N. . . .	Sec. . . .	0·199263
decl. . .	1	36	30 S. . . .	sec. . . .	0·000171
sum. . .	52	24	30	½ hav. (S).	4·920520
zen. dist.	60	21	34	½ hav. (D)	3·840866
sum. . .	112	46	4 (S)	hav. . .	8·960820
diff. . .	7	57	4 (D)		

Apparent time	21^h	39^m	14^s
equation of time		8	45·7+
mean time	21	47	59·7
long. in time		4	24·0 W.
Greenwich mean time . .	21	52	23·7
chronometer showed . .	22	15	47·2
error of chronometer on Greenwich mean time .		23	23·5

The mean time at the place is found to be $21^h\ 47^m\ 59{\cdot}7^s$, but the mean time used for computing the declination and equation of time was $22^h\ 10^m$. Now this has rendered the declination slightly incorrect, and therefore the mean time computed from it. When it is desirable to obtain mean time at the place as correctly as possible, we must recalculate the declination and apparent time, using the approximate mean time for finding a more correct Greenwich date; thus the mean time at the place is found above to be $21^h\ 47^m\ 59{\cdot}7^s$; let us therefore assume the mean time to be $21^h\ 48^m$, obtain in this manner a second Greenwich date, and recompute the sun's declination and hour-angle for this more correct Greenwich date as follows:

Mar. 15, M. T. .	21^h	48^m	
long. in time .		4 W.	
Gr., Mar. 15 .	21	52	
Sun's declination.			
15th . . .	1°	58′	28″S.
16th . . .	1	34	45 S.
		23	43
·04043			
·88022			
·92065		21	36
∴ sun's decl.	1	36	52 S.

Decl. . .	1°	36′	52″ S. . .	Sec. . . .	0·000171
lat. . . .	50	48	0 N. . .	sec. . . .	0·199263
	52	24	52		
zen. dist. .	60	21	34		
sum. . .	112	46	26 (S) . .	½ hav. (S)	4·920540
diff. . .	7	56	42 (D) . .	½ hav. (D)	3·840030
					8·960605

Apparent time $21^h\ 39^m\ 17^s$, which differs 3^s from the previous result; whence the error of chronometer is fast $23^m\ 20{\cdot}5^s$ on Greenwich mean time

397. May 20, 1847, at $5^h\ 20^m$ P.M., mean time nearly, in lat. 47° 20′ N., and long. 94° 30′ E., when a chronometer showed $11^h\ 5^m\ 20^s$, the observed altitude of the sun's lower limb was 20° 0′ 15″, the index correction −4′ 10″, and height of eye above the sea 20 feet: required the error of chronometer on Greenwich mean time. *Ans.* Fast $0^m\ 41{\cdot}4^s$.

398. Feb. 3, 1847, at 10^h 30^m A.M., mean time nearly, in lat. 49° 30′ N., and long. 22° W., when a chronometer showed 0^h 2^m 30^s, the observed altitude of the sun's lower limb was 19° 21′ 30″, the index correction +3′ 20″, and height of eye above the sea 18 feet: required the error of chronometer on Greenwich mean time. *Ans.* Fast 10^m $7{\cdot}4^s$.

399. March 25, 1847, at 3^h 20^m P.M., mean time nearly, in lat. 52° 10′ N., and long. 36° 58′ 15″ W., when a chronometer showed 5^h 40^m 58^s, the observed altitude of the sun's lower limb was 25° 10′ 20″, the index correction −6′ 10″, and height of eye above the sea 20 feet: required the error of chronometer on Greenwich mean time. *Ans.* 9^m $25{\cdot}2^s$ slow.

400. May 19, 1847, at 3^h 0^m P.M., mean time nearly, in lat. 49° 50′ N., and long. 21° 4′ 45″ E., when a chronometer showed 1^h 23^m 20^s, the observed altitude of the sun's lower limb was 42° 50′ 30″, the index correction +4′ 10″, and height of eye above the sea 20 feet: required the error of chronometer on Greenwich mean time. *Ans.* 10^m $37{\cdot}6^s$ slow.

Elements from Nautical Almanac.

	Sun's declination.	Equation of time.	Semi.
May 19......	19° 41′ 37″ N.	3^m 49·6^s sub.	15′ 49″
„ 20......	19 54 26 N.	3 46·2	
Feb. 2	16 54 7 S.	13 58·8 add.	16 14
„ 3	16 36 41 S.	14 5·6	
March 25...	1 40 56 N.	6 13·8 add.	16 3
„ 26...	2 4 29 N.	5 55·2	
May 19......	19 41 37 N.	3 49·5 sub.	15 49
„ 20......	19 54 26 N.	3 46·9	

Rule 45. *To find the error of chronometer.*

Second. *By a* STAR'S ALTITUDE.

1. Get a Greenwich date.

2. Take out of the *Nautical Almanac* the right ascension and declination of the star, and also the right ascension of the mean sun for mean noon of the Greenwich date.

3. Correct the right ascension of mean sun for Greenwich date (p. 86).

4. Correct the observed altitude for index correction, dip, and refraction, and thus get the true altitude, which subtract from 90° for the true zenith distance.

5. *To find star's hour-angle.* First method (using haversines).

Under the latitude put the star's declination; add if the names be unlike, subtract if like. Under the result put the true zenith distance of star, and take the sum and difference.

Add together the log. secants of the two first terms in this form (omitting

the tens in each index), and halves of the log. haversines of the two last; the sum (rejecting 10 in the index) will be the log haversine of hour-angle, to be taken out at top of page if heavenly body be west of meridian, but at bottom if east of meridian.

6. *To find star's hour-angle.* Second method (using sines, &c.).

Proceed as in the corresponding Rule (43) for finding apparent time, p. 164 and example 395.

7. To the hour-angle thus found add the star's right ascension, and from the sum (increased if necessary by 24 hours) subtract the right ascension of mean sun; the remainder is mean time at the place at the instant of observation.

8. Under mean time at place put the chronometer time shown by chronometer; the difference will be the error of chronometer on mean time at place.

To find error on Greenwich *mean time.*

Proceed as in the corresponding rule for the sun, p. 165 and ex. 395.

EXAMPLE.

401. June 3, 1842, at $12^h\ 9^m$ P.M., mean time nearly, in lat. 50° 48′ N., and long. 1° 6′ 3″ W., observed the altitude of α Bootis (west of meridian) to be 89° 53′ 30″ in artificial horizon, when a chro. showed $0^h\ 14^m\ 22{\cdot}3^s$, the index correction was −10″: required the error of the chronometer on mean time at the place, and also on Greenwich mean time.

Ship, June 3 . 12^h 9^m	Star's R. A. . . 14^h 8^m $30{\cdot}5^s$	Ob. alt. . 89° 53′ 30″
long. in time . 4 W.	star's decl. . . . 20° 0′ 15″N.	in cor. . 10
Gr., June 3 . . 12 13	R. A. mean sun 4^h 46^m $7{\cdot}1^s$	2)89 53 20
	cor. for 12^h . . . 1 58·3	44 56 40
	„ 0 13^m 2·1	ref. . . . 58
	∴ R. A. for 12 13 . 4 48 7·5	44 55 42
		90
		zen. dist. 45 4 18

First method.
Hour-angle by haversines.

Lat. . .	50° 48′ 0″N. .	sec. . . .	0·199263
decl. . .	20 0 15 N. .	sec. . . .	0·027026
	30 47 45	½ hav. S.	4·788699
zen.dis.	45 4 18	½ hav. D.	4·094305
sum . .	75 52 3 (S)	hav. . .	9·109293
diff. . .	14 16 33 (D)	h.-ang.	$2^h 48^m 8^s$
star's R. A.			14 8 30·5
			16 56 38·5
R. A. mean sun . .			4 48 7·5
∴ ship mean time .			12 8 31·0
chro. showed (add. 12^h)			12 14 22·3
∴ error of chro. . .			5 51·3 fast.

Second method.
Hour-angle by sines, &c.

Lat. . .	50° 48′ 0″N. .	sec. . . .	0·199263
decl. . .	20 0 15 N. .	sec. . . .	0·027026
	30 47 45	sin. S_1 .	9·788694
zen.dis.	45 4 18	sin. D_1 .	9·094300
sum . .	75 52 3		2)19·109283
diff. . .	14 16 33	sin. . . .	9·554641
½ sum .	37 56 1 (S_1)	½ h.-ang.	$1^h 24^m 4^s$
½ diff. .	7 8 16 (D_1)		2
		∴ hour-angle .	2 48 8

To find the error of chronometer on Greenwich mean time.

Mean time at place	12^h	8^m	$31{\cdot}0^s$
long. in time		4	24·2 W.
Greenwich mean time	12	12	55·2
chronometer showed	12	14	22·5
error of chron. on Gr. mean time		1	27·1 fast.

402. May 4, 1847, at 4^h 40^m A.M., mean time nearly, in lat. 40° 10′ 20″ N., and long. 81° 47′ 15″ E., when a chronometer showed 11^h 13^m 50^s, the observed altitude of α Bootis (west of meridian) was 20° 45′ 45″, the index correction −2′ 10″, and the height of eye above the sea 18 feet: required the error of the chronometer on Greenwich mean time.

Ans. 0^m $35{\cdot}3^s$ slow.

403. Feb. 10, 1847, at 9^h 22^m P.M., mean time nearly, in lat. 28° 30′ N., and long. 27° 15′ W., a chronometer showed 11^h 17^m 20^s, when the observed altitude of α Leonis (east of meridian) was 42° 10′ 0″, the index correction −3′ 20″, and height of eye above the sea 20 feet: required the error of the chronometer on Greenwich mean time. *Ans.* 4^m $57{\cdot}3^s$ fast.

404. April 18, 1848, at 0^h 40^m A.M., mean time nearly, in lat. 46° 32′ N., and long. 43° 36′ 15″ E., when a chronometer showed 10^h 13^m 45^s, the observed altitude of the star α Aquilæ was 14° 45′ 15″ (east of meridian), the index correction +4′ 5″, and height of eye above the sea 18 feet: required the error of the chronometer on Greenwich mean time.

Ans. 19^m $31{\cdot}7^s$ fast.

405. Aug. 11, 1848, at 8^h 10^m P.M., mean time nearly, in lat. 50° 20′ N., and long. 29° 53′ 15″ E., when a chronometer showed 6^h 6^m $20{\cdot}0^s$, the observed altitude of α Bootis (Arcturus) was 39° 5′ 10″ (west of meridian), the index correction −2′ 10″, and height of eye above the sea 18 feet: required the error of the chronometer on Greenwich mean time.

Ans. 11^m $17{\cdot}8^s$ slow.

Elements from Nautical Almanac.

Right ascen. mean sun.					Right ascen. and decl. of star.					
May 3 ...	2^h	43^m	$3{\cdot}3^s$	...... α Bootis ...	14^h	8^m	$43{\cdot}4^s$ ...	19°	58′	46″N.
Feb. 10 ..	21	19	46·0	...... α Leonis ...	10	0	15·3 ...	12	42	30 N.
April 17 ..	1	42	57·3	...... α Aquilæ ...	19	43	22·7 ...	8	28	16 N.
Aug. 11 ...	9	20	17·8	...... α Bootis ...	14	8	45·0 ...	19	58	39 N.

EQUAL ALTITUDES.

When the sun's center is on the meridian of any place, the apparent time is then either 0^h or 24^h. To obtain mean time at the same instant, we have only to apply the equation of time with its proper sign. We thus find mean time at the instant the sun is on the meridian; and if we can also ascertain what a chronometer showed at the same instant, it is manifest that the error of the chronometer on mean time at the place is known, since it will be the difference between the two times.

To find the time shown by the chronometer at apparent noon, we have recourse to the method of equal altitudes, which consists in noting the time shown by the chronometer when the heavenly body has the same altitude on both sides of the meridian; half the interval between the observations being added to what the chronometer showed at the first observation will be the time shown by the chronometer when the heavenly body is on the meridian, if *the declination is supposed to be invariable* in the interval between the observations.

But the sun's declination is not invariable during the interval, but increases or decreases by a small number of minutes; so that the declination at the second equal altitude is not the same as at the first; and therefore half the interval between the observations being added to the time shown by chronometer at the first observation will *not* be the time the chronometer shows when the sun is on the meridian, but will differ by a *few seconds*. This difference or quantity of time is called *the equation of equal altitudes*, and is found by the following rule:

Rule 46. *To find the error of a chronometer on mean time at the place by* EQUAL ALTITUDES OF THE SUN.

1. Find mean time nearly of apparent noon at the place by taking out of the *Nautical Almanac* the equation of time to the nearest minute, and applying it with its proper sign to 0^h or 24^h, according as the *Nautical Almanac* directs it to be added to or subtracted from apparent time, putting the day one back in the latter case.

2. To mean time nearly thus found apply the longitude in time, adding if west, and subtracting if east; the result will be a Greenwich date.

3. Correct the equation of time for this date.

4. From the P.M. time when the second altitude was taken (increased by 12 hours) subtract the A.M. time when the first altitude was taken; the remainder is elapsed time, as shown by the chronometer. Take half the

elapsed time and subtract it from the above date (increased if necessary by 24 hours and the day put one back); the remainder is a second Greenwich date.

5. Take out the sun's declination for this date.

6. *To find the equation of equal altitudes.* Under heads (1) and (2) put down the following quantities:

Under (1) put A, taken from annexed table.
" (2) put B, " "
" (1) put log. cotangent latitude.
" (2) put log. cotangent declination.
" both (1) and (2) put proportional log. change of declination in 24 hours.

7. Add together logarithms under (1) and (2), and reject the tens in the index; look out the result as a proportional logarithm, and take out the seconds and tenths corresponding thereto.

8. Mark the quantities under (1) with the sign plus (+) if the declination is decreasing, and of the same name as the latitude; or if increasing and of a different name. Otherwise mark the quantity (—).

9. Mark the quantity under (2) plus (+) if the declination is increasing, but minus (—) if decreasing.

10. Take the sum or difference of these quantities, according as they have the same or different signs; the result will be the correction or equation of equal altitudes required.

11. Add together A.M. time and half-elapsed time, and to the same apply the correction just found, with its proper sign; the result will be the time shown by the chronometer when the sun's center is on the meridian.

12. Find mean time at the same instant by applying the equation of time to 0^h or 24^h, with the proper sign, as directed in the *Nautical Almanac.*

13. Put down under each other the results determined in (11) and (12), and take the difference, which will be the error of the chronometer on mean time *at the place.*

14. *To find error of the chronometer on Greenwich mean time.* To mean time at the place as found in (12) apply the longitude in time, and thus get mean time at Greenwich, under which put the time shown by chronometer as found in (11); the difference will be the error of the chronometer on *Greenwich* mean time.

EQUATION OF EQUAL ALTITUDES.

Elapsed time.	A	B	Elapsed time.	A	B	Elapsed time.	A	B
1 30	1·97148	1·97991	4 30	1·94886	2·02901	7 30	1·90212	2·15738
1 40	1·97082	1·98123	4 40	1·94692	2·03356	7 40	1·89876	2·16854
1 50	1·97009	1·98272	4 50	1·94490	2·03833	7 50	1·89531	2·18033
2 0	1·96930	1·98435	5 0	1·94281	2·04334	8 0	1·89177	2·19280
2 10	1·96843	1·98614	5 10	1·94064	2·04861	8 10	1·88815	2·20602
2 20	1·96750	1·98808	5 20	1·93840	2·05414	8 20	1·88444	2·22003
2 30	1·96649	1·99017	5 30	1·93608	2·05996	8 30	1·88064	2·23493
2 40	1·96541	1·99243	5 40	1·93368	2·06605	8 40	1·87676	2·25081
2 50	1·96426	1·99484	5 50	1·93122	2·07246	8 50	1·87278	2·26775
3 0	1·96305	1·99743	6 0	1·92866	2·07918	9 0	1·86870	2·28587
3 10	1·96176	2·00019	6 10	1·92604	2·08624	9 10	1·86454	2·30531
3 20	1·96040	2·00312	6 20	1·92333	2·09365	9 20	1·86029	2·32623
3 30	1·95897	2·00623	6 30	1·92054	2·10143	9 30	1·85593	2·34882
3 40	1·95747	2·00954	6 40	1·91767	2·10961	9 40	1·85148	2·37334
3 50	1·95589	2·01303	6 50	1·91473	2·11821	9 50	1·84692	2·40003
4 0	1·95424	2·01671	7 0	1·91170	2·12725	10 0	1·84427	2·42928
4 10	1·95252	2·02060	7 10	1·90859	2·13678	10 10	1·83752	2·46152
4 20	1·95073	2·02470	7 20	1·90539	2·14680	10 20	1·83267	2·49733

EXAMPLE.

406. Aug. 7, 1851, in lat. 50° 48′ N., and long. 1° 6′ W., the sun had equal altitudes at the following times by chronometer:

A.M.	P.M.
$9^h\ 25^m\ 42{\cdot}5^s$	$2^h\ 59^m\ 55{\cdot}6^s$

Required the error of chronometer on mean time at the place, and also on mean time at Greenwich.

August 7	0^h	0^m	apparent time.
equation of time		5+	
	0	5	mean time
long. in time		4	
Greenwich, August 7 ...	0	9	1st date for eq. of time.
½ elapsed time	2	47	
Greenwich, August 6 ...	21	22	2d date for decl.

Equation of time.		Diff. for 1 hour.					
Aug. 7 ... 5^m $33{\cdot}03^s$		10^m is $\frac{1}{6}$	$0{\cdot}3^s$ sub.	P.M.	14^h	59^m	$55{\cdot}6^s$
$\cdot 05$		....	$0{\cdot}05$	A.M.	9	25	42·5
5 32·98 +				elapsed T.	5	34	13·1
				$\frac{1}{2}$ elap. T.	2	47	6·55

Sun's declination.	(1).	(2).
6th............ 16° 49′ 12″ N.	A ....1·93608	B ....2·05996
7th............ 16 32 38 N.	cot. lat. .....9·91147	cot. decl.....0·52631
16 34	prop. log. ...1·03604	prop. log....1·03604
·05048	prop. log. ...2·88359	prop. log. ...3·62231
1·03604	$14{\cdot}2^s$ +	2·55 −
1·08652 14 45	2·55 −	
Declination..16 34 27 N.	Eq. of equal altitudes.. 11·65 +	

A.M.	9^h	25^m	$42{\cdot}5^s$
$\frac{1}{2}$ elapsed time	2	47	6·55
	0	12	49·05
equation of equal altitude			11·65 +
time by chronometer at apparent noon		13	0·70
apparent time at apparent noon...............	0^h	0^m	0^s
equation of time		5	32·98 +
mean time at apparent noon	0	5	32·98
time by chronometer at apparent noon	0	13	0·70
error of chronometer at place		7	27·72 fast.

To find error on Greenwich mean time.

Mean time at apparent noon	0^h	5^m	$32{\cdot}98^s$
longitude in time		4	24·00 +
mean time at Greenwich	0	9	56·98
time by chronometer	0	13	0·70
ERROR OF CHRONOMETER on Gr. mean time...		3	3·72 fast.

407. August 7, 1851, in latitude 50° 48′ N., and longitude 1° 6′ W., the sun had equal altitudes at the following times by chronometer.

A.M.	P.M.
9^h 3^m $42{\cdot}31^s$	3^h 21^m $54{\cdot}22^s$

Required the error of the chronometer on Greenwich mean time.

For Elements from *Nautical Almanac*, see preceding example.

Ans. 3^m $3{\cdot}48^s$ fast.

408. August 21, 1851, in latitude 50° 48′ N., and longitude 1° 6′ W., the sun had equal altitudes at the following times by chronometer.

A.M.	P.M.
$10^h\ 49^m\ 15{\cdot}4^s$	$1^h\ 27^m\ 27{\cdot}6^s$

Required the error of the chronometer on Greenwich mean time.

Ans. $1^m\ 8{\cdot}83^s$ fast.

Elements from Nautical Almanac.

Equation of time $3^m\ 1{\cdot}94^s$ + difference for $1^h\ 0{\cdot}606^s$ —
Declination, 20th, 12° 34′ 59″ N. 21st, 12° 15′ 9″ N.

409. September 10, 1851, in latitude 50° 48′ N., and long. 1° 6′ W., the sun had equal altitudes at the following times by chronometer.

A.M.	P.M.
$9^h\ 45^m\ 55{\cdot}2^s$	$2^h\ 20^m\ 39{\cdot}9^s$

Required the error of the chronometer on Greenwich mean time.

Ans. $3^m\ 47{\cdot}28^s$ slow.

Elements from Nautical Almanac.

Equation of time $2^m\ 58{\cdot}43^s$ + difference in $1^h\ 0{\cdot}866^s$ +
Declination, 9th, 5° 27′ 27″ N. 10th, 5° 4′ 45″ N.

410. May 14, 1844, in latitude 50° 48′ N., and longitude 15° 0′ W., the sun had equal altitudes at the following times by chronometer.

A.M.	P.M.
$10^h\ 46^m\ 57{\cdot}0^s$	$1^h\ 39^m\ 42{\cdot}0^s$

Required the error of the chronometer on the mean time at the place, and also on Greenwich mean time.

Ans. Fast on mean time at place, $17^m\ 4{\cdot}7^s$.
Slow on Greenwich mean time, $42^m\ 55{\cdot}3^s$.

Elements from Nautical Almanac.

Equation of time $3^m\ 53{\cdot}8^s$ — difference in $1^h\ 0{\cdot}01^s$ —
Declination, 13th, 18° 28′ 49″ N. 14th, 18° 43′ 21″ N.

To find the approximate time by chronometer when the P.M. *altitudes should be observed.*

After taking the observations in the morning, it will often be convenient to estimate nearly at what *time by the chronometer* the observer should prepare to take the P.M. sights. To do this the error of the chronometer on mean time *at the place* must be supposed to be known within a few minutes. Thus suppose (as in the last example) a chronometer is known to be about 17 minutes *fast* of mean time at the place, the time of the A.M. observation was by chronometer at $10^h\ 46^m\ 57^s$, equation of time 4 minutes subtractive from apparent time. It is required to find the time the chronometer will show in the afternoon when the sun has the same altitude.

Let a=estimated error of chronometer on mean time at place (supposed fast),

t=time shown by chronometer at A.M. observation.

Then $t-a$=mean time at A.M. observation nearly.

Let E=equation of time (supposed subtractive from apparent time).

$\therefore$ $t-a+\text{E}$=apparent time at A.M. observation.

$\therefore$ $12-(t-a+\text{E})$=apparent time from noon,

=apparent time of P.M. observation.

$\therefore$ $12-(t-a+\text{E})-\text{E}$=mean time of P.M. observation.

And $12-(t-a+\text{E})-\text{E}+a$=mean time of P.M. observation *by chronometer*.

$\therefore$ Mean time of P.M. observation as shown by the chronometer

$$=12-(t-a+\text{E})-\text{E}+a$$
$$=12-t+2\,(a-\text{E}).$$

Thus (see Example) let $t=10^h\ 46^m\ 57^s$, $a=17^m$, $\text{E}=4^m$;

$\therefore$ Time by chronometer$=1^h\ 13^m\ 3^s+26^m=1^h\ 39^m$.

It appears from this that the observer need not prepare to take his P.M. sights until $1^h\ 30^m$ by chronometer.

A similar formula may be made to suit any other case.

A *blank form* for finding the error and rate of a chronometer by equal altitudes is given in page 186, Part II.

The use of blank forms diminishes very much the labour of calculation in some of the problems in Nautical Astronomy, such as the lunar, occultations, double altitudes, &c.

CHAPTER VII.

THE LONGITUDE.

The two principal methods for finding the longitude at sea by astronomical observations, are (1) by means of a chronometer, whose error is known on Greenwich mean time; and (2) by observing the distance of the moon from some well known star, and calculating from thence Greenwich mean time: ship mean time is to be obtained in both methods by the same kind of observation.

To find the longitude by chronometer, an altitude of a heavenly body is to be taken—an operation requiring very little skill in the observer.

To find the longitude by lunar observations, the distance of the moon from some other heavenly body must be observed with considerable accuracy: the skill necessary to do this can only be acquired by practice; for this reason the method of finding the longitude by chronometer is the one chiefly in use, although the longitude deduced from it depends on the regular going of a time-keeper, whose rate from various causes is continually liable to change, while the other, which in fact is (within certain limits) correct and independent of all errors of chronometer, is rarely applied. Another objection usually urged against the use of the method of finding the longitude by lunar observation, is the labour required in reducing the observations; but we will endeavour to show that this ought not to deter the student; for that the work, although certainly more laborious than that required by the other method, is simple, and no ambiguity or distinction of cases need occur to distract the observer. From our own impression of the utility of lunars we feel it right to devote more than usual space to this method of finding the longitude, and we shall therefore give a series of distinct rules to suit every case that can occur.

LONGITUDE BY CHRONOMETER.

When a chronometer is taken to sea, the error on Greenwich mean time and its daily rate are supposed to accompany it: knowing then the error and rate, it is easy to determine the Greenwich mean time *at any instant* after-

wards by applying its original error and the accumulated loss or gain in the interval.

The corresponding mean time at the ship may be found by observing the altitude of the sun, or any other heavenly body, when it bears as nearly east or west as possible.

The difference between the two times is the longitude of ship.

Rule 47. Longitude by sun chronometer.

1. Get a Greenwich date.

2. Find Greenwich mean time at the instant of the observation, by bringing up the error of the chronometer by Rule 42, p. 161.

3. Take out of the *Nautical Almanac* both the declination of the sun and the equation of time, for the noon before and the noon after the Greenwich date; take out also the sun's semidiameter.

4. Correct the declination and equation of time for the Greenwich date (or rather for the Greenwich mean time as shown by the chronometer), either by proportional logarithms or by hourly differences.

5. Correct the observed altitude for index correction, dip, semi., correction in alt., and thus get the true altitude, which subtract from 90° to obtain the zenith distance.

6. *To find ship apparent time.* First method (using log. haversines). Under the latitude put the sun's declination, and, if the names be alike, take the difference; but if unlike take the sum. Under the result put the zenith distance, and find the sum and difference. Add together the log. secants of the two first terms in this form (omitting the tens in each index) and the halves of the log. haversines of the two last, and (rejecting the ten in the index) look out the sum as a log. haversine, *to be taken out at the top of the page if the sun is west of the meridian, but at the bottom of the page if the sun is east of the meridian;* the result is apparent time at the ship at the instant of observation.

7. *To find ship apparent time.* Second method (using log. sines, &c.). Proceed as pointed out in Rule 43, second method, and Example 395.

8. *To find ship mean time.* To the apparent time just obtained, apply the equation of time, with its proper sign as directed in the *Nautical Almanac:* the result is mean time at the ship or place of observation.

9. *To find the longitude.* Under ship mean time put Greenwich mean time as known by the chronometer; the difference is the longitude in time, *west,* if the Greenwich time is greater than ship time, otherwise *east.*

EXAMPLE. SUN WEST OF MERIDIAN.

411. Sept. 23, 1845, at 4^h 42^m P.M. mean time nearly, in latitude 50° 30′ N., and longitude by account 110° 0′ W., when a chronometer showed 11^h 59^m 30^s, the observed altitude of the sun's lower limb was 11° 0′ 50″, the index correction −3′ 20″, and the height of the eye above the sea 20 feet: required the longitude. On August 21 the chronometer was fast on Greenwich mean time 0^m $45{\cdot}5^s$, and its daily rate was 5·7 losing.

		Sun's declination.		Eq. of time.		Semi.
Ship, Sept. 23	4^h 42^m	Sept. 23 . .	0° 6′ 56″ S.	Sept. 23 .	7^m $42{\cdot}0^s$ sub.	15′ 58″
long. in time	7 20 W.	„ 24 . .	0 30 21 S.	„ 24 .	8 2·5	
Gr., Sept. 23 .	12 2		23 25		20·5	
Interval of time from Aug. 21 to Gr. date .	33^d 12^h	·29983 ·88575 1·18558	11 44	·29983 2·72167 3·02150	10·2	
		∴ sun's decl.	18 40 S.		7 52·2	

Daily rate	$5{\cdot}7^s$ losing	Obs. alt.	11° 0′ 50″
	33	index cor.	3 20−
	171		10 57 30
	171	dip	4 24−
	188·1		10 53 6
12^h . . ½	2·8	semi.	15 58
	60) 190·9		11 9 4
accumulated loss .	3^m 10·9	cor. in alt.	4 38−
chron. showed . .	11^h 59 30·0		11 4 26
	12 2 40·9		90
original error . .	0 45·5 fast.	∴ zenith distance . . .	78 55 34
∴ Gr., Sept. 23 . .	12 1 55·4		

Ship apparent time by First Method, using haversines.

Latitude	50° 30′ 0″ N. . .	log. sec.	0·196490
declination . . .	0 18 40 S. . .	„ sec.	0·000006
	50 48 40	„ ½ hav. S . .	4·956810
zenith distance .	78 55 34	„ ½ hav. D . .	4·385440
sum	129 44 14 (S)		9·538746
difference . . .	28 6 54 (D)		

ship app. time	4^h 48^m 7^s
equation of time	7 52·2
ship mean time	4 40 14·8
Greenwich mean time . . .	12 1 55·4
longitude in time	7 21 40·6
∴ LONGITUDE	110° 25′ 9″ W.

Or thus: Ship apparent time, by second method, using sines, &c. only.

Latitude . . .	50°	30′	0″ N.	. . log. sec.		0·196490
declination . .	0	18	40 S.	. . „ sec.		0·000006
	50	48	40	„ sin. S_1 . . .		9·956803
zenith distance .	78	55	34	„ sin. D_1 . . .		9·385445
sum	129	44	14		2)	19·538744
difference . . .	28	6	54	„ sin.		9·769372
½ sum	64	52	7 (S_1)			36° 4 0″
½ difference . .	14	3	27 (D_1)		or	2h 24m 4s
						2
				∴ ship app. time		4 48 8

EXAMPLE. SUN EAST OF MERIDIAN.

412. April 18, 1844, at 9^h 18^m A.M. mean time nearly, in latitude 50° 48′ N., and longitude by account 1° 0′ W., when a chronometer showed 9^h 27^m 48^s, the observed altitude of the sun's lower limb was 76° 16′ 46″ (in artificial horizon), index correction 3′ 46″—: required the longitude. On April 1 the chronometer was fast 1^m $58{\cdot}7^s$ on Greenwich mean time, and its mean daily rate was $11{\cdot}2^s$ gaining.

			Sun's declination.				Eq. of time.		Semi.
Ship April 17	21h	18m	April 17 . .	10° 36′ 49″ N		17 . .	0′ 31·3″ sub.		15′ 56″
long. in time		4W	„ 18 . .	10 57 46 N		18 . .	0 45·1		
Gr., April 17	21	22		20 57			13·8		
			·05048			·05048			
Interval from			·93409			2·89354			
Ap. 1 at noon			·98457 .	18 39		2·94402	12·3		
to April 17, 16d 21¼h			sun's decl. .	10 55 28 N.			0 43·6		

Daily rate			11·2s gaining.	
			16	
			672	
			112	
			179·2	
	12h . . ½		5·6	
	6 . . ¼		2·8	
	3 . . ⅛		1.4	
	¼ . . ⅙		·2	
		60	189·2	
accum. gain . .		3m	9·2s	
chro. showed . .	9	27	48·0	
	9	24	38·8	
original error . .		1	58·7 fast.	
	9	22	40·1 A.M.	
	12			
Gr., April 17 . .	21	22	40·1	

Obs. alt.		76°	16′	46″
index cor.			3	46—
	2)	76	13	0
		38	6	30
semi			15	56
		38	22	26
cor. in alt.			1	7—
sun's true alt.		38	21	19
		90		
„ zenith distance . .		51	38	41

Ship apparent time. First method, using haversines.

Latitude	50°	48′	0″ N. . .	log. sec. . . .	0·199263
declination . . .	10	55	28 N. . .	„ sec. . . .	0·007943
	39	52	32	„ ½ hav. S. .	4·855173
zenith distance .	51	38	41	„ ½ hav. D. .	4·010890
sum	91	31	13 (S)	„ hav. . . .	9·073269
difference . . .	11	46	9 (D)		

ship apparent time	21^h	19^m	0^s
equation of time			43·6—
ship mean time	21	18	16·4
Greenwich mean time . . .	21	22	40·1
longitude in time		4	23·7
or LONGITUDE . . .	1° 6′ W.		

Or thus: Ship apparent time. Second method, using sines, &c. only.

Latitude . . .	50°	48′	0″ N. . .	log. sec.	0·199263
declination . .	10	55	28 N. . .	„ sec. . . .	0·007943
	39	52	32	„ sin. S_1 . .	9·855158
zenith distance .	51	38	41	„ sin. D_1 . .	9·010737
sum	91	31	13		2) 19·073101
difference . . .	11	46	9		9·536500
½ sum	45	45	36 (S_1)		20° 7′ 15″
½ difference . .	5	53	4 (D_1)		or 1^h 20^m 29^s
					2
					2 40 58
					24
∴ ship apparent time . . .					21 19 2

413. Sept. 25, 1845, at 4^h 20^m P.M. mean time nearly, in latitude 59° 30′ N., and longitude by account 112° 30′ W., when a chronometer showed 11^h 44^m 20^s, the observed altitude of the sun's lower limb was 10° 50′ 10″, the index correction +6′ 10″, and height of eye above the sea 18 feet: required the longitude. On Sept. 20 the chronometer was fast on Greenwich mean time 0^m $30·7^s$, and its daily rate was $10·5^s$ losing.

Ans. 112° 33′ W.

414. May 30, 1845, at 3^h 10^m P.M. mean time nearly, in latitude 30° 12′ 0″ S., and longitude by account 156° 0′ E., the observed altitude of the sun's lower limb was 21° 8′ 40″ when a chronometer showed 4^h 44^m 56^s, the index correction —1′ 10″, and the height of eye above the sea 30 feet: required the longitude. On May 19 the chronometer was fast 5^m 16^s on Greenwich mean time, and its daily rate was $3·5^s$ gaining. *Ans.* 156° 23′ E.

415. July 8, 1849, at 1^h 40^m P.M. mean time nearly, in latitude 50° 48′ N., and longitude by account 1° 1′ W., the observed altitude of the sun's lower limb, taken by the artificial horizon, was 109° 54′ 44″, the chronometer

showed $1^h\ 44^m\ 14^s$, the index correction $+1'\ 25''$: required the longitude. On July 1 the chronometer was slow on Greenwich mean time $8^m\ 18{\cdot}4^s$, and its daily rate was $3{\cdot}5^s$ losing.

Ans. 1° 6′ 0″ W.

416. January 20, 1846, at $6^h\ 40^m$ A.M. mean time nearly, in latitude 56° 20′ S., and longitude by account 83° 10′ W., when a chronometer showed $0^h\ 14^m\ 50^s$, the observed altitude of the sun's lower limb was 20° 20′ 30″, the index correction $-1'\ 30''$, and the height of the eye above the sea 20 feet: required the longitude. On Jan. 2 the chronometer was fast on Greenwich mean time $5^m\ 20^s$, and on Jan. 6 it was fast $4^m\ 52^s$, from which may be found its mean daily rate.

Ans. 83° 5′ 0″ W.

417. Feb. 10, 1846, at $7^h\ 50^m$ A.M. mean time nearly, in latitude 50° 48′ N., and longitude by account 170° 30′ E., when a chronometer showed $9^h\ 59^m\ 25^s$, the observed altitude of the sun's lower limb was 51° 9′ 10″, the index correction $-\ 3'\ 20''$, and the height of eye above the sea 16 feet: required the longitude. On Jan. 31, at Greenwich noon, the chronometer was fast $1^h\ 34^m\ 43^s$, and its daily rate was $20{\cdot}6^s$ losing.

Ans. 170° 34′ 15″ E.

418. May 14, 1859, at $7^h\ 20^m$ A.M. mean time nearly, in lat. 50° 50′ N., and longitude by account 4° 10′ E., when a chronometer showed $7^h\ 0^m\ 20^s$, the observed altitude of the sun's lower limb was 27° 20′ 10″, the index correction $-7'\ 20''$, and the height of the eye above the sea 19 feet: required the longitude. On April 24, at noon, the chronometer was slow on Greenwich mean time $1^m\ 10{\cdot}5^s$, and its daily rate was $5{\cdot}7^s$ losing.

Ans. 4° 6′ E.

Elements from Nautical Almanac.

	Sun's declination.	Equation of time.	Semi.
Sept. 25	0° 53′ 47″ S.	8^m 23·0 sub.	15′ 59″
„ 26	1 17 12 S.	8 43·4	
May 29	21 38 43 N.	2 56·4 sub.	15 47
„ 30	21 47 47 N.	2 48·5	
July 8	22 29 9 N.	4 40·9 add.	15 45
„ 9	22 22 7 N.	4 50·1	
Jan. 20	20 8 22 S.	11 19·2 add	16 16
Feb. 9	14 41 33 S.	14 31·0 add	16 13
„ 10	14 22 11 S.	14 32·0	
May 13	18 19 29 N.	3 52·4 sub.	15 51
„ 14	18 23 13 N.	3 53·2	

Rule 48. LONGITUDE BY STAR CHRONOMETER

1. Get a Greenwich date.

2. Find Greenwich mean time by bringing up the error of the chronometer to the instant of observation by Rule 42.

3. Take out of the *Nautical Almanac* the right ascension and declination of the star, and also the right ascension of the mean sun (called in *Nautical Almanac* sidereal time) for mean noon of the Greenwich date.

4. Correct the right ascension of the mean sun for Greenwich date (or rather for the Greenwich mean time as shown by the chronometer).

5. Correct the observed altitude for index correction, dip, and refraction, and thus get the true altitude, which subtract from 90° to obtain the zenith distance.

6. *To find the star's hour-angle.* First method, using log. haversines. Under the latitude put the star's declination; add if the names be unlike, subtract if like; under the result put star's zenith distance, and take the sum and difference. Add together the log. secants of the two first terms in this form (omitting the tens in each index), and the halves of the log. haversines of the two last; the sum, rejecting ten in the index, will be the log. haversines of star's hour-angle, to be taken out at top of page if heavenly body be west of meridian, but at bottom if east of meridian.

7. *To find star's hour-angle.* Second method, using sines, &c. Proceed as pointed out in Rule 43, Ex. 395.

8. *To find mean time at ship.* To the hour-angle thus found add the star's right ascension, and from the sum, increased if necessary by 24 hours, subtract the right ascension of the mean sun; the remainder is mean time at the place at the instant of observation.

9. *To find the longitude.* Under ship mean time put Greenwich mean time as known by the chronometer; the difference is the longitude in time, *west* if Greenwich time is greater than ship time, otherwise *east.*

EXAMPLE. STAR WEST OF MERIDIAN.

419. Sept. 10, 1844, at $7^h\ 15^m$ P.M. mean time nearly, in latitude 48° 20′ N., and longitude by account 32° E., when a timekeeper showed $5^h\ 1^m\ 28^s$, the observed altitude of α Bootis (Arcturus) W. of meridian was

31° 5′ 40″, the index correction −4′ 10″, and height of eye above the sea 20 feet: required the longitude. On Aug. 25 the chronometer was slow on Greenwich mean time 2^m 40^s, and its daily rate was $4{\cdot}3^s$ gaining.

Ship, Sep. 10 . .	7^h	15^m	
long. in time . .	2	8	E.
Gr., Sep. 10 . .	5	7	
Star's R. A.	14^h	8^m	$34{\cdot}65^s$
Star's decl. .	19°	59′	44″N.
R. A. mean sun.			
10 . . .	11^h	18^m	$28{\cdot}15^s$
cor. 5^h . .			49·28
3^m .			·49
R. A. mean sun . .	11	19	17·92

Daily rate . .		$4{\cdot}3^s$ gaining.
interval 16^d 5^h.		16
		258
		43
		68·8
	4 . . $\frac{1}{6}$	0·7
	1 . . $\frac{1}{4}$	·2
	60	69·7
accum. rate		1^m $9{\cdot}7^s$
chro. show.	5	1 28·0
	5	0 18·3
orig. error .		2 40·0 slow.
Gr. M. T. .	5	2 58·3

Obs. alt. . . .	31°	5′	40″
in. cor. . . .		4	10−
	31	1	30
dip		4	24−
	30	57	6
ref.		1	37−
	30	55	29
	90		
zen. dist. . .	59	4	31

Hour-angle by First Method, using haversines.

Latitude . . .	48°	20′	0″N. . .	log. sec. . .	0·177312
declination . .	19	59	44 N. . .	„ sec. . .	0·027003
	28	20	16	„ $\frac{1}{2}$ hav. (S)	4·839453
zenith distance .	59	4	31	„ $\frac{1}{2}$ hav. (D)	4·423295
sum	87	24	47 (S)		9·467063
diff.	30	44	15 (D)	∴ hour-angle .	4^h 22^m 15^s
				star's R. A. . .	14 8 35
					18 30 50
				R. A. mean sun .	11 19 18
				ship mean time	7 11 32
				Gr. mean time .	5 2 58
				long. in time .	2 8 34
				∴ long. . . .	32° 8′ 30″ E.

Hour-angle by Second Method, using sines, &c., only.

Latitude . . .	48°	20′	0″N. . .	log. sec. . . .	0·177312
declination . .	19	59	44 N. . .	„ sec. . . .	0·027003
	28	20	16	„ sin. (S_1) .	9·839470
zenith distance .	59	4	31	„ sin. (D_1) .	9·423238
sum	87	24	47		19·467023
diff.	30	44	15	„ sin. . . .	9·733511
$\frac{1}{2}$ sum	43	42	23 (S_1)		32° 46′ 45″
$\frac{1}{2}$ diff. . . .	15	22	7 (D_1)	or	2^h 11^m 7^s
					2
				∴ star's hour-angle	4 22 14

EXAMPLE. STAR EAST OF MERIDIAN.

420. May 24, 1844, at 11ʰ 11ᵐ P.M. mean time nearly, in latitude 50° 48′ N., and longitude by account 1° 0′ W., when a timekeeper showed 11ʰ 12ᵐ 11·8ˢ, the observed altitude of α Lyræ (Vega) E. of meridian was 109° 29′ 18″ in artificial horizon, the index correction −3′ 46″: required the longitude. On May 14, at Greenwich mean noon, the chronometer was slow 1ᵐ 15·8ˢ, and its mean daily rate was 7·4ˢ losing.

Ship, May 24 . .	11ʰ	11ᵐ	
long. in time . .		4	
Gr., May 24 . .	11	15	
star's R. A.	18ʰ	31ᵐ	42·2ˢ
star's decl. .	38°	38′	24″N.
Right ascension mean sun.			
24th . . .	4ʰ	8ᵐ	43·56ˢ
11ʰ . .		1	48·42
14ᵐ . .			2·30
45ˢ . .			·13
	4	10	34·4

Daily rate . . .			7·4ˢ
interval, 10ᵈ 11ʰ 15ᵐ			10
			74·0
8ʰ is ⅓			2·5
2 „ ¼			·6
1 „ ½			·3
15ᵐ „ ¼			·1
		60)	77·5 lost
Accum. rate.		1ᵐ	17·5ˢ
chro. showed	11ʰ	12	11·8
	11	13	29·3
origl. error. .		1	15·8
Gr. M. T. . .	11	14	45·1

Obs. alt. . .	109°	29′	18″
index cor. .		3	46−
2)	109	25	32
	54	42	46
ref. . . .			41−
	54	42	5
	90		
zen. dist. .	35	17	55

Hour-angle by First Method, using haversines.

Latitude	50°	48′	0″ N. . .	log. sec. . . .	0·199263
declination . . .	38	38	24 N. . .	„ sec. . . .	0·107300
	12	9	36	„ ½ hav. (S) .	4·604673
zenith distance .	35	17	55	„ ½ hav. (D) .	4·302209
Sum	47	27	31 (S)		9·213445
Difference . . .	23	8	19 (D)		

Hour-angle	20ʰ	49ᵐ	13ˢ
star's right ascension . . .	18	31	42·2
	39	20	55·2
right ascension mean sun .	4	10	34·4
ship mean time	11	10	20·8
Greenwich mean time . . .	11	14	45·1
longitude in time		4	24·3 = 1° 6′ W.

Hour-angle by Second Method, using sines, &c., only.

Latitude . . .	50°	48′	0″ N. . .	log. sec.	0·199263
declination . .	38	38	24 N. . .	„ sec.	0·107312
	12	9	36	„ sin. (S_1) . .	9·604673
zenith distance .	35	17	55	„ sin. (D_1) . .	9·302200
sum	47	27	31		19·213458
diff.	23	8	19	„ sin.	9·606729
½ sum	23	43	45 (S_1)		23° 51′ 0″
½ diff.	11	34	9 (D_1)	or	1ʰ 35ᵐ 24ˢ
					2
					3 10 48
					24
				∴ star's hour-angle . .	20 49 12

421. Aug. 20, 1845, at 0^h 30^m A.M. mean time nearly, in lat. 50° 20′ N., and long. by account 142° 0′ E., when a chronometer showed 2^h 41^m 12^s, the observed altitude of the star α Aquilæ (Altair) was 36° 59′ 50″ west of the meridian, the index correction + 6′ 30″, and height of eye above the sea 20 feet: required the longitude. On August 1 the chronometer was slow on Greenwich mean time 17^m $45{\cdot}0^s$, and its daily rate was $4{\cdot}3^s$ losing.

Ans. 142° 14′ 15″ E.

422. Sept. 10, 1844, at 4^h 21^m A.M. mean time nearly, in lat. 40° 36′ N., and longitude by account 73° E., when a chronometer showed 11^h 21^m 56^s, the observed altitude of β Geminorum (Pollux) was 39° 0′ 10″ east of meridian, the index correction —4′ 10″, and height of eye above the sea 20 feet: required the longitude. On Aug. 20 the chronometer was slow on Greenwich mean time 3^m $19{\cdot}9^s$, and its daily rate was $9{\cdot}3^s$ gaining.

Ans. 72° 45′ 45″ E.

423. January 16, 1845, at 8^h 0^m P.M. mean time nearly, in latitude 49° 56′ 50″ N., and longitude by account 90° 30′ E., when a chronometer showed 2^h 24^m 30^s, the observed altitude of α Leonis (Regulus) was 8° 4′ 20″ east of meridian, the index correction —4′ 20″, and height of eye above the sea 25 feet: required the longitude. On January 1 the chronometer was fast on Greenwich mean time 5^m $30{\cdot}5^s$, and its daily rate was $5{\cdot}5^s$ losing.

Ans. 86° 6′ E.

424. January 20, 1846, at 8^h 30^m P.M. mean time nearly, in latitude 50° 48′ N., and long. by account 7° 10′ W., when a chronometer showed 8^h 32^m 50^s, the observed altitude of ϵ Leonis was 28° 0′ 10″ east of meridian, the index correction —6′ 20″, and height of eye above the sea 20 feet: required the longitude. On January 2 the chronometer was fast on Greenwich mean time 30^m 30^s, and its mean daily rate was $15{\cdot}5^s$ losing.

Ans. 7° 19′ E.

Elements from Nautical Almanac.

	Right ascen. mean sun.			Star's right ascen.			Star's decl.			
Aug. 19	9^h	50^m	$46{\cdot}5^s$	19^h	43^m	$17{\cdot}0^s$	8°	28′	7″	N.
Sept. 9	11	14	31·6	7	35	48·6	28	23	40	N.
Jan. 16	19	43	7·2	10	0	8·8	12	43	6	N.
Jan. 20	19	57	55·9	9	37	8·3	24	28	33	N.

LONGITUDE BY LUNAR OBSERVATIONS.

The *time at the ship* is obtained by the same kind of observation as that for finding the longitude by chronometer, selecting of the two bodies observed that whose bearing is the greatest. The *time at Greenwich* is found by calculating the *true distance* of the moon from the sun or some other heavenly body at the moment of observation, and comparing it with the true distance of the moon from the same heavenly body as recorded in the *Nautical Almanac* for some given time at Greenwich.

TO CALCULATE THE TRUE DISTANCE.

The true distance is found by clearing the observed distance of the effects of parallax and refraction, by the following or some other similar methods.

In *Nav.* Part II. are given the investigations of several methods of clearing the distance. We will here confine ourselves to two (the first and sixth in Part II.); the former, because it is the most simple method when Inman's Tables are at hand; the latter, because it is adapted to any book of logarithms that contains tables of log. sines, &c.

The practical inconvenience of this last method arises from the necessity of taking out the log. sines, &c. to the nearest second, a work of considerable labour with the common tables of log. sines, &c., which seldom give the arcs nearer than a minute or 15″. We have also to add together the unusual number of six logarithms, and this can seldom be done with accuracy unless great attention is given to the addition. This last objection may be somewhat removed by making use of a small auxiliary table, investigated in *Nav.* Part II. (which we will reprint here and call table log. C). The correction taken out of this table is equivalent to the sum of two of the above six logarithms; but notwithstanding this simplification the method is tedious, the labour of proportioning for seconds not being got rid of. For these reasons the first method is to be preferred, being simple and direct; and as it is made to depend on the table of versines, which is computed to seconds, all the quantities can be taken out *by inspection*, thereby entirely avoiding the trouble of proportioning.

TO CLEAR THE LUNAR DISTANCE.

First method: by Inman's Tables.

In *Nav.* Part II. it is proved that

$$\text{vers. } \mathrm{D} = \text{vers. } (z+z_1) + \text{ver. } (a+a_1+\mathrm{A}) + \text{ver. } (a+a_1 \backsim \mathrm{A}) \\ + \text{ver. } (d+\mathrm{A}) + \text{ver. } (d \backsim \mathrm{A}) - 4{,}000{,}000\,;$$

where z and z_1 are the true zenith distances, a and a_1 are the apparent altitudes, d the apparent distance of the centers of the two heavenly bodies observed, $\mathrm{A} = \frac{1}{2} \cdot \dfrac{\text{sin. } z \text{ sin. } z_1}{\text{cos. } a \cos a_1}$ a quantity tabulated and called the auxiliary

angle A, and D the true distance to be found. From this formula the following rule may be deduced.

Rule 49. To CLEAR THE LUNAR DISTANCE.

1. Under the sun's or star's true zenith distance put the moon's true zenith distance; take the sum, which mark vers.

2. Under the apparent distance of the two centers put the auxiliary angle A; take their sum and difference, against both of which mark vers.

3. Under the sun's or star's apparent altitude put the moon's apparent altitude, and take their sum; under which put the auxiliary angle A: take the sum and difference, against both of which mark vers.

4. Add together the five last figures of the versines of the quantities marked vers., rejecting all but the last five in the result, which look for in the column of versines under the apparent distance, or under the adjacent one: take out the arc corresponding thereto, which will be the true distance required.

NOTE. The auxiliary angle A is found in the Nautical Tables of Inman, Riddle, Norie, and others.

The student will be able to determine the relative value of the two methods by working an example by each.

EXAMPLE.

425. Required the true distance of the moon from the sun, having given

App. dis. of the centers	35°	47′	24″	True alt. sun	34°	20′	14″
app. alt. sun	34	21	32	true alt. moon	57	40	11
app. alt. moon	57	11	25	auxiliary angle A ...	60	25	16

Sun's true zen. dist. z.	55°	39′	46″				Parts for seconds.
moon's true zen. dis. z_1.	32	19	50				
$z+z_1$	87	59	36	vers.	964810	......	174
					1107999	......	195
apparent distance d .	35	47	24		90885	......	104
aux. angle A	60	25	16		1882674	......	30
$d+$A	96	12	40	vers.	143883	......	102
$d\sim$A	24	37	52	vers.	4190251	......	605
					605		
moon's app. alt. a	57	11	25		4190856		
sun's app. alt. a_1	34	21	32		4000000		
	91	32	57		vers. 190856		
auxiliary angle A	60	25	16				
$a+a_1+$A	151	58	13	vers.	∴ True dist. ... 35° 59′ 15″		
$a+a_1\sim$A	31	7	41	vers.			

In practice it is not necessary to take from the table of versines more than the last five figures, rejecting also all but these last five in the sum, since the true distance will be always either in the same column with the apparent distance or the adjacent one. Thus, taking the above example, it may be worked thus :

				Vers.		Parts for seconds.
55°	39′	46″				
32	19	50				
87	59	36	vers.	64810	....	174
				07999	....	195
35	47	24		90885	....	104
60	25	16		82674	....	30
96	12	40	vers.	43883	....	102
24	37	52	vers.	90251	....	605
				605		
57	11	25		90856	......	35° 59′
34	21	32		812		
91	32	57		44	......	15″
60	25	16		True dist.	......	35 59 15
151	58	13	vers.			
31	7	41	vers.			

NOTE. The figures 90856 are looked for in the table of versines in the column under the degrees of the apparent distance, 35°, or the one adjacent.

TO CLEAR THE LUNAR DISTANCE.

Second method: by the common table of sines, &c.

In *Nav.* Part II. it is proved that

$$\text{M}=\sqrt{\cos.\ \text{A}_1 \sec.\ a_1 \cos.\ \text{A} \sec a \cos.\ \tfrac{1}{2}(a+a_1+d) . \cos.\ \tfrac{1}{2}(a+a_1-d)}$$

$$\tan\theta=\frac{\text{M}}{\sin.\ \frac{1}{2}(\text{A}+\text{A}_1)},\ \text{and}\ \cos.\frac{\text{D}}{2}=\frac{\text{M}}{\sin.\ \theta}.$$

where A and A_1 are the true altitudes of the moon and other heavenly body observed; a and a_1 their apparent altitudes; d the apparent distance of the centers of the two heavenly bodies observed, and D the true distance to be found.

From the above formulæ the following method may be deduced for clearing the lunar distance:

1. Add together the true altitudes, and divide by 2: call the result $\frac{1}{2}(\text{A}+\text{A}_1)$.

2. Add together the apparent altitudes, under which put the apparent distance. Take the sum and difference, and divide each by 2, and call the result the half-sum and half-difference.

3. Add together the following six logarithms:

log. cosine of the half-sum,
,, cosine of the half-difference,
,, cosine of moon's true altitude,
,, secant of moon's apparent altitude (rejecting 10 in index),
,, * cosine of sun or star's true altitude,
,, * secant of sun or star's app. altitude (rejecting 10 in index).

Divide the sum by 2, and call the result log. M.

4. Put down log. M a second time, a little to the right.

5. From log. M subtract log. sin. $\frac{1}{2}(A+A_1)$; the remainder will be the log. tangent of an arc, which take from the tables and call θ.

6. From log. M subtract log. sine of θ: the remainder is the log. sine of half the true distance. Take this from the tables, and multiplied by 2 will be the TRUE DISTANCE required.

TABLE LOG. C.

FOR THE *SUN.* ARGUMENT. SUN'S APP. ALT.		FOR A *STAR* OR *PLANET.* ARGUMENT. STAR'S APP. ALT.	
App. alt.	Log. C.	App. alt.	Log. C.
90	0·000104	90	0·000123
75	0·000105	30	0·000122
65	0·000106	20	0·000121
60	0·000107	15	0·000120
55	0·000108	12	0·000119
50	0·000109	10	0·000118
45	0·000110	9	0·000117
40	0·000111	8	0·000116
35	0·000112	7	0·000114
30	0·000113	6	0·000111
25	0·000114	5	0·000107
20	0·000115	4	0·000100
10	0·000114	3	0·000090
8	0·000113		
7	0·000112		
6	0·000109		
4	0·000100		

Use of Table log. C.

Instead of taking out the last two of the above six logarithms (viz. those marked thus *), enter table log. C with the sun or star's apparent altitude, and take out the corresponding logarithm. This quantity, added to the four logarithms already taken out (the index 10 of sec. being retained), will give the same result as the sum of the six logarithms.

426. The apparent distance of centers of sun and moon, 35° 47′ 24″; sun's apparent altitude, 34° 21′ 32″; moon's apparent altitude, 57° 11′ 25″; sun's true altitude, 34° 20′ 14′; moon's true altitude, 57° 40′ 10″: required the true distance.

Apparent dist. (d) . . 35° 47′ 24″

Sun's app. alt. (a_1)	34°	21′	32″	Sun's true alt. (A_1)	34°	20′	14″
moon's app. alt. (a)	57	11	25	moon's true alt. (A)	57	40	10
$a+a_1$	91	32	57	$A+A_1$	92	0	24
d	35	47	24	$\therefore \frac{1}{2}(A+A_1)$	46	0	12
$a+a_1+d$	127	20	21				
$a+a_1-d$	55	45	33				
$\therefore \frac{1}{2}(a+a_1+d)$	63	40	10	log. cos.	9·646939		
$\frac{1}{2}(a+a_1-d)$	27	52	46	„ cos.	9·946419		
				„ cos. A	9·728194		
				„ sec. a	0·266121		
				„ cos. A_1	9·916840		
				„ sec. a_1	0·083273		
					39·587786		

$\therefore$ log. M	19·793893	. . . . .	19·793893
„ sin. $\frac{1}{2}(A+A_1)$	9·856959	log. sin. θ	9·815670
„ tan. θ	9·936934	„ cos. $\frac{D}{2}$	9·978223
$\therefore \theta$	40° 51′ 16″	$\therefore \frac{D}{2}$	17° 59′ 36″
			2
		$\therefore$ TRUE DISTANCE =	35 59 12

To find log. M *by Table log.* C.

	9·646939
	9·946419
	9·728194
	10·266121
log. C	112
	39·587785
log. M	19·793893
	as before.

EXAMPLES.

Find the TRUE DISTANCE in the following examples:

427. Sun's app. alt. is 30° 29′ 48″; moon's app. alt., 50° 54′ 38″; moon's true zen. dist., 38° 30′ 40″; sun's true zen. dist., 59° 31′ 44″; the auxiliary angle A, 60° 24′ 12″; and app. dist., 88° 49′ 58″.

Ans. True distance, 88° 24′ 17″.

428. Sun's app. alt. is 54° 29′ 33″; moon's app. alt., 5° 25′ 59″; moon's true zen. dist., 83° 48′ 29″; sun's true zen. dist., 35° 31′ 3″; auxiliary angle A, 60° 2′ 11″; and app. dist., 105° 5′ 47″.

Ans. True distance, 104° 26′ 18″

429. Sun's app. alt. is 17° 39′ 31″; moon's app. alt., 24° 13′ 45″; moon's zen. dist., 64° 56′ 45″; sun's zen. dist., 72° 23′ 22″; auxiliary angle A, 60° 12′ 33″; and app. dist., 111° 20′ 45″.

Ans. True distance, 110° 56′ 0″.

430. Sun's app. alt. is 54° 47′ 4″; moon's app. alt., 21° 20′ 1″; moon's zen. dist., 67° 51′ 5″; sun's zen. dist., 35° 13′ 32″; auxiliary angle A, 60° 10′ 44″; and app. dist., 71° 16′ 44″.

Ans. True distance, 70° 38′ 5″.

431. Sun's app. alt. is 12° 19′ 30″; moon's app. alt., 20° 40′ 18″; moon's

zen. dist., 68° 28′ 19″; sun's zen. dist., 77° 44′ 42″; auxiliary angle A, 60° 10′ 53″; and app. dist., 124° 44′ 32″.

Ans. True distance, 124° 19′ 11″.

432. Sun's app. alt. is 57° 53′ 52″; moon's app. alt., 35° 3′ 2″; moon's zen. dist., 54° 11′ 56″; sun's zen. dist., 32° 6′ 40″; auxiliary angle A, 60° 17′ 54″; and app. dist., 65° 34′ 42″.

Ans. True distance, 64° 58′ 10″.

433. Sun's app. alt. is 15° 43′ 48″; moon's app. alt., 16° 5′ 5″; moon's zen. dist., 73° 1′ 32″; sun's zen. dist., 74° 19′ 28″; auxiliary angle A, 60° 8′ 36″; and app. dist., 119° 44′ 31″.

Ans. True distance, 119° 19′ 51″.

434. App. alt. of a star is 20° 13′ 26″; moon's app. alt., 31° 17′ 22″; star's zen. dist., 69° 49′ 11″; moon's zen. dist., 57° 57′ 44″; auxiliary angle A, 60° 15′ 21″; and app. dist., 72° 42′ 16″.

Ans. True distance, 72° 33′ 4″.

435. App. alt. of a star is 29° 59′ 16″; moon's app. alt., 32° 30′ 10″; star's zen. dist., 60° 2′ 24″; moon's zen. dist., 56° 41′ 33″; auxiliary angle A, 60° 17′ 23″; and app. dist., 58° 44′ 19″.

Ans. True distance, 58° 30′ 21″.

Rule 50. TO FIND THE LONGITUDE BY LUNAR OBSERVATIONS.

Objects observed, sun and moon. Altitudes taken. Ship mean time determined from *sun's altitude.*

1. Get a Greenwich date.

2. Take from the *Nautical Almanac* and correct for Greenwich date the following quantities:

Sun's declination and semidiameter.

Equation of time (noting whether it is to be added to or subtracted from the ship apparent time).

Moon's semidiameter and horizontal parallax.

3. Correct the sun's apparent altitude for index correction, dip, semidiameter, correction in altitude, and thus get the sun's apparent and true altitudes. Subtract the true altitude from 90° for sun's zenith distance.

4. Correct the moon's observed altitude for index correction, dip, semidiameter (augmented), correction in altitude, and thus get the moon's apparent and true altitude. Subtract the true altitude from 90° for moon's zenith distance.

5. When the moon's correction in altitude is taken out of the Tables, take out also at the same opening the auxiliary angle A.

6. Correct the observed distance for index correction, and to the result add the semidiameter of the sun and moon (augmented), and thus get the apparent distance of the centers.

7. *To find ship mean time*, by first method, using haversines.*

Under sun's declination put the latitude of the ship; take the *sum* if their names be *unlike*, the *difference* if the names be *alike*. Under the result put the sun's zenith distance; take the sum and difference of the last two lines put down. Add together the log. secants of the two first quantities in this form (omitting to put down the tens in the index), and half of the log. haversines of each of the two last quantities. The sum will be the log. haversine of the ship apparent time. When the sun is west of the meridian, the time corresponding to the haversine must be taken out at the top of the page; but when the sun is east it must be taken out at the bottom. The result is apparent time at the ship: to this apply the equation of time with its proper sign, and the result will be the ship mean time.

8. *To calculate the true distance.* First method, using versines.†

Add together the zenith distances of the sun and moon, and mark the sum *v*.

Add together the apparent altitudes of the sun and moon, and under the sum put the auxiliary angle A: take the sum and difference of the last two quantities, and mark each with the letter *v*.

Under the apparent distance of the centers put the auxiliary angle A, and take the sum and difference, and mark each result with the letter *v*.

Add together the *five last figures* of the versines of each of the quantities marked *v*. The five last figures in the sum being looked for in the column of versines under the apparent distance or in the adjacent column, the arc corresponding thereto will be the true distance at the time of the observation.

9. *To find Greenwich mean time corresponding to this true distance.*

Take out of the *Nautical Almanac* two distances of the sun and moon three hours apart, between which is the true distance just calculated: place the first distance taken out under the true distance, and the one three hours after under the other distance taken out. Take the difference between the first and second, and also between the second and third. From the proportional logarithm of the first difference subtract the proportional logarithm of the second difference; the remainder is the proportional logarithm of a portion of time, which take from the table, and add thereto the hours corresponding to the first distance taken out of the *Nautical Almanac.* The result is Greenwich mean time when the observation was taken.

The difference between ship mean time found above and Greenwich mean time is the longitude in time: turn it into degrees, and mark it "east if the Greenwich time is the least, and west if the Greenwich time is best."

* To find ship mean, using only the common tables of sines, &c., see second method p. 164, ex. 395.

† To find true distance, using sines, &c., see second method, p. 189, ex. 426.

EXAMPLES.

436. Feb. 12, 1848, at $2^h\ 36^m$ P.M. mean time nearly, in lat. 53° 30′ N., and long. by account 15° 45′ E., the following lunar observation was taken:

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	29° 17′ 26″	25° 40′ 20″	99° 27′ 30″
Index cor.	2 10−	1 10−	0 50−

The height of the eye above the sea was 20 feet: required the longitude.

Ship, February 12	$2^h\ 36^m$
Longitude in time	1 3 E.
Greenwich, February 12 . .	1 33

Sun's declin.		Eq. of time.	Moon's semi.			Hor. par.
12th . 13° 52′ 18″S. . .		$14^m\ 33^s$ add	12th noon . 15′ 58″			58′ 36″
13th . 13 32 21 S. . .		14 32	,, mid. . 15 54			58 23
19 57		1	4			13
1·18985		∴ cor. 0		·88885	·88885	
·95533		14 33		3·43136	2·91948	
2·14518	1 17			4·32021	0·4 3·80833	1·7
13 51 1 S.			15 57·6			58 34·3
sun's semi. . 16′ 13″			Aug. . . 7·4			
			16 5			

Sun's alt.		Moon's alt.		Distance.	
Obs. alt. . .	29° 17′ 26″	Obs. alt. . .	25° 40′ 20″		99° 27′ 30″
index cor. .	2 10−	index cor. .	1 10−	index cor. .	0 50−
	29 15 16		25 39 10		99 26 40
dip. . . .	4 24−	dip . . .	4 24−	sun's semi. .	16 13
	29 10 52		25 34 46	moon's semi.	16 5
semi. . . .	16 13	semi. . . .	16 5		99 58 58
	29 27 5		25 50 51	Aux. angle.	
cor. in alt. .	1 35−	cor. in alt. .	50 13		60° 13′ 40″
	29 25 30		31		9
	90		26 41 35		3
sun's Z. D. .	60 34 30		90		60 13 52
		moon's Z. D.	63 18 25		

To find ship mean time.

Sun's declination . . .	13° 51′ 1″ S.	. . sec. . .	0·012814
latitude	53 30 0 N.	. . sec. . .	0·225612
	67 21 1		
sun's zenith distance .	60 34 30		
sum	127 55 31	. . ½ hav. .	4·953521
difference	6 46 31	. . ½ hav. .	3·771503
		hav. . .	8·963450
Apparent time	$2^h\ 21^m\ 12^s$		
equation of time	14 33+		
ship mean time	2 35 45		

To find Greenwich mean time.

Sun's zenith distance .	60°	34′	30″
moon's zenith distance	63	18	25
	123	52	55 vers.
sun's app. alt. . . .	29	27	5
moon's app. alt. . . .	25	50	51
sum	55	17	56
auxiliary angle A . .	60	13	52
sum	115	31	48 vers.
difference	4	55	56 vers.
apparent distance . .	99°	58	58
auxiliary angle A . .	60	13	52
sum	160	12	50 vers.
difference	39	45	6 vers.
	1^h	32^m	33^s
	0		
Greenwich mean time	1	32	33 . . . . ·28893
ship mean time . . .	2	35	45
longitude in time . .	1	3	12
longitude . . .	15°	48′	0″ E.

Versines.		True distance.	
57262	. . 222		
30774	. . 210		
03680	. . 19		
40881	. . 83		
31158	. . 18		
63755	. . 552		
552			
64307	. .	99° 27 25	
187		98 38 0	0 hours.
120		100 14 7	3 hours.
		0 49 25	
		1 36 7	
		·56140	
		·27247	
		·28893	

437. March 25, 1847, at 3^h 30^m P.M. mean time nearly, in lat. 52° N., and long. by account 33° W., the following lunar was taken:

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	23° 10′ 20″	23° 50′ 10″	112° 56′ 30″
Index cor.	6 10−	5 0+	4 20−

The height of the eye above the sea was 20 feet: required the longitude.

Ans. Hour-angle, 3^h 32^m 9^s; true dist. 112° 52′ 45″; longitude, 32° 59′ 38″ W.

438. April 20, 1847, at 2^h 0^m P.M. mean time nearly, in lat. 50° 50′ N., and long. by account 1° 40′ E., the following lunar was taken:

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	43° 16′ 30″	24° 39′ 20″	69° 12′ 0″
Index cor.	0 10+	0 20−	0 50−

The height of the eye above the sea was 20 feet: required the longitude.

Ans. Hour-angle, 2^h 1^m 25^s; true dist. 69° 11′ 37″; longitude, 1° 20′ 37″ E.

439. May 19, 1847, at 2^h 50^m P.M. mean time nearly, in lat. 51° 30′ N., and long. by account 20° 40′ E., the following lunar was taken :

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	42° 50′ 30″	25° 10′ 20″	61° 40′ 20″
Index cor.	4 10+	6 10−	2 10+

The height of the eye above the sea was 20 feet: required the longitude.

Ans. Hour-angle, 2^h 57^m 20^s; true dist. 61° 44′ 38″; longitude, 20° 41′ 26″ E.

440. Feb. 6, 1851, at 3^h 30^m P.M. mean time nearly, in lat. 60° 20′ N., and long. by account 26° 45′ E., the following lunar was taken :

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	24° 20′ 0″	33° 10′ 0″	57° 30′ 10″
Index cor.	2 30+	1 20+	2 0+

The height of the eye above the sea was 11 feet: required the longitude.

Ans. Hour-angle, 3^h 22^m 21^s; true dist. 57° 56′ 43″; longitude, 28° 34′ 38″ E.

441. Feb. 20, 1850, at 3^h 50^m P.M. mean time nearly, in lat. 10° 20′ N., and long. by account 7° W., the following lunar was taken :

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	30° 15′ 40″	24° 10′ 10″	100° 55′ 10″
Index cor.	3 10−	1 10+	0 30+

The height of the eye above the sea was 20 feet: required the longitude.

Ans. Hour-angle, 3^h 44^m 12^s; true dist. 100° 54′ 54″; longitude, 7° 3′ 45″ W.

442. Jan. 9, 1851, at 2^h 50^m P.M. mean time nearly, in lat. 56° 10′ 20″ N., and long. by account 20° 40′ E., the following lunar was taken :

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	19° 10′ 20″	25° 30′ 10″	77° 10′ 20″
Index cor.	2 10+	1 20−	2 20−

The height of the eye above the sea was 20 feet : required the longitude.

Ans. Hour-angle, 2^h 45^m 40^s; true dist. 77° 27′ 33″; longitude, 20° 35′ E.

443. July 16, 1869, at 4^h 15^m P.M. mean time nearly, in lat. 25° 30′ N., and longitude by account 48° 18′ E., the following lunar was taken :

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	34° 9′ 27″	45° 42′ 40″	93° 21′ 52″
Index cor.	2 10−	2 10+	1 15+

The height of the eye above the sea was 20 feet : required the longitude.

Ans. 49° 49′ E.

Elements from Nautical Almanac.

	Sun's declin.	Eq. of time.	Moon's semi.	Hor. par.	Sun's semi.
Mar. 25,	1° 40′ 56″N. . .	6^m $13{\cdot}8^s$ add .	. 14′ 59″ .	. 54′ 59″ .	. 16′ 3″
„ 26,	2 4 29 N. . .	5 55·2 „ .	. 14 55 .	. 54 44	
	Distance at 3 hours, 111° 33′ 34″; at 6 hours, 112° 57′ 16″.				
Apr. 20,	11 23 54 N. . .	1 3·4 sub. .	. 15 23·7 .	. 56 29·8 .	. 15 56
„ 21,	11 44 26 N. . .	1 16·3 „ .	. 15 17·0 .	. 56 5·3	
	Distance at noon, 68° 14′ 43″; at 3 hours, 60° 43′ 48″.				
May 19,	19 41 37 N. . .	3 49·58 sub. .	. 15 11·6 .	. 55 45·5 .	. 15 49
„ 20,	19 54 26 N. . .	3 46·90 „ .	. 15 6·3 .	. 55 25·7	
	Distance at noon, 61° 0′ 58″; at 3 hours, 62° 27′ 35″.				
Feb. 6,	15 42 19 S. . .	14 22·51 add .	. 14 55·0 .	. 54 44·3 .	. 16 14
„ 7,	15 23 35 S. . .	14 26·16 „ .	. 14 59·0 .	. 54 59·1	
	Distance at 0 hours, 57° 9′ 21″; at 3 hours, 58° 32′ 36″.				
Feb. 20,	10 55 53 S. . .	14 1·7 add .	. 16 4·4 .	. 58 59·2 .	. 16 11
„ 21,	10 34 16 S. . .	13 54·7 „ .	. 16 9·6 .	. 59 18·0	
	Distance at 3 hours, 100° 7′ 50″; at 6 hours, 101° 45′ 50″.				
Jan. 9,	22 9 2 S. . .	7 19·4 add .	. 14 55·8 .	. 54 47·2 .	. 16 17
„ 10,	22 0 23 S. . .	7 44·1 „ .	. 15 0·2 .	. 55 3·6	
	Distance at 0 hours, 76° 45′ 42″; at 3 hours, 78° 8′ 46″.				
July 16,	21 20 13 N. . .	5 45·0 add .	. 16 5·1 .	. 58 55·7 .	. 15 46
hourly difference	. . +25″.	. +0·24ˢ.	. 16 1·4 .	. 58 42·2	
	Distance at 0 hours, 92° 49′ 59″; at 3 hours, 94° 27′ 44″.				

When the sun or star is near the meridian, the ship mean time must be obtained by computing the hour-angle of the moon, and deducing from thence the ship mean time. This may be done by the following rule:

Rule 51. To FIND THE LONGITUDE BY LUNAR OBSERVATIONS.

Objects observed, moon and sun. Altitudes taken, and ship mean time obtained from *moon's altitude.*

1. Get a Greenwich date.

2. Take out of *Nautical Almanac* and correct for Greenwich date the following quantities: Sun's semidiameter and right ascension of mean sun; right ascension and declination of moon; semidiameter and horizontal parallax of moon.

3. Correct the sun's altitude for index correction, dip, semidiameter, and thus get the apparent altitude: from the apparent altitude subtract correction in altitude; the result is sun's true altitude, which subtract from 90° for sun's true zenith distance.

4. Correct the moon's altitude for index correction, dip, semidiameter

(augmented); the result is the moon's apparent altitude. To the apparent altitude add the correction in altitude, the result subtract from 90° for the moon's true zenith distance.

5. When the moon's correction in altitude is taken out, take out also at the same opening of the book the auxiliary angle A.

6. Correct the observed distance for index and semidiameter.

7. *To find ship mean time from moon's altitude.* First method, using haversines.*

Under the moon's declination put the latitude of ship: take the difference if the names be alike, but their sum if the names be unlike: under the result put the moon's zenith distance, and take the sum and difference. Add together the log. secants of the two first quantities in this form (rejecting the tens in index), and the halves of the log. haversines of the two last; the sum is the log. haversine of the moon's hour-angle, to be taken out at the top of the page if the moon is west of the meridian, but at the bottom of the page if the moon is east of meridian. To the hour-angle thus found add the moon's right ascension, and from right sum (increased if necessary by 24 hours) subtract the ascension of the mean sun; the remainder (rejecting 24 hours if greater than 24 hours) is ship mean time at the instant of observation.

8. Then proceed to calculate the true distance and Greenwich mean time as pointed out in p. 193, arts. 8, 9.

EXAMPLES.

444. May 22, 1844, at $11^h\ 15^m$ A.M. mean time nearly, in lat. 50° 48′ N., and long. by account 1° W., the following lunar observation was taken:

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L. E. of meridian.	Obs. dist. N. L.
	57° 53′ 0″	22° 53′ 2″	56° 26′ 6″
Index cor.	0 35+	0 20−	0 35−

The height of the eye above the sea was 24 feet: required the longitude.

			R. A. mean sun.
Ship, May 21	$23^h\ 15^m$	May 21	$3^h\ 56^m\ 53{\cdot}8^s$
longitude in time	4 W.	cor. 23^h	3 46·7
Greenwich, May 21	23 19	19^m	3·1
sun's semi. . . 15′ 49″			4 0 43·6

* To find ship mean time using only the common tables of sines, see second method, p. 164, ex. 395.

Moon's R. A.		Moon's decl.
23h	$7^h\ 55^m\ 24^s$	17° 5′ 12″ N.
0	7 57 29	16 57 11 N.
logistic logs.	2 5	logistic logs. 8 1
·49940		·49940
1·93651		1·35128
2·43591	0 40	1·85068 2 32
	7 56 4	17 2 40 N

Moon's semi.		Moon's hor. par.
21, mid.	15′ 1·3″	55′ 7·6″
22, noon	15 5·7	55 23·7
	4·4	16·1
·02546		·02546
3·38997		2·82930
3·41543	4·2	2·85476 15·1
	15 5·5	55 22·7
aug.	5·5	
moon's semi	15 11·0	

Sun's alt.		Moon's alt.			
Ob. alt.	57° 53′ 0″	Obs. alt.	22° 53′ 2″	Obs. dist.	56° 26′ 6″
in. cor.	35+	in. cor.	0 20−	in. cor.	35−
	57 53 35		22 52 42		56 25 31
dip	4 49−	dip	4 49−	sun's semi	15 49
	57 48 46		22 47 53	moon's semi	15 11
semi	15 49	semi	15 11	M. app. dist.	56 56 31
	58 4 35		23 3 4		
cor. in alt.	32−	cor.	48 21	Aux. ang. A.	
			21		60° 11′ 31″
sun's true alt.	58 4 3				5
	90	M. true alt.	23 51 46		3
sun's zen. dis.	31 55 57		90		60 11 39
		M. zen. dist	66 8 14		

Moon's hour-angle. First method, using haversines.*

Moon's decl.	17° 2′ 40″ N.	log. sec.		0·019510
latitude	50 48 0 N.	„ sec.		0·199263
	33 45 20	„ ½ hav. (S)		4·883909
moon's zen. dist.	66 8 14	„ ½ hav. (D)		4·445373
sum	99 53 34 (S)	„ hav.		9·548055
difference	32 22 54 (D)			
∴ hour-angle		$19^h\ 8^m\ 17^s$		
moon's R. A.		7 56 4		
		27 4 21		
R. A. mean sun		4 0 44		
∴ ship mean time		23 3 37		

* If no haversines, then find hour-angle by second method. (See ex. 395, p. 164.)

Greenwich mean time. First method, using versines.*

Zenith distance	31°	55′	57″		
zenith distance	66	8	14		
	98	4	11 vers.		
apparent altitude	58	4	35		
apparent altitude	23	3	4		
sum	81	7	39	**Versines.**	
auxiliary angle A	60	11	39	40325 . .	53
sum	141	19	18 vers.	80612 . .	55
difference	20	56	0 vers.	66003 . .	0
				56063 . .	43
apparent distance	56	56	31	01608 . .	2
auxiliary angle A	60	11	39	44611 . .	153
sum	117	8	10 vers.	153	
difference	3	15	8 vers.	44764	
				672	
true distance	56°	16′	23″	92	
" at 21h . . .	55	15	36		
	56	41	20		
·47149 . . .	1	0	47		
·32212 . . .	1	25	44		
·14937					

	2^h	7^m	37^s
	21		
Greenwich mean time . . .	23	7	37
ship mean time	23	3	37
longitude in time		4	0
or longitude	1°	0′ W.	

445. May 16, 1850, at 0^h 50^m P.M. mean time nearly, in lat. 42° 30′ N., and long. by account 29° 6′ W., the following lunar was taken:

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L. E. of meridian.	Obs. dist. N. L.
	61° 44′ 30″	39° 30′ 20″	63° 10′ 0″
Index cor.	2 10−	1 10−	0 20+

The height of the eye above the sea was 20 feet: required the longitude.

Ans. Hour-angle, 20^h 32^m 26^s; true dist. 63° 4′ 0″; longitude, 29° 5′ 0″ W.

446. May 18, 1850, at 1^h 0^m P.M. mean time nearly, in lat. 43° N., and long. by account 41° 36′ W., the following lunar was taken:

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L. E. of meridian.	Obs. dist. N. L.
	64° 30′ 10″	18° 10′ 20″	90° 20′ 10″
Index cor.	2 10+	1 10−	1 20+

The height of the eye above the sea was 15 feet: required the longitude.

Ans. Hour-angle, 18^h 57^m 17^s; true dist. 90° 2′ 57″; longitude, 41° 33′ 45″ W.

* If no versines, then find true distance by second method. (See ex. 426, p. 189.)

Elements from Nautical Almanac.

Mean sun's R.A.	Moon's semi.	Hor. par.	Moon's R.A.	Moon's decl.	S. semi.
16th, $3^h 35^m 22{\cdot}14^s$	Noon, 16′ 16·4″	59′ 43·0″	$7^h 58^m 16{\cdot}79^s$	18° 51′ 56″ N.	15′ 50″
	Mid. 16 13·9	59 34·0	8 0 48·01	18 47 43 N.	
Distance at noon, 61° 27′ 26″; at 3 hours, 63° 7′ 50″.					
18th, 3 43 15·25	Noon, 16 4·3	58 58·8	9 57 4·66	13 17 16 N.	15 49
	Mid. 16 0·6	58 45·3	9 59 23·58	13 8 8 N.	
Distance at 3 hours, 89° 31′ 39″; at 6 hours, 91° 9′ 1″.					

Rule 52. To FIND THE LONGITUDE BY LUNAR OBSERVATIONS.

Objects observed, moon and star. Altitudes taken and ship mean time obtained from star's altitude.

1. Get a Greenwich date.

2. Take out of the *Nautical Almanac* and correct for Greenwich date the following quantities: Star's right ascension and declination; right ascension of mean sun; moon's semidiameter and horizontal parallax.

3. Correct the star's observed altitude for index correction and dip; the result is the star's apparent altitude; from this subtract refraction; the remainder is the star's true altitude, which take from 90° to find star's true zenith distance.

4. Correct the moon's altitude for index correction, dip, semidiameter (augmented), and thus get the moon's apparent altitude; to this add the correction in altitude: the result is the moon's true altitude. Subtract the moon's true altitude from 90°; the remainder is the moon's true zenith distance.

5. When the correction of the moon's altitude is taken out, take out also at the same opening of the book the auxiliary angle A.

6. *To find ship mean time from star's altitude.* First method, using haversines.*

Under star's declination put latitude of ship: take the sum if the names be unlike, but if the names be like take the difference. Under the result put the star's true zenith distance; take the sum and difference of the two last quantities put down.

Add together the log. secants of the two first quantities in this form (rejecting the tens in the index), and the halves of the log. haversines of the two last; the sum will be the log. haversine of the star's hour-angle, to be taken out at the top of the page when the star is *west* of the meridian, and at bottom when east. To the hour-angle thus found add the star's right ascension, and from the sum (increased if necessary by 24 hours) subtract

* To find ship mean time, using the common tables of sines, &c., see second method, p. 164, and ex. 395.

the right ascension of the mean sun: the result (rejecting 24 hours if greater than 24 hours) will be ship mean time.

Then proceed to calculate the true distance and Greenwich mean time, as pointed out in p. 193, arts. 8, 9.

EXAMPLES.

447. June 2, 1849, at $10^h\ 17^m$ P.M. mean time nearly, in lat. 50° 50′ N., and long. by account 41° W., the following lunar observation was taken:

	Obs. alt. Regulus. W. of meridian.	Obs. alt. moon's L. L.	Dist. N. L.
	20° 21′ 40″	31° 11′ 0″	72° 36′ 30″
Index cor.	3 50–	4 10–	9 10–

The height of the eye above the sea was 20 feet: required the longitude.

Ship, June 2	10^h	17^m
longitude in time	2	44 W.
Greenwich, June 2	13	1

Right. asc. mean sun.		Moon's semi.	Hor. par.
2d ... $4^h\ 43^m\ 21{\cdot}2^s$	Star's R.A. $10^h\ 0^m\ 20^s$	2d, mid. 14′ 49″ ...	54′ 23″
13^h 2 8·1	Star's decl. 12 42 4 N.	3d, noon 14 47 ...	54 15
4 45 29·3		2	8
		cor. ...... 0 ...	0
		14 49	54 23
		aug. 7	
		14 56	

Star's altitude.		Moon's altitude.		Observed distance.	
Obs. alt. ...	20° 21′ 40″	Obs. alt. ...	31° 11′ 0″		72° 36′ 30″
in. cor.	3 50–	in. cor.	4 10–	in. cor.	9 10–
	20 17 50		31 6 50		72 27 20
dip	4 24–	dip	4 24	semi.	14 56
app. alt. ...	20 13 26		31 2 26	app. dist. ...	72 42 16
cor. in alt..	2 37–	semi.	14 56		
	20 10 49	app. alt. ...	31 17 22		Aux. angle A.
	90	cor. in alt..	44 34	....	60° 15′ 14″
			20	....	7
zen. dist. ...	69 49 11		32 2 16		60 15 21
			90		
		zen. dist. ...	57 57 44		

To find ship mean time.

Star's declination ...	12°	42′	4″N.	...... Sec.	0·010765
latitude	50	50	0 N.	...... sec.	0·199573
	38	7	56		
star's zenith dist. ...	69	49	11		
sum	107	57	7	...... ½ hav.	4·907820
difference	31	41	15	...... ½ hav.	4·436186
				Hav.	9·554344

Star's hour-angle	4^h	54^m	11^s
star's right ascension	10	0	20
	14	54	31
right ascension mean sun	4	45	29
ship mean time	10	9	2

To find Greenwich mean time.

Star's zen. dist....	69°	49′	11″	
moon's zen. dist..	57	57	44	
	127	46	55	vers.
star's app. alt. ...	20	13	26	
moon's app. alt. ..	31	17	22	
	51	30	48	
aux. angle A......	60	15	21	
sum................	111	46	9	vers.
difference	8	44	33	vers.
app. dist.	72	42	16	
aux. A.	60	15	21	
sum................	132	57	37	vers.
difference	12	26	55	vers.

Versines.				
81360 131				
23453 56				
12447 212				
70828 41				
11594 24				
99682 464				
464				
00146 ...	72°	33′	5″	
127	72	19	35	at 12 hours
19	73	49	27	at 15 hours
1·12548		13	29	
·30167	1	29	52	
·82381 ...	0^h	27^m	0^s	
	12			

Greenwich mean time	12	27	0
ship mean time	10	9	2
longitude in time	2	17	58
Longitude .... 34°	30′	10″ W.	

448. Jan. 8, 1851, at $7^h\ 0^m$ P.M. mean time nearly, in lat. 50° 40′ N., and long. by account 4° E., the following lunar was taken:

	Obs. alt. Aldebaran. E. of meridian.	Obs. alt. moon's L. L.	Obs. dist. F. L.
	45° 20′ 10″	30° 30′ 0″	71° 31′ 10″
Index cor.	2 20−	2 10+	3 30−

The height of eye above the sea was 18 feet: required the longitude.

Ans. Hour-angle, $21^h\ 36^m\ 57^s$: true dist. 70° 42′ 50″; longitude, 3° 46′ 30″ E.

449. Jan. 9, 1851, at $7^h\ 50^m$ P.M. mean time nearly, in lat. 49° 40′ N., and long. by account 10° E., the following lunar was taken:

	Obs. alt. Pollux. E. of meridian.	Obs. alt. moon's L. L.	Obs. dist. F. L.
	37° 10′ 10″	31° 50′ 10″	103° 20′ 0″
Index cor.	1 10−	1 20+	1 30+

The height of eye above the sea was 18 feet: required the longitude.

Ans. Hour-angle, $19^h\ 39^m\ 52^s$; true dist. 102° 28′ 3″; longitude, 10° 20′ E.

450. April 18, 1850, at $9^h\ 40^m$ P.M. mean time nearly, in lat. 56° 10′ N., and long. by account 23° E., the following lunar was taken:

	Obs. alt. α Virginis. E. of meridian.	Obs. alt. moon's L. L.	Obs. dist. F. L.
	19° 40′ 0″	48° 40′ 20″	91° 30′ 20″
Index cor.	1 10+	1 20+	0 30−

The height of eye above the sea was 20 feet: required the longitude.

Ans. Hour-angle, $22^h\ 8^m\ 50^s$; true dist. 90° 55′ 16″; longitude, 23° 3′ 30″ E.

451. April 17, 1850, at $8^h\ 45^m$ P.M. mean time nearly, in lat. 51° 20′ N., and long. by account 5° 10′ E., the following lunar was taken:

	Obs. alt. α Virginis. E. of meridian.	Obs. alt. moon's L. L.	Obs. dist. F. L.
	18° 15′ 30″	36° 25′ 10″	105° 33′ 28″
Index cor.	1 10+	1 20−	0 20+

The height of eye above the sea was 20 feet: required the longitude.

Ans. 5° 8′ E.

452. January 11, 1868, at $5^h\ 40^m$ P.M. mean time nearly, in latitude 50° 48′ N., and long. by account 70° W., the following lunar was taken:

	Obs. alt. α Tauri. E. of meridian.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	36° 25′ 50″	30° 39′ 30″	71° 50′ 55″
Index cor.	5 30+	2 10−	2 15−

The height of eye above the sea was 20 feet: required the longitude.

Ans. 69° 58′ W.

453. March 20, 1845, at 7^h 50^m P.M. mean time nearly, in lat. 49° 50′ N., and long. by account 1° 30′ E., the following lunar was taken:

	Obs. alt. Aldebaran. W. of meridian.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	37° 4′ 30″	38° 17′ 20″	75° 15′ 10″
Index cor.	2 4−	1 5+	0 20+

The height of eye above the sea was 20 feet: required the longitude.

Ans. 1° 34′ 30″ E.

Elements from Nautical Almanac.

Right ascension mean sun.	Moon's semi.	Hor. par.	
Jan. 8, 19^h 9^m $45{\cdot}72^s$. .	Noon, 14′ 48·8″. .	54′ 21·8″	Star's R.A. . 4^h 27^m $22{\cdot}7^s$
	Mid. 14 52·0 . .	54 33·3	Star's decl. . 16° 12′ 13″ N.
Distance at 6 hours, 71° 1′ 57″; at 9 hours, 69° 32′ 16″.			
Jan. 9, 19 13 42·27 . .	Noon, 14 55·8 . .	54 47·2	Star's R.A. . 7^h 36^m 12^s
	Mid. 15 0·2 . .	55 3·6	Star's decl. . 28° 22′ 46″ N.
Distance at 6 hours, 103° 8′ 4″; at 9 hours, 101° 37′ 51″.			
Ap. 18, 1 44 58·62 . .	Noon, 16 8·8 . .	59 15·4	Star's R.A. . 13^h 17^m 19^s
	Mid. 16 8·7 . .	59 14·8	Star's decl. . 10° 22′ 43″ S.
Distance at 6 hours, 92° 10′ 13″; at 9 hours, 90° 24′ 32″.			
Ap. 17, 1 41 2·07 . .	Noon, 16 8·2 . .	59 13·1	Star's R.A. . 13^h 17^m 19^s
	Mid. 16 8·7 . .	59 14·9	Star's decl. . 10° 22 43 S.
Distance at 6 hours, 106° 16′ 11″; at 9 hours, 104° 30′ 26″.			
Jan. 11, 19 21 8·50 . .	Noon, 16 41·6 . .	61 9·8	Star's R.A. . 4^h 28^m 21^s
	Mid. 16 37·2 . .	60 53·6	Star's decl. . 16° 14′ 21″ N.
Distance at 9 hours, 70° 16′ 10″; at 12 hours, 72° 8′ 51″.			
Mar. 20, 23 51 30·11 . .	Noon, 15 12·2 . .	55 47·6	Star's R.A. . 4^h 27^m 3^s
	Mid. 15 17·4 . .	56 6·7	Star's decl. . 16° 11^m 30^s N.
Distance at 6 hours, 74° 7′ 12″; at 9 hours, 75° 42′ 9″.			

Rule 53. To FIND THE LONGITUDE BY LUNAR OBSERVATIONS.

Objects observed, moon and star. Altitudes taken and ship mean time obtained from moon's altitude.

This differs very little from Rule 51, p. 197.

1. Get a Greenwich date.

2. Take out of the *Nautical Almanac* and correct for Greenwich date the following quantities: Right ascension of mean sun; moon's right ascension and declination; semidiameter, and horizontal parallax.

3. Correct the star's altitude for index correction, dip, and thus get the apparent altitude. From the star's apparent altitude subtract refraction: the result is the true altitude, which take from 90° for the star's true zenith distance.

Then proceed to calculate the moon's zenith distance, &c., ship mean time, true distance, and Greenwich mean time, as pointed out in p. 197.

Rule 54.

Objects observed, moon and planet. Altitudes taken and ship mean time from planet's altitude.

1. Get a Greenwich date.

2. Take out of the *Nautical Almanac* and correct for Greenwich date the following quantities: Right ascension of mean sun; planet's right ascension and declination; planet's horizontal parallax (if great accuracy is required); moon's semidiameter and horizontal parallax.

3. Correct the planet's observed altitude for index correction and dip, and thus get the apparent altitude. From the apparent altitude subtract the refraction, and add the parallax in altitude (usually neglected, being very small); the result is the planet's true altitude, which subtract from 90° to get the true zenith distance.

4. Correct the moon's altitude as in 4, p. 201.

5. Get the auxiliary angle A, as in 5, p. 201.

6. Find ship mean time, as in 6, p. 201, using planet's declination and right ascension instead of star's.

7. Then proceed to calculate the true distance and Greenwich mean time, as described in arts. 8, 9, p. 193.

EXAMPLE.

454. September 24, 1849, at $7^h\ 50^m$ P.M. mean time nearly, in latitude 47° 50′ N., and longitude by account 2° 30′ W., the following lunar observation was taken:

	Obs. alt. Saturn. E. of meridian.	Obs. alt. moon's L. L.	Obs. dist. F. L.
	16° 55′ 0″	15° 30′ 45″	89° 51′ 36″
Index cor.	3 5 −	3 10 −	3 0 +

The height of eye above the sea was 20 feet: required the longitude.

Ship, Sept. 24	$7^h\ 50^m$
long. in time	10 W.
Greenwich, Sept. 24.........	8 0

Right ascen. mean sun.		Planet's right ascen. and declination.			
24th	$12^h\ 12^m\ 48{\cdot}4^s$	24th...	$0^h\ 21^m\ 50{\cdot}2^s$	....	0° 31′ 23″S.
8^h ...	1 18·8	25th...	0 21 33·0	....	0 33 17 S.
	12 14 7·2		17·2		1 54
		Cor. ...	6·0	Cor...	38
Planet's hor. par. 1·0″.			0 21 44		0 32 1 S.

					Moon's hor. par.				Moon's semi.		
Obs. dist..	89°	51′	36″	Noon	54′	13·6″	....	14′	46·6″		
index cor..		3	0+	Mid.	54	16·2	....	14	47·3		
	89	54	36			2·6			0·7		
semi.......		14	51−	cor..........		1·7	cor.		0·5		
app. dist..	89	39	45		54	15·3		14	47·1		
							aug....		3·9		
								14	51		

Planet's altitude.				Moon's altitude.				Angle A.		
Obs. alt.	16°	55′	0″	Obs. alt...	15°	30′	45″			
index cor.		3	5−	index. cor.		3	10−			
	16	51	55		15	27	35			
dip		4	24−	dip.		4	24−			
app. alt.	16	47	31		15	23	11			
refr. 3′ 11″− ...				semi.......		14	51+			
par. in alt. 1″+		3	10−		15	38	2			
	16	44	21	cor. in alt.		48	36...	60°	7′	29″
	90						14			2
										0
zenith dist.......	73	15	39		16	26	52			
					90			60	7	31
				zen. dist..	73	33	8			

To find ship mean time.

Latitude	47°	50′	0″ N.		Sec.	0·173090
declination	0	32	1	S.	sec.	0·000019
					½ hav.	4·941037
	48	22	1		½ hav.	4·333480
zenith distance ...	73	15	39			
					hav.	9·447626
sum...................	121	37	40			
difference............	24	53	38			

Hour-angle	19^h	44^m	16^s
planet's right ascension ...	0	21	44
	20	6	0
right ascension mean sun..	12	14	7·2
ship mean time	7	51	52·8

To find Greenwich mean time.

Zenith dist.......	73°	15′	39″	
zenith dist.	73	33	8	
	146	48	47	vers.
App. alt.........	16	47	31	
app. alt.	15	38	2	
	32	25	33	
aux. angle	60	7	31	
sum..............	92	33	4	vers.
difference	27	41	58	vers.
App. dist.	89	39	45	
aux. angle	60	7	31	
sum..............	149	47	16	vers.
difference	29	32	14	vers.

Versines.					
36764 126					
44491 19					
14471 130					
64128 39					
29931 33					
89785 347					
347					
90132 ...	89°	26′	5″	
110	90	26	30	at 6 hours
	88	57	4	
22				
·47412	1	0	25	
·30377	1	29	26	
·17035	2^h	1^m	36^s	
	6			
Greenwich mean time	8	1	36	
ship mean time..............	7	51	53	
longitude in time		9	43	W.

Longitude...... 2° 25′ 45″ W.

LONGITUDE BY LUNAR.—ALTITUDES CALCULATED.

To find ship mean time.

The error of the chronometer on ship mean time is found a little before or after the lunar distance is taken. For this purpose the observer selects any heavenly body whose bearing is nearly east or west, so that the error in the altitude may produce the smallest error in the resulting hour-angle (see Rule 43). Then the time being noted by the same chronometer when the distance is taken, ship mean time is known at the same instant by applying the error found by the above observation.

Rule 55. *To find longitude by lunar observations.*

Objects observed, moon and sun. Altitudes calculated.

1. Get a Greenwich date.

2. Take from the *Nautical Almanac* and correct for the Greenwich date the following quantities: Sun's declination, equation of time and semidiameter, right ascension of mean sun. Moon's right ascension and declination, moon's semidiameter and horizontal parallax.

3. *To find the sun's hour-angle.*

To the time shown by chronometer at the observation apply the error of chronometer with its proper sign, and thus get ship mean time; to this apply equation of time: the result is ship apparent time, and also the sun's hour-angle.

4. *To calculate the sun's true altitude.*

Under the latitude* put down the sun's declination; take the sum if the names be unlike, but the difference if the names be alike; call the result v; add together constant log. 6·301030, log. cos. latitude, log. cos. sun's declination, and log. haversine sun's hour-angle; reject 30 in the index, and look out the result as a logarithm, and take its natural number to the nearest unit. (If no haversines, find sun's and moon's true alt. by Rule IX., *Trig.* Part I., viz. two sides and included angle given, to find third side.)

Add together this natural number and the versine of the quantity v found above: the sum is the versine of the sun's *true zenith distance*, which find in the tables and subtract from 90°: the result is the sun's true altitude.

To find the sun's apparent altitude.

To the true altitude just found *add* correction in altitude (for parallax and refraction): the result will be the sun's apparent altitude very nearly.†

5. *To find the moon's hour-angle.*

To right ascension of mean sun add ship mean time, and from the sum (increased if necessary by 24 hours) subtract the moon's right ascension: the result is the moon's hour-angle (rejecting 24 hours if greater than 24 hours).

6. *To find the moon's true altitude.*

Under the latitude put the moon's declination; take the sum if the names be unlike, and the difference if the names be alike; call the result v. Add together the constant quantity 6·301030, log. cos. latitude, log. cos. moon's declination, and log. haversine of moon's hour-angle; reject 30 from the index, and look out the result as a logarithm, and take its natural number. To this natural number add the versine of the quantity v, found as above; the sum is the versine of the moon's true zenith distance, which find in the table and subtract from 90°: the result is the moon's true altitude.

* When great accuracy is required, the latitude and horizontal parallax should be corrected for the spheroidal figure of the earth.

† In strictness, the table for correction of altitude ought to have been entered with the *apparent* altitude, instead of the *true*, to get the correction in altitude; but in the case of the sun's altitude the above is sufficiently correct. A more exact method is followed in finding the *moon's* apparent altitude from the true (p. 210).

To find the moon's apparent altitude.

Consider the moon's true altitude just found as the apparent altitude; enter the table with it, and take out the correction in altitude thus approximately; subtract this correction from the moon's true altitude, and thus get the moon's apparent altitude nearly. Then enter the table again with this *corrected altitude,* and thus take out the correction in altitude more exactly; subtract this correction from the moon's *true* altitude, and the result will be the moon's apparent altitude very nearly.

Take out at the same opening of the table the auxiliary angle A.

Correct the moon's semidiameter for augmentation. Then proceed as in arts. 6, 8, 9, p. 193.

EXAMPLE.

455. August 19, 1843, in lat. 50° 48′ N., and long. by account 1° 6′ W., when a chronometer showed $11^h\,10^m\,19{\cdot}8^s$ A.M., the observed distance of the nearest limb of the sun and moon was 76° 51′ 26″, the error of the chronometer on ship mean time being fast $7^m\,29{\cdot}3^s$, and the index correction 1′ 57″+ : required the longitude.

Time by chro...	11^h	10^m	$19{\cdot}8^s$
error of chro....		7	29·3 fast
ship mean time	23	2	50·5
long. in time ...		4	24·0 W.
Gr., Aug. 18 ...	23	7	
Ship mean time	23	2	50·5
equation of time		3	30·6 sub.
sun's hour-angle	22	59	19·9

Sun's declination.				Eq. of time.	
18th	13°	15′	12″ N.	3^m	$43{\cdot}2^s$ sub.
19th	12	55	47 N.	3	30·1
		19	25		13·1
·01629				·01692	
·96710				2·91615	
·98339		18	42	2·93244	12·6
	13	56	30 N.	3	30·6

Sun's semi.... 15′ 49″

Moon's right ascen.			
18th at 23^h.	4^h	27^m	27^s
„ 24 .	4	29	41
·93305		2	14
1·42920			
2·36225			0·16
	4	27	43

Right ascen. mean sun.			
18th	9^h	44^m	$48{\cdot}22^s$
		3	46·84
			1·14
	9	48	36·20
ship M.T.	23	2	50·50
	32	51	26·70
M.'s R.A.	4	27	43·0
M.'s H.A.	4	23	43·70

Moon's declin.			
	23°	43′	13″ N.
	23	43	27 N.
		0	14
·93305			
2·41018			
3·34323		0	2
	23	43	15 N.

		Moon's semi.			Moon's hor. par.	
18th, at mid.........		14′	59·7″	....	55′	1·8″
19th, at noon		15	4·5	....	55	19·1
			4·8			17·8
	·03321			·03321		
	3·35218			2·79538		
	3·38539		4·4	2·82859		16·0
		15	4·1		55	17·8
aug.			8·2			
moon's semi.........		15	12·3			

To calculate sun's altitude.

Latitude	50°	48′	0″N.
sun's declination...	13	56	30 N.
	36	51	30 *v.*

Const. log.	6·301030
cos. lat......................	9·800737
cos. sun's decl.	9·987014
hav. sun's hour-angle ...	8·240938
log.	4·329719
nat. no......................	21366
vers. *v.*	199792
	87
ver. sun's zen. dist.	0221245
	209
	36

Sun's zenith distance	38°	51′	12″
	90		
sun's true alt..........	51	8	48
cor. in alt.		0	42
sun's app. alt..........	51	9	30

To calculate moon's altitude.

Latitude	50°	48′	0″N.
moon's decl..........	23	43	15 N.
	27	4	45 *v,*

Const. log.	6·301030
cos. lat......................	9·800737
cos. moon's decl.	9·961666
hav. moon's hour-angle	9·470952
log.	5·534872
nat. no......................	342667
vers. *v,*	0109522
	99
vers. moon's zen. dist. ...	0452288
	193
	95

Moon's zenith dist...	56°	47′	23″
	90		
moon's true alt.	33	12	37
cor. in alt.		44	0
moon's ap. alt. nearly	32	28	0

Obs. dist. .	76° 51′ 26″	Aux. angle A.			44′ 53″
	1 57+	60° 16′ 4″			15
		4			
	76 53 23	3		True cor. in alt.	45 8
sun's semi.	15 49			moon's true alt.	33 12 37
moon's „	15 12	60 16 11 arc A.			
				moon's app. alt.	32 27 29
app. dist...	77 24 24				

To find Greenwich mean time.

					Versines.				
Sun's zenith dist...	38°	51′	12″						
moon's zenith dist.	56	47	23	98462 ... 169					
				07819 ... 40					
sum..................	95	38	35 *v.*	81784 ... 83					
				39239 ... 128					
Sun's app. alt.......	51	9	30	44378 ... 13					
moon's app. alt. ...	32	27	29						
				71382	433				
sum..................	83	36	59	433					
arc A.	60	16	5				True dist.		
				71815	....		76°	48′	35″
sum..................	143	53	14 *v.*	649			77	48	59 at 21^h
difference...........	23	20	44 *v.*				76	24	3
				166					
App. dist.	77	24	24		·47424		1	0	24
arc A.	60	16	15		·32619		1	24	56
sum..................	137	40	39 *v.*		·14805		2^h	8^m	0^s
difference...........	17	8	9 *v.*				21		

Greenwich mean time	23	8	0
ship mean time	23	2	50
longitude in time		5	10

Longitude ...... 1° 17′ 30″ W.

456. September 16, 1843, in latitude 50° 48′ N., and long. by account 1° 6′ W., when a chronometer showed 9^h 34^m $6{\cdot}6^s$ A.M., the observed distance of the nearest limb of the sun and moon was 96° 26′ 18″, the index correction being + 1′ 32″, and the error of chronometer on ship mean time being fast 7^m 59^s: required the longitude. *Ans.* 1° 35′ 30″ W.

457. October 14, 1843, in lat. 50° 48′ N., and long. by account 1° 6′ W.,

when a chronometer showed $9^h\ 53^m\ 57{\cdot}1^s$ A.M., the observed distance of the nearest limb of the sun and moon was 114° 58′ 22″, the error of the chronometer being slow $3^m\ 27^s$, and the index correction + 1′ 32″ : required the longitude. *Ans.* 1° 30′ 30″ W.

548. October 16, 1843, in lat. 50° 48′ N., and long. by account 1° 6′ W. when a chronometer showed $9^h\ 58^m\ 9{\cdot}8^s$ A.M., the observed distance of the nearest limb of the sun and moon was 91° 45′ 38″, the error of the chronometer being slow $3^m\ 26{\cdot}5^s$, and the index correction + 1′ 30″: required the longitude. *Ans.* 1° 31′ 30″ W.

459. August 17, 1843, in lat. 50° 37′ 30″ N., and long. by account 1° 6′ W., when a chronometer showed $10^h\ 42^m\ 28{\cdot}7^s$ A.M., the observed distance of the nearest limb of the sun and moon was 99° 22′ 35″, the error of the chronometer on ship mean time being fast $7^m\ 44^s$, and the index correction + 1′ 55″: required the longitude. *Ans.* 1° 17′ W.

460. May 25, 1843, in lat. 50° 48′ N., and long. by account 1° 6′ W., when a chronometer showed $11^h\ 19^m\ 15{\cdot}1^s$ A.M., the observed distance of the nearest limb of the sun and moon was 42° 48′ 48·3ˢ, the error of the chronometer on ship mean time being slow $3^m\ 29{\cdot}7^s$, and the index correction + 3′ 30″: required the longitude. *Ans.* 1° 5′ 45″ W.

461. May 25, 1843, in lat. 50° 37′ 30″ N., and long. by account 1° 6′ W., when a chronometer showed $11^h\ 4^m\ 12{\cdot}2^s$ A.M., the observed distance of the sun and moon's nearest limb was 43° 1′ 3″, the error of chronometer on ship mean time being fast $12^m\ 53{\cdot}5^s$, and the index correction + 0′ 57″: required the longitude. *Ans.* 1° 33′ W.

Elements from Nautical Almanac.

Sun's declination.	Equation of time.	Mean sun's right ascen.
Sept. 15... 3° 11′ 23″N.	... $4^m\ 42{\cdot}5^s$ add	... 15th... $11^h\ 35^m\ 11{\cdot}72^s$
„ 16... 2 48 15 N.	... 5 3·6	Sun's semi. 15′ 56″

	Moon's right ascen.	Moon's declin.	Moon's semi.	Hor. par.
15th, at 21^h...	$4^h\ 59^m\ 11{\cdot}7^s$ ...	23° 46′ 38·6″...	Mid. ... 14′ 58·2″ ...	54′ 56·2″
„ 22^h...	5 1 25·7 ...	23 47 9·0 ...	Noon... 15 2·8 ...	55 13·1

Distance at 21 hours, 96° 42′ 14″; at noon, 95° 17′ 50″.

Sun's declination.	Equation of time.	Mean sun's right ascen.
Oct. 13... 7° 38′ 12″S.	... $13^m\ 36{\cdot}0^s$ add	... 13th... $13^h\ 25^m\ 35{\cdot}20^s$
„ 14... 8 0 40 S.	... 13 50·2	Sun's semi. 16′ 4″

	Moon's right ascen.	Moon's declin.	Moon's semi.	Hor. par.
13th, at 22^h...	$5^h\,40^m\,1{\cdot}1^s$ ...	23° 21′ 26″N....	Mid. ... 14′ 57·8″...	54′ 54·7″
„ 23^h...	5 42 14·6 ...	23 19 43 N....	Noon... 15 2·1 ...	55 10·3

Distance at 21 hours, 115° 27′ 42″; at noon, 114° 3′ 26″.

	Sun's declination.	Equation of time.	Mean sun's right ascen.
Oct. 15...	8° 23′ 1·4″S. ...	$14^m\,3{\cdot}8^s$ add ...	15th... $13^h\,33^m\,28{\cdot}31^s$
„ 16...	8 45 15·6 S. ...	14 16·8	Sun's semi. 16′ 4″

	Moon's right ascen.	Moon's declin.	Moon's semi.	Hor. par.
15th, at 22^h...	$7^h\,26^m\,58{\cdot}8^s$...	19° 45′ 6″N....	Mid. ... 15′ 18·0″...	56′ 8·7″
„ 23^h...	7 29 11·9 ...	19 37 45 N....	Noon... 15 24·3 ...	56 32·1

Distance at 21 hours, 92° 27′ 48″; at noon, 90° 58′ 57″.

	Sun's declination.	Equation of time.	Mean sun's right ascen.
Aug. 16...	13° 53′ 24″N. ...	$4^m\,7{\cdot}9^s$ sub. ...	16th... $9^h\,36^m\,55{\cdot}12^s$
„ 17...	13 34 24 N. ...	3 55·8	Sun's semi. 15′ 49″

	Moon's right ascen.	Moon's declin.	Moon's semi.	Hor. par.
16th, at 22^h...	$2^h\,42^m\,36^s$...	$19^h\,40^m\,25^s$N....	Mid. 14′ 47·5″...	54′ 16·8″
„ 23^h...	2 44 29 ...	19 47 38 N....	Noon 14 49·5 ...	54 24·3

Distance at 21 hours, 99° 59′ 6″; at noon, 98° 37′ 11″.

	Sun's declination.	Equation of time.	Mean sun's right ascen.
May 24...	20° 42′ 10·8″N. ...	$3^m\,30{\cdot}9^s$ add ...	24th... $4^h\,5^m\,44{\cdot}32^s$
„ 25...	20 53 16·6 N. ...	3 25·6	Sun's semi. 15′ 48″

	Moon's right ascen.	Moon's declin.	Moon's semi.	Hor. par.
24th, at 23^h...	$1^h\,8^m\,43{\cdot}11^s$...	12° 39′ 15·8″...	Mid. 14′ 44·8″...	54′ 6·8″
„ 24^h...	1 10 36·96 ...	12 49 47·9 ...	Noon 14 45·7 ...	54 10·3

Distance at 21 hours, 44° 0′ 59″; at 0 hours, 42° 39′ 15″.

Rule 56. Longitude by lunar.

Objects observed, moon and star. Altitudes calculated.

1. Get a Greenwich date.

2. Take from the *Nautical Almanac*, and correct for the Greenwich date, the following quantities: Right ascension of mean sun; moon's right ascension and moon's declination; moon's semidiameter and moon's horizontal parallax. Take out also the star's right ascension and declination.

3. *To find star's hour-angle.*

To the right ascension of mean sun add mean time at ship,* and from the sum subtract the star's right ascension : the remainder is the hour-angle of the star.

4. *To find the moon's hour-angle.*

From the same sum (viz. right ascension of mean sun and ship mean time) subtract the moon's right ascension ; the remainder is the hour-angle of the moon.

5. *To calculate the star's true altitude.*

Proceed as in 4, p. 209, using the star's declination instead of the sun's.

To find star's apparent altitude.

To the true altitude just found add the refraction ; the result is the star's apparent altitude.

6. *To find the moon's true and apparent altitude* proceed as in 6, p. 209.

7. Proceed then as in former rule (arts. 6, 8, 9, p. 193).

EXAMPLE.

462. May 16, 1842, in lat. 50° 37′ 30″ N., and long. by account 1° 6′ W., when a chronometer showed $10^h\ 39^m\ 26^s$ P.M., the observed distance of the star α Virginis from the moon's farthest limb was 64° 1′ 50″, index correction +0′ 40″, the error of the chronometer on ship mean time being fast $4^m\ 14^s$: required the longitude.

In this example the lat. (50° 48′ N.) and horizontal par. (59′ 8″ and 59′ 12·6″) have been corrected for the spheroidal figure of the earth (see *Nav.* Part II. pp. 122, &c.).

Time by chro. .	10^h	39^m	26^s	R. A. mean sun.				Star's R. A. and decl.			
error fast		4	14	16 ...	$3^h\ 35^m$		$9·00^s$	13^h	16^m	$55·7^s$	
ship mean time	10	35	12			1	38·6	10°	20′	25″ S.	
long. in time .		4	24	cor. ...			6·5				
Gr., May 16 .	10	40			3	36	54·1				
	Ship mean time				10	35	12·0				
					14	12	6·1	....	14^h	12^m	$6·1^s$
	star's right ascen. ...				13	16	55·7	....	9	19	40·0
	∴ star's hour-angle .				0	56	10·4	M.'s H.A.	4	52	26·1

* Ship mean time is usually found by an altitude of a heavenly body taken a little before or after the lunar as directed, p. 164, ex. 395.

	Moon's R. A.	Moon's decl.	Moon's semi.	M.'s H. P.
At 10h ...	9h 18m 10·0s ...	13° 25′ 13″N. ...	Noon... 16′ 7·0″......	59′ 1·0″
„ 11h ...	9 20 24·5 ...	13 11 47 N. ...	Mid. ... 16 8·1	59 5·6
	2 14·5	13 26	1·1	4·6
·17609		·17609	·05115	·05115
1·42920		·64997	3·99203	3·37067
1·60529...	1 30	·82606 8 56	4·04318 1·0	3·42182 4·1
	9 19 40	13 16 17 N.	16 8·0	59 5·1
			aug. 5·8	
			16 13·8	

To calculate star's altitude.		*To calculate moon's altitude.*		
Lat..................	50° 37′ 30″N.	Lat.	50° 37′ 30″N.	
decl.	10 20 25 S.	decl.	13 16 17 N.	
	60 57 55 v.		37 21 13 $v_{,}$	
Const. log.	6·301030	Const. log..........	6·301030	
cos. lat................	9·802359	cos. lat.	9·802359	
cos. star's decl.......	9·992887	cos. moon's decl...	9·988245	
hav. star's H. A....	8·158830	hav. moon's H. A.	9·549884	
	4·255106		5·641518	
	17993		438044	
ver. v.	514427	ver. $v_{,}$	205056	
	232		38	
ver. zen. dist.	532652	ver. zen. dist.......	643138	
	584		2990	
62° 8′ 16″	68	69° 5′ 33″	148	
∴ star's zen. dist..	62° 8′ 16″	∴ moon's zen. dist.	69° 5′ 33″	
	90		90	
∴ star's true alt....	27 51 44	∴ moon's true alt.	20 54 27	
cor. in alt. + ...	1 50	cor. in alt. (nearly)	52 −	
∴ star's app. alt...	27 53 34	app. alt. (nearly)..	20 2 0	**Aux. angle A.**
obs. dist.............	64 1 50	cor. in alt.	52 48	60° 10′ 47″
index correction ...	0 40 +		5	1
				0
	64 2 30	true cor. in alt....	52 53	
semi.	16 14 −	moon's true alt. ...	20 54 27	60 10 48
app. dist.............	63 46 16	moon's app. alt. ...	20 1 34	

To find true distance and Greenwich mean time.

Star's zen. dist...	62°	8′	16″
moon's zen. dist.	69	5	33
	131	13	49 *v.*
Star's app. alt....	27	53	34
moon's app. alt..	20	1	34 *v.*
sum..............	47	55	8
arc A.	60	10	48
sum..............	108	5	56 *v.*
diff..............	12	15	40 *v.*
App. dist.	63	46	16
arc A.	60	10	48
sum..............	123	57	4 *v.*
diff.	3	35	28 *v.*

Versines			True dist.		
58908 ... 179					
10400 ... 258					
22769 ... 40					
58469 ... 16					
01955 ... 7					
52501					
500					
53001	....		63°	26′	55″
2761			64	24	13 at 9^h
			62	38	17
240					
	49712	...	0	57	18
	23024		1	45	56
	26688		1^h	37^m	22^s
			9		
Gr. mean time ...			10	37	22
ship mean time..			10	35	12
long. in time ...				2	10

∴ longitude 0° 32′ 30″ W.

463. April 20, 1847, in lat. 50° 37′ 12″ N., and long. by account 1° 6′ W., when a chronometer showed $8^h\ 58^m\ 45^s$ P.M., the observed distance of the star α Leonis from the moon's farthest limb was 46° 2′ 12″, index correction +0′ 30″, the error of chronometer being fast $3^m\ 22^s$: required the longitude. *Ans.* 0° 55′ 45″ W.

464. December 10, 1845, in lat. 50° 37′ 30″ N., and long. by account 1° 6′ W., when a chronometer showed $9^h\ 24^m\ 48{\cdot}3^s$ P.M., the observed distance of the star Pollux from the moon's farthest limb was 65° 28′ 30″, index correction +0′ 30″, the error of the chronometer on ship mean time being fast $12^m\ 50{\cdot}8^s$: required the longitude. *Ans.* 0° 44′ 15″ W.

465. April 19, 1847, in lat. 50° 48′ N., and long. by account 1° 6′ W., when a chronometer showed $8^h\ 40^m\ 18^s$ P.M., the observed distance of the star Regulus from the moon's farthest limb was 59° 11′ 1·6″, index correction +30″, the error of the chronometer on ship mean time being fast $9^m\ 30{\cdot}4^s$: required the longitude. *Ans.* 1° 7′ W.

466. September 1, 1843, in lat. 50° 37′ 30″ N., and long. by account 1° 6′ W., when a chronometer showed $8^h\ 2^m\ 54{\cdot}4^s$ P.M., the observed distance of the planet Jupiter from the moon's farthest limb was 64° 19′ 57″, index

correction $+1' 50''$, the error of the chronometer on ship mean time being fast $2^m 2{\cdot}6^s$: required the longitude. *Ans.* $0° 35' 15''$ W.

467. September 5, 1843, in lat. $50' 48''$ N., and long. by account $1° 6'$ W., when a chronometer showed $8° 52' 39''$ P.M., the observed distance of the planet Mars from the moon's nearest limb was $45° 11' 23{\cdot}3''$, index correction $+1' 50''$, the error of the chronometer being fast $4^m 47{\cdot}4^s$: required the longitude. *Ans.* $1° 19' 45''$ W.

Elements from Nautical Almanac.

	Star's declin.	Star's right ascen.	Mean sun's R. A.
April 20...	$12° 42' 30''$ N. ...	$10^h 0^m 15{\cdot}35^s$ ...	$1^h 51^m 48{\cdot}09^s$

	Moon's right ascen.	Moon's declin.	Moon's semi.	Hor. par.
20th, at 8^h...	$6^h 51^m 3{\cdot}8^s$...	$17° 39' 51''$N. ...	Noon. $15' 23{\cdot}7''$...	$56' 29{\cdot}8''$
„ 9^h...	6 53 16·8 ...	17 36 40 N. ...	Mid... 15 17·0 ...	56 5·3

Distance at 6 hours, $46° 51' 27''$; at 9 hours, $45° 16' 23''$.

	Star's declin.	Star's right asc.	Mean sun's right asc.
10th...	$28° 23' 22''$N.	$7^h 35^m 54{\cdot}9^s$	$17^h 16^m 17{\cdot}09^s$

	Moon's right ascen.	Moon's decl.	Moon's semi.	Hor. par.
10th, at 9^h...	$2^h 54^m 19{\cdot}38^s$...	$16° 40' 52''$N. ...	Noon $15' 11{\cdot}5''$...	$55' 45{\cdot}0''$
„ 10^h...	2 56 27·74 ...	16 47 6 N. ...	Mid.. 15 7·7 ...	55 30·9

Distance at 9 hours, $65° 12' 56''$; at mid., $63° 41' 12''$.

	Star's declin.	Star's right ascen.	Mean sun's right asc.
19th...	$12° 42' 30''$N.	$10^h 0^m 15{\cdot}35^s$	$1^h 47^m 51{\cdot}54^s$

	Moon's right ascen.	Moon's declin.	Moon's semi.	Hor. par.
19th, at 8^h...	$5^h 56^m 41{\cdot}12^s$...	$18° 28' 9{\cdot}6''$N. ...	Noon $15' 38{\cdot}4''$...	$57' 23{\cdot}5''$
„ 9^h...	5 58 59·68 ...	18 27 17·2 N. ...	Mid. 15 30·9 ...	56 56·1

Distance at 6 hours, $59° 46' 25''$; at 9 hours, $58° 8' 6''$

	Planet's declin.	Planet's right ascen.	Mean sun's right asc.
1st...	$15° 46' 6{\cdot}8^s$ S.	$21^h 33^m 3{\cdot}13^s$	$10^h 39^m 59{\cdot}98^s$
2d...	15 48 22·9 S.	21 32 35·33	

	Moon's right asc.	Moon's declin.	Moon's semi.	Hor. par.
1st, at 8^h...	$17^h 0' 32{\cdot}9^s$...	$23° 56' 5{\cdot}7''$S. ...	Noon $15' 54{\cdot}8''$...	$58' 24{\cdot}0''$
„ 9^h...	17 3 2·2 ...	23 56 34·4 S. ...	Mid.. 15 49·5 ...	58 4·5

Distance at 6 hours, $65° 7' 44''$; at 9 hours, $63° 24' 57''$.

	Planet's declin.	Planet's right ascen.	Mean sun's right asc.
5th......	$26° 34' 9{\cdot}5''$S.	$17^h 32^m 5{\cdot}8^s$	$10^h 55^m 46{\cdot}19^s$
6th......	26 34 29·5 S.	17 34 27·8	

	Moon's right ascen.	Moon's declin.	Moon's semi.	Hor. par.
5th, at 8^h...	$20^h 41^m 16{\cdot}9^s$...	$15° 1' 2''$S. ...	Noon.. $15' 15{\cdot}7''$...	$56' 0{\cdot}4''$
„ 9^h...	20 43 20·7 ...	14 50 47 S. ...	Mid.... 15 11·7 ...	55 45·5

Distance at 6 hours, $44° 14' 50''$; at 9 hours, $45° 45' 38''$.

THE VARIATION OF THE COMPASS.

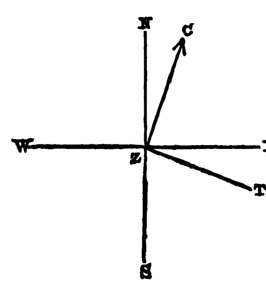
The deflection of the magnetic needle from the true North, or, as it is usually called, the *variation of the compass*, is found at sea either by computing the true bearing of the sun from an observed altitude, the compass bearing being noted at the time of the observation; or, without taking an altitude, determining the true bearing of the sun when in the horizon, its compass bearing being observed at the same instant. The difference between the compass bearing and true bearing thus found is *the variation of the compass.*

Sometimes it is necessary to correct the variation of the compass determined as above for the deviation of the compass itself, arising from the following local causes. The iron on board draws the needle to the east or west of the magnetic meridian; and this effect is greater or less on the needle according as the iron is distributed more or less unequally on different sides of the magnetic meridian. The deviation of the compass due to this cause is discovered, previously to the ship going to sea, by swinging her round and noting the deflection of the needle from the magnetic meridian on different points; a table is then formed similar to the one in p. 221, from which the correction of the compass for different positions of the ship's head may be readily found. The method of determining whether the variation of the compass is east or west will be best seen by means of the following examples.

EXAMPLES.

468. Suppose the true bearing of the sun was found by observation to be N. 100° 10′ E., when the compass bearing was N. 90° 42′ E.: required the variation of the compass, the ship's head being N.E.

Let N represent the true north point of the horizon, and NS the true meridian; measure (roughly) 100° 10′ from north towards east as the angle NZT; then T represents the place of the sun when the observation was taken. From T measure back towards N the compass bearing 90° 42′, as TZC; then ZC is the direction of the needle, and the angle NZC is the variation of the compass, which is evidently easterly, since the compass north is to the east of the true north: hence in this example the variation is said to be east, thus:



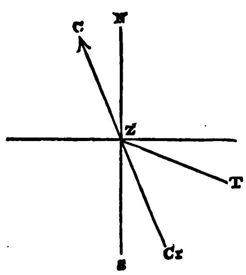
True bearing NZT	N.	100° 10′	E.
compass bearing CZT............	N.	90 42	E.
apparent variation		9 28	E.

Now if the iron on board had no effect on the needle, this would be the true variation; but referring to the table, it appears that the needle itself is drawn or deflected 10° to the east, in consequence of the disturbing effects

of the iron, when the direction of the ship's head is N.E. Placing the 10° 0′ E. under the above 9° 28′ E., and subtracting, we have the true variation of the compass corrected for local deviation, thus:

Observed variation............................	9° 28′ E.
deviation.......................................	10 0 E.
true variation..................................	0 32 W.

469. The true bearing of the sun was found by observation to be S. 60° 42′ E., when the compass bearing was S. 50° 10′ E.: required the variation of the compass, the ship's head being N.E.



Let NZS represent as before the true meridian; draw ZT, making the angle SZT equal to 60° 42′ (roughly); then T represents the true place of the sun; from T measure back towards S the compass bearing TZC′, equal 50° 10′; then ZC′ is the direction of the magnetic needle, and the angle SZC′ or NZC is the observed variation of the compass, to be corrected for deviation (if any). Thus:

True bearing...............................	S. 60° 42′ E.
compass bearing..........................	S. 50 10 E.
observed variation.......................	10 32 W.
deviation..................................	10 0 E.
true variation.............................	20 32 W.

For by the table it appears that the needle, by the effects of the iron, is drawn 10° to the eastward; if there had been no iron on board the needle would have been directed 10° to the westward of its observed place. Hence may be deduced the following rule to find the variation of the compass.

Rule 57. *Given the* TRUE BEARING *and* COMPASS BEARING *and* DEVIATION, *to find the* VARIATION OF THE COMPASS.

1. Reckon the compass bearing and the true bearing from the same point, north or south.

2. Take the difference of the two bearings when measured towards the same point, but the sum when measured towards different points; the result is the apparent variation of the compass; east when the true bearing is to the right of the compass bearing, west if the true bearing is to the left of the compass; the observer being supposed to be placed in the centre of the compass, and looking towards the heavenly body.

NOTE.—The name of the variation, whether east or west, may also be readily found by making a figure similar to those in the preceding examples.

3. If there be no deviation to be allowed for local attraction, the above is the true variation.

4. *To correct for local deviation* (if any). Under the apparent variation just found, put the correction from the table of deviation, with the *opposite* letter to that given in the table.

5. When the names put down are alike add, putting the common letter to the result: if the names put down be unlike, subtract the less from the greater, putting to the remainder the name of the greater. The result will be the variation of the compass corrected for deviation, and therefore the true variation.

DEVIATION OF THE COMPASS OF H.M.S. ——,

(Caused by the local attraction of the Ship) for given positions of the Ship's head.

Direction of Ship's Head.	Deviation of Compass.	Direction of Ship's Head.	Deviation of Compass.
N.	2° 45′ E.	S.	3° 0′ W.
N. by E.	4 57	S. by W.	4 20
N.N.E.	7 30	S.S.W.	5 0
N.E. by N.	9 0	S.W. by S.	6 7
N.E.	10 0	S.W.	7 0
N.E. by E.	10 55	S.W. by W.	7 27
E.N.E.	10 40	W.S.W.	7 50
E. by N.	9 55	W. by S.	8 20
E.	8 50	W.	8 50
E. by S.	7 15	W. by N.	8 10
E.S.E.	5 35	W.N.W.	6 50
S.E. by E.	3 40	N.W. by W.	5 40
S.E.	1 50	N.W.	4 50
S.E. by S.	0 20 E.	N.W. by N.	3 20
S.S.E.	0 56 W.	N.N.W.	1 40 W.
S. by E.	2 20	N. by W.	1 10 E.

EXAMPLES.

470. The true bearing of the sun is N. 117° 39′ E., and compass bearing S. 71° 10′ E. : required the true variation. The ship's head being S.b.E., and therefore the deviation of the compass 2° 20′ W. (see Table). The compass bearing reckoned from the same point as the true bearing is, N. 108° 50′ E.

True bearing	N. 117°	39′	E.
compass bearing	N. 108	50	E.
apparent variation	8	49	E.
deviation	2	20	E.
true variation	11	9	E.

471. The true bearing is E. 10° N., when the compass bearing is E. 8° S. : required the true variation, the ship's head being S.W.

True bearing	E.	10° N.
compass bearing	E.	8 S.
apparent variation		18 W.
deviation		7 E.
true variation		11 W.

472. The true bearing is S. 80° W., when the compass bearing is N. 108° W. : required the true variation, the ship's head being S.W. b.W., and therefore the deviation by Table $7\frac{1}{2}$° W.

True bearing	S.	80° W.
compass bearing	S.	72 W.
apparent variation		8 E.
deviation		$7\frac{1}{2}$ E.
true variation		$15\frac{1}{2}$ E.

473. The true bearing of the sun was N. 36° E., when the compass bearing was N. 24° E., the ship's head being W. $\frac{1}{2}$ N. : required the variation of the compass. *Ans.* $20\frac{1}{2}$° E.

474. The true bearing was N. 110° 42′ W., when the compass bearing was N. 90° 24′ W., the ship's head being S.S.W. : required the variation of the compass. *Ans.* 15° 18′ W.

475. The true bearing was S. 48° 30′ W., when the compass bearing was N. 132° 33′ W., the ship's head being S.W. b. W. : required the variation of the compass. *Ans.* 8° 30′ E.

476. The true bearing of the sun was E. 20° 20′ N., when the compass bearing was E. 32° 45′ N., the ship's head being W.: required the variation of the compass. *Ans.* 21° 15′ E.

477. The true bearing of the sun was W. 12° 32′ S., the compass bearing was W. 2° 10′ N., the ship's head being W. b. S.: required the variation of the compass. *Ans.* 6° 22′ W.

478. The true bearing of the sun was W. 30° 10′ N., the compass bearing was W. 20° 42′ N., the ship's head being N. b. E.: required the variation of the compass. *Ans.* 4° 31′ E.

479. The true bearing of the sun was W. 30° 30′ N., the compass bearing was W. 28° 15′ N., the ship's head being N.E. b. N.: required the variation of the compass. *Ans.* 6° 45′ W.

The variation of the compass is found at sea by either of the following problems:

1. Given the latitude of the ship and the sun's declination when in the horizon, to find the bearing or amplitude.

2. Given the latitude of the ship, and the altitude of the sun and declination, to find the true bearing or azimuth.

3. Given the latitude and time at the ship and the sun's declination, to find the true bearing or azimuth.

The compass bearing being observed at the time of observation, the difference of compass and true bearing—that is, the variation of the compass—is readily found by the preceding rules.

Rule 58. Variation by amplitude.

1. Get a Greenwich date.

2. Take out of the *Nautical Almanac* the sun's declination for this date.

3. Add together the log. sin. of the declination and log. secant of latitude; the sum, rejecting 10 in the index, is the log. sin. of amplitude, which take from the tables.

4. If the body is rising, mark it east: if setting, west: mark it also north or south, according as the declination is north or south.

5. The result is the amplitude or true bearing.

6. Under the true bearing put the compass bearing, and determine the variation of the compass by the preceding rule.

EXAMPLE.

480. September 19, 1851, at 5^h 51^m A.M. mean time nearly, in latitude 47° 25′ N., and longitude 72° 15′ W., the sun rose by compass E. 12° 10′ N.: required the variation, the ship's head being E. b. S.

Ship, Sept. 18	17^h	51^m
longitude in time	4	51 W.
Greenwich, Sept. 18	22	42

Sun's declination.								
18th	2°	0′	31″N.	Sin. decl.		8·457103		
19th	1	37	14 N.	sec. lat.		0·169628		
		23	17	sin. ampl.		8·626731		
·02419				true bearing	E.	2°	25′	N.
·88823				comp. bearing	E.	12	10	N.
·91242		22	2	app. variation		9	45	E.
declination	1	38	29 N.	deviation		7	15	W.
				true variation		2	30	E.

481. May 6, 1846, at 5^h 30^m A.M. mean time nearly, in lat. 50° 48′ N., and long. 47° 12′ E., the sun rose by compass E. 2° 10′ S.: required the variation, the ship's head being S. b. W. *Ans.* 24° 21′ 30″ W.

482. Nov. 14, 1846, at 6^h 45^m P.M. mean time nearly, in lat. 32° 14′ S., and long. 100° E., the sun set by compass W. 15° 40′ S.: required the variation, the ship's head being N.E. *Ans.* 16° 1′ 30″ W.

483. Jan. 10, 1846, at 6^h 58^m A.M. mean time nearly, in lat. 31° 56′ N., and long. 75° 30′ W., the sun rose by compass E. 30° 10′ S.: required the variation, the ship's head being N.E. b. E. *Ans.* 14° 55′ W.

484. March 21, 1846, at 6^h 0^m A.M. mean time nearly, in lat. 42° 13′ N., and long. 90° E., the sun rose by compass E. 11° 40′ S.: required the variation, the ship's head being W. b. S. *Ans.* 3° 20′ 15″ W.

485. March 31, 1850, at 6^h 0^m P.M. mean time nearly, in lat. 42° 13′ N., and long. 124° W., the sun set by compass W. 11° 30′ S.: required the variation, the ship's head being N. *Ans.* 14° 38′ 15″ E.

486. Dec. 4, 1851, at 7^h 50^m A.M. mean time nearly, in lat. 50° 40′ N., and long. 94° W., the sun rose by compass E. 10° 42′ S.: required the variation of the compass, the ship's head being N.E. *Ans.* 15° 57′ E.

Elements from Nautical Almanac.

Sun's declination.

May 5	16°	13′	22″N.	6	16°	30′	22″N.
Nov. 14	18	13	22 S.	15	18	28	54 S.
Jan. 10	21	58	29 S.				
March 20	0	11	37 S.	21	0	12	5 N.
March 31	4	7	44 N.	32	4	30	54 N.
Dec. 4	22	13	9 S.	5	22	21	7 S.

Rule 69. Variation by azimuth and sun's altitude.

Given the altitude of the sun, and the compass bearing, to find the variation of the compass.

1. Get a Greenwich date.

2. Take out of the *Nautical Almanac* the sun's declination for this date, and also the sun's semidiameter.

3. Find the polar distance by adding 90° to the declination, when the latitude and declination have different names, or by subtracting the declination from 90° when the latitude and declination have the same names.

4. Correct the observed altitude for index correction, dip, semidiameter, and correction in altitude, and thus get the true altitude.

5. *To find the true bearing.* First method, by haversines.

Put down the latitude under the altitude, and take their difference; under which put the polar distance: take the sum and difference.

6. To the log. secants of the two first terms in this form (omitting the tens in the index) add the halves of the log. haversines* of the two last: the result, rejecting 10 in the index, is the log. haversine of the true bearing or azimuth, which find from the table.

7. *To find the true bearing.* Second method, by log. sines, &c.

Put down the latitude under the altitude, and take their difference; under which put the polar distance: take the sum and difference, and the half-sum and half-difference.

8. To the log. secants of the two first terms in this form (omitting the tens in the index) add the sines of the two last, and divide the sum by 2; the result is the log. sine of half the true bearing, which find from the tables and multiply by 2 for the true bearing or azimuth required.

9. Mark the true bearing N. or S., according as the latitude is N. or S.; mark it also E. or W., according as the heavenly body is E. or W. of the meridian.

* If the student have no table of haversines, the true bearing may be found by the second method.

10. With the true bearing thus found, and the compass bearing, find the variation of the compass by Rule 57, p. 220.

EXAMPLE.

487. June 7, 1851, at 5h 50m A.M. mean time nearly, in lat. 50° 47′ N., and long. 99° 45′ W., when the sun bore by compass S. 92° 36′ E., the observed altitude of the sun's lower limb was 18° 35′ 20″, index correction + 3′ 10″, and the height of the eye above the level of the sea was 19 feet: required the variation, the ship's head being N.E.

Ship, June 6	17h 50m
longitude in time	6 39 W.
Greenwich, June 7	0 29

	Sun's declin.	Sun's semi.		Obs. alt.
7th	22° 43′ 49″ N.	15′ 46″		18° 35′ 20″
8th	22 49 34 N.		In. cor.	3 10+
	5 45			18 38 30
1·69597			dip	4 17−
1·49560				18 34 13
3·19157 . .	0 7		semi	15 46
	22 43 56 N.		app. alt.	18 49 59
	90		cor. in alt.	2 41−
pol. dist.	67 16 4		true alt.	18 47 18

First method. True bearing by haversines.

Latitude	50° 47′ 0″	. . Sec. . .	0·199108
altitude	18 47 18	. . sec. . .	0·023779
	31 59 42		
polar distance	67 16 4		
sum	99 15 46 (S)	. . ½ hav. S.	4·881893
difference	35 16 22 (D)	. ½ hav. D.	4·481384
			9·586164

True bearing	N. 76° 46′ 30″ E.
compass bearing	N. 87 24 0 E.
app. variation	10 37 30 W.
deviation	10 0 0 W.
true variation	20 37 30 W.

Second method. True bearing by sines, &c.

Latitude	50° 47′ 0″	. . Sec. . .	0·199108
altitude	18 47 18	. . sec. . .	0·023779
	31 59 42	sin. S_1 .	9·881880
polar distance	67 16 4	sin. D_1 .	9·481434
sum	99 15 46		2)19·586201
difference	35 16 22		9·793100
½ sum	49 37 53 (S_1)		38° 23′ 15″
½ difference	17 38 11 (D_1)		2
			76 46 30

True bearing as before.

When the ship is in harbour, or in any position where the sight of the horizon is intercepted by land, or obscured by fog, so that the altitude of the sun cannot be taken, the preceding methods are inapplicable. The following rule may then be used, in which it is supposed that the hour-angle at the ship is known, or can be found by means of the chronometer, or the time at the place.

Rule 60. VARIATION BY AZIMUTH AND SHIP MEAN TIME.

Given mean time at ship and the compass bearing, to find the variation of the compass.

1. Get a Greenwich date.

2. Take out of the *Nautical Almanac* for this date the equation of time and sun's declination.

3. Find the polar distance, by adding 90° to the declination, when the latitude and declination have different names, or by subtracting the declination from 90° when the latitude and declination have the same name.

4. Under the colatitude (found by subtracting the latitude from 90°) put the polar distance, take the sum and difference, and the half sum and half difference.

5. *To find the hour-angle from ship mean time.*

Correct the time shown by chronometer when the compass bearing was observed for its error on Greenwich mean time, and thus get the mean time at Greenwich; to mean time apply the equation of time, to obtain apparent time; under this put the longitude in time, adding if east, and subtracting if west; the result will be ship apparent time, and also the hour-angle if P.M.; but if A.M. at ship, subtract the apparent time from 24 hours, the remainder will then be the hour-angle required.

6. Divide the hour-angle by 2. Then under heads (1) and (2) put down the following quantities.

7. Under both (1) and (2) put log. cotangent of half hour-angle,

Under (1) log. cosine }
(2) log. sine. } of half difference of polar distance and colatitude.

(1) log. sec. }
(2) log. cosec. } of half sum of polar distance and colatitude.

8. Add together the log. under (1) and (2) separately, and take out the angles corresponding to each as a log. tangent. Put one under the other, and take their sum, if the polar distance is greater than the colatitude, or their difference, if the polar distance is less than the colatitude: the result will be the true bearing of the sun at the time of observation.

9. Then proceed to find the variation as in Rule 57.

EXAMPLES.

488. June 23, 1847, at $10^h\ 58^m$ A.M. mean time nearly, in lat. 50° 48′ N., and long. 1° 6′ W., when a chronometer showed $11^h\ 3^m\ 37^s$, the bearing of the sun was observed to be N. 173° 10′ E., the error of the chronometer on Greenwich mean time being $0^m\ 54^s$ fast: required the variation.

				Equation of time.				Sun's declination.		
June 22, at . . .	22^h	58^m		22d . . .		$1^m\ 29·72^s$		23°	27′	21″ N.
long. in time . .		4+		23d . . .		1 42·64 sub. . . .		23	26	59 N.
Gr., June 22 . . .	23	2				12·92				22
					·01786		·01786			
colat.	39°	12′	0″		2·92283		2·69100			
pol. dist.	66	33	0		2·94069	12·40	2·70886			21
sum	105	45	0			1 42·12		23	27	0
difference	27	21	0							
½ sum	52	52	30			Polar dist. .		66	33	0
½ difference . . .	13	40	30							

					(1)		(2)
Time by chro. . .	11^h	3^m	37^s+12^h	Cot. ½ h.-ang.	10·856573	. . . .	10·856573
error on Gr. M. T. .		0	54 fast.	cos. ½ diff. .	9·987511	Sin. ½ diff.	9·373674
Greenwich, 22d . .	23	2	43	sec. ½ sum .	10·219283	cos. ½ sum	10·098367
equation of time .		1	42 sub.	tan. . . .	11·063367	tan. . .	10·328614
apparent time . .	23	1	1				
longitude in time .		4	24 W.		85° 3′ 30″		64° 51′ 45″
ship app. time . .	22	56	37		64 51 45		
	24			True b. N.	149 55 15 E.		
hour-angle . . .	1	3	23	comp. b. N.	173 10 0 E.		
½ hour-angle . . .	0	31	41	variation .	23 14 45 W.		

489. April 27th, 1847, at $1^h\ 10^m$ P.M. mean time nearly, in lat. 50° 48′ N., and long. 1° 6′ W., when a chronometer showed $1^h\ 15^m\ 51^s$, the bearing of the sun was observed to be S. 51° 55′ W., the error of the chronometer on Greenwich mean time being $1^m\ 18^s$ fast: required the variation.

Ans. 23° 46′ W.

490. December 14, 1847, at $10^h\ 22^m$ A.M. mean time nearly, in lat. 52° 10″ N., and long. 1° 30′ W., when a chronometer showed $10^h\ 30^m$ 48, the bearing of the sun was observed to be N., 179° 20′ E., the error of the chronometer on Greenwich mean time being $3^m\ 38^s$ fast: required the variation.

Ans. 21° 13′ 15″ W.

491. December 14, 1847, at $1^h\ 55^m$ P.M. mean time nearly, in lat. 48° 50′ N., and long. 1° 30′ W., when a chronometer showed $1^h\ 59^m\ 55^s$, the bearing of the sun was observed to be S. 51° 40′ W., the error of the chronometer oh Greenwich mean time being $0^m\ 5^s$ fast: required the variation.

Ans. 22° 44′ W.

492. December 14, 1848, at $11^h\ 11^m$ A.M. mean time nearly, in lat. 39° 40′ N., and long. 0° 40′ E., when a chronometer showed $11^h\ 19^m\ 43^s$, the bearing of the sun was observed to be N. 167° 50′ E., the error of the chronometer on Greenwich mean time being $3^m\ 38^s$ fast: required the variation. *Ans.* 2° 50′ 13″ E.

493. March 7, 1844, at $9^h\ 59^m$ A.M. mean time nearly, in lat. 49° 48′ N., and long. 1° 10′ E., when a chronometer showed $10^h\ 24^m\ 8^s$, the bearing of the sun was observed to be N. 164° 51′ 40″ E., the error of the chronometer on Greenwich mean time being fast $20^m\ 48^s$: required the variation. *Ans.* 20° 26′ 40″ W.

494. May 26, 1851, at $9^h\ 48^m$ A.M. mean time nearly, in lat. 50° 48′ N., and long. 1° 6′ W., when a chronometer showed $9^h\ 47^m\ 37^s$, the bearing of the sun was observed to be S. 31° 7′ E., the error of the chronometer on Greenwich mean time being $3^m\ 17^s$ fast: required the variation. *Ans.* 23° 35′ 15″ W.

Elements from Nautical Almanac.

	Sun's declination.				Equation of time.	
April 27	13°	43′	22″ N.	27	2^m	$24{\cdot}56^s$ add
,, 28	14	2	25 N.	28	2	34·29
Dec. 13	23	8	42 S.	13	5	43·66 add
,, 14	23	12	38 S.	14	5	15·08
Dec. 14	23	12	38 S.	14	5	15·08 add
,, 15	23	16	7 S.	15	4	46·23
Dec. 13	23	11	43 S.	13	5	23·23 add
,, 14	23	15	18 S.	14	4	54·52
March 6	5	30	13 S.	6	11	25·48 sub.
,, 7	5	6	55 S.	7	11	10·81
May 25	20	53	42 N.	25	3	25·40 add
,, 26	21	4	25 N.	26	3	19·55

TO FIND THE TIME OF HIGH WATER.

The Change Tide, upon which the rule is made to depend, is that tide which takes place P.M. on the day the moon changes, or is at full. The time of high water at change of the moon is given at different places in apparent time; and indeed cannot be generally expressed in mean time. If the tide be given A.M. on that day as the *change tide,* it should be reduced to P.M. by adding 18 minutes, which may be considered as an average difference on that day.

Rule. *Given the apparent time of change tide, and the longitude of the place, to find the mean time of high water* A.M. *and* P.M.

Take out of the *Nautical Almanac* the moon's meridian passage on the given day, and also on the preceding day; also the moon's semidiameter and equation of time for the given day (roughly).

Under heads (1), (2), and (3) (see Examples), put down the following quantities:

Under (1), the time of moon's meridian passage on proposed day, as found in the *Nautical Almanac*.

" (3), the meridian passage on preceding day.

" (2), put down half the sum of these times. (See Examples.)

Correct quantity under (1) by table (*k*), p. 5 of Inman's Tables, by entering with longitude of place at top, and difference of the times under (1) and (3) at the side: thus find the time of moon's meridian passage at the place.

Take out the correction from table (*l*), p. 5, and place it under (1). This correction is found as follows: Enter the table at top with moon's semidiameter, and at the side with meridian passage under (1), corrected by equation of time to nearest minute, so as to reduce the time of moon's meridian passage (which is given in mean time) to apparent time.

Apply the correction thus found with its proper sign, and to the result add the given apparent time of change tide.

1. When the quantity under (1) is *less* than 12 hours.

The time thus found is the mean time of high water P.M. for the proposed day. (See Example 1.)

2. When the quantity under (1) is *greater* than 12 hours, and less than 24 hours.

Work as described above with the meridian passage under (2). Then, if the result is greater than 12 hours, reject 12 hours; the remainder is mean time of high water on the proposed day, P.M. (See Example 3.) But if the result be less than 12 hours, it will be the mean time of high water A.M. on the proposed day. (See Example 5.)

3. When the quantity under (1) is greater than 24 hours.

Work as described above, with the meridian passage under (3). Then, if the result be greater than 24 hours, reject 24 hours; the remainder will be the mean time of high water P.M. on the proposed day. (See Example 7.) But if the result be less than 24 hours, and greater than 12 hours, reject 12 hours; the remainder will be the mean time of high water A.M. on the proposed day. (See Example 11.)

To find the next time of high water A.M. *or* P.M.

If the time of high water found as above is the P.M. time, subtract therefrom the difference between meridian passages under (1) and (2); the remainder will be the mean time of high water A.M. on the proposed day.

If the time of high water is the A.M. time, add thereto the difference between the meridian passages under (1) and (2), and the sum will be the mean time of high water P.M. on the proposed day.

If it be necessary to add 12 hours before this difference can be subtracted, in that case the remainder will be the mean time of high water P.M. on the preceding day; there will be no high water A.M. on the proposed day. And if in adding the difference the sum be greater than 12 hours, this sum, rejecting 12 hours, will be the mean time of high water A.M. on the following day; there will be no high water P.M. on the proposed day.

EXAMPLES.

495. Find the time of high water on January 3, 1857. Change tide 2^h 10^m P.M. apparent time; long. 50° W.

Moon's mer. pass. Jan 3, 6^h 7^m	☾ semi... 16′ 10″	
" " 2, 5 19	Eq. of time 4^m 54^s −	
48		
24		

(1)	(2)	(3)
6^h 7^m	5 43	5^h 19^m
7+		
6 14		
1 0−		
5 14		
2 10+		
7 24 P.M.	January 3	
24−		
7 0 A.M.		

496. Find the time of high water on May 28, 1857. Change tide 5^h 30^m P.M. app. time; long. 75° E.; moon's mer. pass. on 28, 4^h 51^m; on 27, 3^h 58^m; ☾ semi. 15′ 41″; eq. of time 3^m 1^s+.

Ans. 9^h 1^m P.M.; 8 35^m A.M.

497. Find the time of high water on January 12, 1857. Change tide 1^h 30^m P.M. app. time; long. 60° E.

Moon's mer. pass. Jan. 12,	$14^h\ 28^m$	☾ semi. 15′ 22′	
" " " 11,	13 40	Eq. of time $8^m\ 42^s-$	
	48		
	24		

(1)	(2)	(3)
$14^h\ 28^m$	14 4	$13^h\ 40^m$
8 −	8 −	
14 20	13 56	
0 38 −	0 29 −	
13 42	13 27	
1 30 +	1 30 +	
15 12	14 57	
Greater than 12 hours.	2 57 P.M.	January 12.
	24 −	
	2 33 A.M.	

498. Find the time of high water on June 8, 1857. Change tide $4^h\ 20^m$ P.M. app. time; long. 40° W.; moon's mer. pass. on June 8, $13^h\ 4^m$; on 7, $12^h\ 10^m$; ☾ semi. 14′ 58″; eq. of time $1^m\ 17^s+$.

Ans. $5^h\ 5^m$ P.M.; $4^h\ 38^m$ A.M.

499. Find the time of high water on February 5, 1857. Change tide, $2^h\ 10^m$ P.M. app. time; long. 70° E.

Moon's mer. pass. Feb. 5,	$9^h\ 37^m$	☾ semi. 15′ 49″	
" " " 4,	8 37	Eq. of time $14^m\ 19^s-$	
	60		
	30		

(1)	(2)	(3)
$9^h\ 37^m$	9 7	$8^h\ 37^m$
12 −	12 −	
9 25	8 55	
0 27 +	0 20 +	
9 52	9 15	
2 10 +	2 10 +	
12 2	11 25 A.M.	February 5.
Greater than 12 hours.	30 +	
	11 55 P.M.	

500. Find the time of high water on September 26, 1857. Change tide $6^h\ 40^m$ P.M. app. time; long $20°$ W.; moon's mer. pass. on Sept. 26, $6^h\ 13^m$; on 25, $5^h\ 20^m$; ☾ semi. $14'\ 59''$; eq. of time $8^m\ 43^s+$.

Ans. $11^h\ 32^m$ A.M.; $11^h\ 58^m$ P.M.

501. Find the time of high water on January 24, 1857. Change tide $5^h\ 0^m$ P.M. app. time; long. $30°$ W.

Moon's mer. pass. Jan. 24, $23^h\ 51^m$		☾ semi. $15'\ 41''$
" " " 23, 22 54		Eq. of time $12^m\ 27^s-$
57		
28		
(1)	(2) 23 23	(3)
$23^h\ 51^m$		$22^h\ 54^m$
5+		5+
23 56		22 59
0 11+		0 20+
24 7		23 19
5 0+		5 0+
29 7		28 19
Greater than 24 hours.		January 24 { 4 19 P.M.
		28−
		3 51 A.M.

502. Find the time of high water on June 19, 1857. Change tide $3^h\ 40^m$ P.M. app. time; long. $120°$ E.; moon's mer. pass. on June 19, $22^h\ 28^m$; on 18, $21^h\ 27^m$; ☾ semi. $16'\ 32''$; eq. of time $0^m\ 58^s-$.

Ans. $1^h\ 19^m$ P.M.; $0^h\ 49^m$ A.M.

503. Find the time of high water on March 4, 1857. Change tide $5^h\ 0^m$ P.M. app. time; long. $45°$ W.

Moon's mer. pass. March 4, $7^h\ 32^m$	☾ semi. $15'\ 48''$
" " " 3, 6 31	Eq. of time $11^m\ 54^s-$
61	
30	
7 2	

(1)	(2)	(3)
7^h 32^m	7^h 2^m	6^h 31^m
7+	7+	
7 39	7 9	
0 15−	0 34−	
7 24	6 35	
5 0+	5 0+	
12 24	11 35 A.M. March 4; no P.M. tide.	
Greater than 12 hours.	30+	
	0 5 A.M. March 5.	

504. Find the time of high water on July 30, 1857. Change tide 5^h 30^m P.M. app. time; long. 90° W.; moon's mer. pass. on July 30, 7^h 6^m; on 29, 6^h 20^m; ☾ semi. 14′ 49″; eq. of time 6^m 8^s—.

Ans. 11^h 48^m A.M. July 30; no P.M. tide.

505. Find the time of high water on March 21, 1857. Change tide 3^h 0^m P.M. app. time; long. 100° W.

Moon's mer. pass. March 21, 21^h 11^m ☾ semi. 15′ 50″
" " " 20, 20 17 Eq. of time 7^m 16^s—

54

27

(1)	(2)	(3)
21^h 11^m	20 44	20^h 17^m
14+		14+
21 25		20 31
0 27+		0 13+
21 52		20 44
3 0+		3 0+
24 52		March 21, A.M. 11 44; no P.M. tide.
Greater than 24 hours.		27+
		March 22, A.M. 0 11

506. Find the time of high water on November 12, 1857. Change tide 2^h 40^m P.M. app. time; long. 70° W.; moon's mer. pass. on November 12, 21^h 24^m; on 11th, 20^h 43^m; ☾ semi. 15′ 0″; eq. of time 15^m 40^s+.

Ans. 11^h 49^m A.M. November 12; no P.M. tide.

ROYAL NAVAL COLLEGE EXAMINATION PAPERS.

WE will conclude this treatise of the practical part of Nautical Astronomy with a series of Examination Papers given at the Royal Naval College to candidates passing for lieutenants' and masters' commissions in the Royal Navy.

Questions.—No. I.

1. Required the course and distance from A to B.

Lat. A......	56° 35′ S.	Long. A......	2° 15′ E.
„ B......	51 10 S.	„ B......	3 10 W.

2. Required the course and distance from A to B.

Lat. A......	61° 10′ N.	Long. A......	8° 40′ E.
„ B......	61 10 N.	„ B......	15 10 E.

3. On May 8, at noon, a point of land in lat. 48° 10′ N., and long. 2° 2′ W., bore by compass E. by S. ½ S. distant 20 miles (variation 3¼ E.); afterwards sailed as by the following log. account: required the latitude and longitude in at noon on May 9.

Hours.	Knots.	$\frac{1}{10}$ths.	Course.	Wind.	Leeway.	—
1	3	0	N.N.W.½W.	N.E.	2¼	P.M.
2	3	2				
3	3	4				
4	2	7				
5	2	4	E. by S.¾S.	Do.	3	
6	2	4				Variation 3¼ E.
7	3	6				
8	3	2				
9	3	2				
10	4	0	E.S.E.	South.	2¾	
11	4	2				
12	4	5				
						Remarks in H.M.S. May 9, 1870.
1	5	2	N.N.E.¼E.	N.W.	1¾	A.M.
2	6	2				
3	7	2				
4	7	5				
5	8	0				
6	5	2	W.S.W.	Do.	3¼	
7	6	2				
8	6	4				
9	5	4				
10	6	2				
11	6	3				
12	7	0				

4. What bright star will pass the meridian of Canton in China the first after mean midnight on June 15, 1835, and how far N. or S. of the zenith?

5. June 15, in long. 100° 32′ E., the observed meridian altitude of the sun's lower limb was 20° 15′ 40″ (zenith S. of the sun), the index correction was +2′ 50″, and the height of the eye above the sea was 14 feet: required the latitude.

6. April 23, at 9^h A.M., mean time nearly, in long. 5° 10′ W., the observed meridian altitude of the moon's lower limb was 38° 40′ 45″ (zenith N. of the moon), the index correction was −2′ 50″, and the height of eye above the sea was 20 feet: required the latitude.

7. June 18, the observed meridian altitude of the star α Scorpii (Antares) was 20° 10′ 50″ (zenith north of the star), the index correction was +4′ 50″, and the height of the eye above the sea was 18 feet: required the latitude.

8. June 12, the observed meridian altitude under the S. Pole of α^2 Crucis was 6° 40′ 10″, the index correction was +3′ 40″, and the height of the eye above the sea was 18 feet: required the latitude.

9. December 10, at 2^h 10^m A.M., mean time nearly, in long. 76° 12′ E., the observed altitude of α Ursæ Minoris (Polaris) was 47° 50′ 25″, the index correction was −4′ 10″, and the height of the eye above the sea was 13 feet: required the latitude.

10. September 15, observed the following double altitude of the sun:

Mean time nearly.	Chronometer.	Obs. alt. sun's L. L.	True bearing.
10^h 40^m A.M.	10^h 22^m 36^s	40° 34′ 30″	S.b.E.½E.
11 40 A.M.	11 22 45	42 12 0	S.½E.

The run of the ship in the interval was E.b.N. 12 miles, the index correction was +3′ 50″, and the height of the eye above the sea was 18 feet: required the true latitude, the latitude by account being 51° N., and the longitude 50° 10′ W.

11. March 2, at 7^h 44^m P.M., mean time nearly, in lat. 44° 25′ N., and long. by account 58° E., a chronometer showed 5^h 10^m $42{\cdot}5^s$, and the observed altitude of α Arietis was 30° 10′ 40″ W. of meridian, the index correction was +4′ 20″, and the height of eye above the sea was 18 feet: required the true longitude.

February 24, at Greenwich mean noon, the chronometer was fast on Greenwich mean time 1^h 11^m 22^s, and its daily rate was $2{\cdot}2^s$ losing.

12. Sept. 3, at 9^h 10^m P.M., mean time nearly, in lat. 30° 10′ N., and long. by account 91° 5′ E., the following lunar observation was taken:

Obs. alt. α Arietis E. of meridian.	Obs. alt. moon's L. L.	Obs. dist. F. L.
10° 15′ 40″	36° 12′ 30″	99° 27′ 50″
+1 40	−1 10	−0 30

The height of the eye above the sea was 12 feet: required the true longitude.

13. July 5, at $7^h\ 0^m$ P.M., mean time, in lat. 50° 53′ N., and long. 120° 10′ E., the compass bearing of the sun was W. 10° 15′ N., and the observed altitude of its lower limb was 9° 40′ 0″, the index correction was + 3′ 50″, and the height of the eye above the sea was 18 feet: required the variation of the compass.

14. On Dec. 20, at $4^h\ 30^m$ P.M., mean time nearly, in lat. 41° 12′ N., and long. 110° 45′ E., the sun set by compass S.W.: required the variation.

15. Required the time of high water at A on June 10, A.M. and P.M.

Change tide... $3^h\ 40^m$ A.M. app. time. Long. A.... 65° W.

NOTE.—In this and the following Examination Papers the compass is supposed to have no deviation arising from local attraction.

Elements from Nautical Almanac and Answers.

1. N. 30° 29′ 15″ W. 377·2 miles.

2. E. 188·1 miles.

3. Corrected courses, N.W. $\frac{3}{4}$ N. 20′; departure course, N.b.W. $\frac{1}{2}$ W. 12·3′; S. 14·8′; S.E. b.E. $\frac{1}{2}$ E. 12·7′; E. $\frac{3}{4}$ N. 34·1′; W.S.W. 42·7′. Lat. in 48° 6′ N., long. 2° 18′ W.

4. Right ascension mean sun on June 14, at Greenwich mean noon, $5^h\ 32^m\ 11{\cdot}85^s$. γ Draconis, 28° 23′ N. of zenith.

5. Sun's declination on June 14, at Greenwich mean noon, 23° 15′ 26″ N.; on June 15, 23° 18′ 24″ N., semidiameter 15′ 46″. Lat. 46° 14′ 16″ S.

6. Moon's declination on April 22, at 21^h Greenwich mean time, 11° 35′ 58″ S., at 22^h, 11° 23′ 53″ S.; moon's horizontal semidiameter April 22, at Greenwich mean midnight, 15′ 4·1″, April 23, at Greenwich mean noon, 15′ 0″; corresponding horizontal parallax, 55′ 17·8″ and 55′ 2·8″. Lat. 38° 57′ 50″ N.

7. Declination α Scorpii (Antares), 26° 3′ 34″ S. Lat. 43° 47′ 34″ N.

8. Declination α^2 Crucis, 62° 11′ 25″ S. Lat. 34° 20′ 28″ S.

9. Right ascension mean sun on June 16, at Greenwich mean noon, $17^h\ 10^m\ 2{\cdot}21^s$. Lat. 47° 51′ N.

10. Sun's declination on September 15, at Greenwich mean noon,

2° 49′ 54″ N., on September 16, 2° 26′ 42″ N.; semidiameter, 15′ 56″. Lat. 50° 20′ N.

11. Right ascension mean sun March 2, at Greenwich mean noon, 22^h 38^m $13{\cdot}55^s$. Right ascension α Arietis, 1^h 57^m $51{\cdot}5^s$; declination α Arietis, 22° 40′ 42″ N.; hour-angle, 4^h 37^m 23^s W. Long. 59° 12′ 15″ E.

12. Right ascension mean sun September 3, at Greenwich mean noon, 10^h 47^m $36{\cdot}37^s$. Right ascension α Arietis, 1^h 57^m $55{\cdot}3^s$; declination α Arietis, 22° 40′ 56″ N. Horizontal semidiameter moon September 3, at Greenwich mean noon, 15′ 51·7″, at Greenwich mean midnight, 15′ 48·3″; corresponding horizontal parallax, 58′ 12·5″ and 57′ 59·9″. True distance, 98° 59′ 4″; distance from *Nautical Almanac*, at iii., 99° 2′ 53″, at vi., 97° 22′ 28″; hour-angle, 17^h 54^m 56^s W. Long. 89° 28′ 30″ E.

13. Sun's declination on July 4, at Greenwich mean noon, 22° 56′ 37″ N., on July 5, 22° 51′ 24″ N.; semidiameter, 15′ 45″; true bearing, N. 65° 41′ W. Variation, 14° 4′ E.

14. Sun's declination on December 19, at Greenwich mean noon, 23° 25′ 35″ S., on December 20, 23° 26′ 46″ S.; true bearing, W. 31° 55′ 30″ S. Variation, 13° 4′ 30″ E.

15. Moon's Greenwich meridian passage June 10, 12^h 2^m, June 9, 11^h 0^m; moon's semidiameter, 16′ 36″; equation of time, 1^m S. from apparent time. High water, 3^h 2^m A.M. and 3^h 33^m P.M.

NOTE.—The right ascension of mean sun is found in the *Nautical Almanac* in page ii. of each month, under the heading of "Sidereal Time."

Questions.—No. II.

1. Required the course and distance from A to B.

Lat. A......	40° 25′ N.	Long. A......	2° 10′ E.
„ B......	35 32 N.	„ B......	1 40 W.

2. Required the course and distance from A to B.

Lat. A......	50° 48′ N.	Long. A......	100° E.
„ B......	50 48 N.	„ B......	101 E.

3. May 10, at noon, a point of land in lat. 38° 17′ N. and long. 56° 19′ W. bore by compass W. b. S. ½ S. distant 17½ miles (variation of compass 2¾ E.); afterwards sailed as by the following log account: required the latitude and longitude in, May 11, at noon.

Hours.	Knots.	$\frac{1}{10}$ths.	Course.	Wind.	Leeway.	—
1	5	4	S.S.E.	E.	$2\frac{1}{4}$	P.M.
2	5	6				
3	4	9				
4	4	8				
5	4	8	S.S.W.$\frac{1}{2}$W.	W.	$2\frac{3}{4}$	
6	4	7				Variation $2\frac{3}{4}$ E.
7	5	3				
8	5	2				
9	5	1				
10	6	0	W.S.W.	S.	$2\frac{1}{2}$	
11	6	4				
12	6	8				
						H.M.S. May 11, 1870.
1	6	7				A.M.
2	5	9				
3	5	8	W.$\frac{1}{2}$N.	N.N.E.	0	
4	4	6				During the last 7 hours a current set the ship 3 miles an hour N.N.E. by compass.
5	4	8				
6	5	9				
7	4	8				
8	3	7				
9	3	6	E.	S.S.E.	$2\frac{1}{2}$	
10	3	4				
11	3	5				
12	2	9				

4. What bright star will pass the meridian of Greenwich the first after 10^h P.M. on October 20, and how far N. or S. of the zenith?

5. October 19, in longitude 88° 49′ E., the observed meridian altitude of the sun's lower limb was 58° 37′ 56″ (zenith N. of the sun), the index correction was +8′ 38″, and the height of the eye above the sea was 17 feet: required the latitude.

6. August 10, at 6^h 40^m P.M., mean time, in long. 50° 17′ E., the observed meridian altitude of the moon's lower limb was 45° 47′ 39″ (zenith N. of the moon), the index correction was −3′ 18″, and the height of the eye above the sea was 24 feet: required the latitude.

7. June 3, the observed meridian altitude of the star α Canis Majoris was 43° 29′ 47″ (zenith S. of the star), the index correction was −3′ 14″, and the height of the eye above the sea was 16 feet: required the latitude.

8. February 18, the observed meridian altitude of the star α Ursæ Majoris under the North Pole was 53° 28′ 47″, the index correction was −3′ 49″, and the height of the eye above the sea was 18 feet: required the latitude.

9. February 9, at 10^h 20^m P.M., mean time, in long. 85° 32′ W., the observed altitude of α Ursæ Minoris (Polaris) was 50° 25′ 30″, the index correction was −4′ 10″, and the height of the eye above the sea was 15 feet: required the latitude.

10. June 9, the following double altitude of the sun was observed:

Mean time nearly.	Chronometer.	Obs. alt. sun's L. L.	True bearing.
1^h 3^m P.M.	1^h 10^m 50^s	52° 5′ 40″	S.S.W.
7 6 P.M.	7 12 48	14 57 30	W.N.W.

The run of the ship in the interval was N.N.E. 5 miles, the index correction was −1′ 20″, and the height of the eye above the sea was 17 feet: required the true latitude at the second observation, the latitude by account being 59° N., and longitude 47° 18′ E.

11. August 25, at 9^h 45^m P.M., in lat. 60° 2′ N., and longitude by account 59° 15′ E, when a chronometer, No. 10, showed 5^h 42^m 16^s, the observed altitude of the star α Andromedæ was 39° 32′ 28″ E. of the meridian, the index correction was +5′ 17″, and the height of the eye above the sea was 15 feet: required the true longitude.

On May 15, at Greenwich mean noon, No. 10 was *slow* on Greenwich mean time 8^m $40{\cdot}5^s$, and its daily rate was $7{\cdot}8^s$ *losing*.

12. May 14, at 2^h 20^m P.M., mean time nearly, in lat. 50° 48′ N., and longitude by account 60° 52′ E., the following lunar observation was taken:

	Obs. alt. sun's L. L.	Moon's L. L.	Obs. dist. N. L.
	46° 48′ 7″	45° 47′ 38″	108° 58′ 45″
Index cor.	+3 10	−1 12	+2 18

The height of the eye above the sea was 10 feet: required the true longitude.

13. May 20, at 4^h 47^m A.M., mean time nearly, in 18° 42′ S. and long. 160° E., the sun rose by compass E. 21° 18′ 30″ N.: required the variation.

14. March 7, at 2^h 50^m P.M., mean time nearly, in lat. 51° 10′ N. and long. 86° E., the compass bearing of the sun was S. 74° 42′ W., and at the same time the observed altitude of the sun's lower limb was 21° 40′ 45″, the index correction was −2′ 18″, and the height of the eye above the sea was 14 feet: required the variation.

15. Required the time of high water at A on August 27, A.M. and P.M.

Change tide at A... 5^h 18^m P.M. app. time. Long. A... 93° E.

Elements from Nautical Almanac and Answers.

1. S. 31° 43′ 30″ W. 344·5′.
2. E. 37·9′.

3. Corrected courses, E. b. S. ½ S. 17·5′; S.W. b. S. 20·7′; S.S.W. ½ W. 25·1′; N.W. ¾ W. 31·8′; N.W. ¾ W. 29·6′; E. ½ S. 13·4′; N.E. ¾ E. 21′. Lat. in 38° 21′ N.; long. in 56° 52′ W.

4. α Andromedæ 23° 17′ 10″ S. of zenith.

5. Sun's declination on October 18, at Greenwich mean noon, 9° 39′ 16″ S.; on October 19, 10° 1′ 3″ S.; semidiameter, 16′ 5″. Lat. 21° 6′ 29″ N.

6. Moon's declination on August 10 at 3^h, Greenwich mean time, 23° 15′ 12″ S.; on August 10 at 4^h, 23° 24′ 37″ S.; moon's horizontal semidiameter on August 10, at Greenwich mean noon, 15′ 43·8″; on August 10, at Greenwich mean midnight, 15′ 51·3″; corresponding horizontal parallax, 57′ 43·5″ and 58′ 11·0″. Lat. 20° 7′ 1″ N.

7. Declination of α Canis Majoris, 16° 29′ 49″ S. Lat. 63° 8′ 14″ S.

8. Declination of α Ursæ Majoris, 62° 37′ 42″ N. Lat. 80° 42′ 22″ N.

9. Right ascension mean sun on February 9, 1837, at Greenwich mean noon, $21^h\ 17^m\ 28{\cdot}28^s$. Lat. 50° 34′ N.

10. Sun's declination on June 8, at Greenwich mean noon, 22° 51′ 58″ N.; on June 9, 22° 57′ 10″ N.; and on June 10, 23° 1′ 58″ N.; semidiameter, 15′ 46″. Arc (1), 81° 40′ 15″; Arc (2), 68° 32′ 15″; Arc (3), 38° 4′ 0″. Lat. 60° 11′ 51″ N.

11. Right ascension mean sun on August 25, at Greenwich mean noon, $10^h\ 14^m\ 9{\cdot}84^s$; declination of α Andromedæ, 28° 11′ 40″ N.; right ascension, $0^h\ 0^m\ 1^s$; hour-angle, $20^h\ 4^m\ 23^s$ W. Long. 56° 15′ 0″ E.

12. Sun's declination on May 13, at Greenwich mean noon, 18° 24′ 17″ N.; on May 14, 18° 38′ 55″ N.; corresponding eq. of time, $3^m\ 55{\cdot}3^s$ S. and $3^m\ 55{\cdot}9^s$ S.; moon's horizontal semidiameter on May 13, at Greenwich mean midnight, 14′ 55·2″; on May 14, at Greenwich mean noon, 14′ 59″; corresponding horizontal parallax, 54′ 45·1″ and 54′ 58·9″. True distance, 108° 37′ 59″; distance at xxi., 108° 4′ 49″; distance on 14, at Greenwich mean noon, 109° 28′ 44″; hour-angle, $2^h\ 24^m\ 5^s$. Long. 62° 15′ E.

13. Sun's declination on May 19, at Greenwich mean noon, 19° 47′ 14″ N.; on May 20, 19° 59′ 54″ N. True bearing, E. 20° 59′ 45″ N. Variation, 0° 18′ 45″ E.

14. Sun's declination on March 6, at Greenwich mean noon, 5° 37′ 27″ S.; on March 7, 5° 14′ 8″ S.; semidiameter, 16′8″. True bearing, N. 130°56′30″ W. Variation, 25° 38′ 30″ W.

15. Moon's Greenwich meridian passage on August 27, $22^h\ 8{\cdot}7^m$; August 26, $21^h\ 19{\cdot}9^m$; moon's semidiameter, 14′ 45″. Equation of time, $1^m\ 19^s$ S. from mean time. High water, $2^h\ 18^m$ A.M. and $2^h\ 42^m$ P.M.

Questions.—No. III.

1. Required the course and distance from A to B.

Lat. A...... 70° 15′ S. Long. A...... 3° 10′ W.
,, B...... 75 20 S. ,, B...... 2 15 E.

2. How many miles are there in 10° of longitude in the latitude of Portsmouth?

3. March 4, at noon, a point of land in lat. 50° 48′ N., and long. 1° 6′ W., bore by compass N.N.E. ½ E., distant 15 miles (variation 2¼ W.), afterwards sailed as by the following log account: required the latitude and longitude in, on March 5 at noon.

Hours.	Knots.	$\frac{1}{10}$ths.	Course.	Wind.	Leeway.	
1	3	5	N.N.W. ½ W.	N.E.	1¾	P.M.
2	4	1				
3	4	3				
4	2	7	E.S.E.	Do.	2	
5	3	0				
6	3	2				Variation 2¼ W.
7	4	0				
8	5	6	S. ¾ W.	E.S.E.	2¼	
9	5	2				
10	5	5				
11	4	5				
12	4	6				
						Remarks in H.M.S. Mar. 5, 1870.
1	4	7	N.E. ¼ N.	Do.	1½	A.M.
2	4	2				
3	4	4				
4	3	7				
5	3	2				A current set the ship N.E. the last 6 hours at the rate of 3¼ miles per hour by compass.
6	3	5	W. ½ N.	S.S.W.	1¼	
7	4	2				
8	3	6				
9	3	4				
10	9	5	N. by E.	South.	0	
11	10	2				
12	10	3				

4. At what time will the star α Lyræ pass the meridian of Portsmouth on May 11, and how far N. or S. of the zenith?

5. March 8, in long. 89° 48′ E., the observed meridian altitude of the sun's lower limb was 51° 49′ 30″, zenith north of the sun, the index correction was −3′ 17″, and the height of the eye above the sea 15 feet: required the latitude.

6. March 16, at 8ʰ 2 P.M. mean time nearly, in long. 110° E., the observed meridian altitude of the moon's lower limb was 48° 47′ 36″, zenith

north of the moon, the index correction was $-2' 47''$, and the height of the eye above the sea was 13 feet: required the latitude.

7. July 7, the observed meridian altitude of the star α Cygni was $53° 29' 38''$, zenith north of the star, the index correction was $-5' 12''$, and the height of the eye above the sea was 16 feet: required the latitude.

8. Oct. 16, the observed meridian altitude of the star α Ursæ Majoris under the North Pole was $5° 26' 10''$, the index correction was $-2' 10''$, and the height of the eye above the sea was 17 feet: required the latitude.

9. Sept. 10, 1837, at $3^h 42^m$ A.M., longitude $83° 14'$ E., the observed altitude of α Ursæ Minoris was $39° 47' 48''$, the index correction was $+3' 45''$, and the height of the eye above the sea was 17 feet: required the latitude.

10. April 10, the following double altitude of the sun was observed:

Mean time nearly.	Chronometer.	Obs. alt. sun's L. L.	True bearing.
$10^h\ 14^m$ A.M.	$10^h\ 9^m\ 40^s$	$41°\ 15'\ 45''$	S.E.
11 47 A.M.	11 43 28	46 43 12	S. by E.

The run of the ship in the interval was N.W. 6 miles, the index correction was $-4' 24''$, and the height of the eye above the sea was 20 feet: required the true latitude at the second observation, the latitude by account being $51°$ N., and the longitude $1° 6'$ W.

11. May 10, at $3^h 10^m$ P.M. mean time, in lat. $48° 12'$ N., and long. by account $45° 10'$ E., when a chronometer showed $0^h 10^m 42^s$, the observed altitude of the sun's lower limb was $37° 20' 10''$, the index correction was $+3' 10''$, and the height of the eye above the sea was 18 feet: required the true longitude.

On May 1, at Greenwich mean noon, the chronometer was fast on Greenwich mean time $9^m 50^s$, and its daily rate was $3{\cdot}2^s$ gaining.

12. January 16, at $3^h 4^m$ P.M. mean time nearly, in lat. $50° 50'$ N., and long. by account $65°$ E., the following lunar observation was taken:

	Obs. alt. sun's L. L.	Obs. alt. moon's L. L.	Obs. dist. N. L.
	$8°\ 32'\ 20''$	$15°\ 42'\ 30''$	$121°\ 10'\ 30''$
Index cor.	+1 10	+5 47	−5 47

The height of the eye above the sea was 16 feet: required the true longitude.

13. May 18, at $4^h 50^m$ A.M. mean time nearly, in lat. $18° 45'$ S., and long. $99° 18'$ E., the sun rose by compass S. $80° 12'$ E.: required the variation.

14. March 7, at $9^h 10^m$ A.M. mean time nearly, in lat. $51° 10'$ N., and long. $89° 12'$ E., the compass bearing of the sun was S. $74° 50'$ E., and at the same time the observed altitude of the sun's lower limb was $21° 40' 43''$, the index correction was $-2' 18''$, and the height of the eye above the sea was 14 feet: required the variation.

15. Required the time of high water at A on March 10, A.M. and P.M.

Change tide at A... $6^h 45^m$ P.M. app. time. Long. A... $98°$ E.

Elements from Nautical Almanac and Answers.

1. S. 17° 23′ E., 319·4′.

2. 379·2′.

3. Corrected courses, S. ¼ W. 15′; departure course, W. b. N. ½ N. 11·9′; E.S.E. ¼ E. 12·9; S. ¾ W. 25·4′; N. 20·2′; W. ½ S. 14·7′; N. b. W. ¼ W. 30′; N. b. E. ¾ E. 21′. Latitude in 51° 14′ 54″ N.; longitude in 1° 35′ 24″ W.

4. At $15^h\ 12^m\ 42^s$: 12° 10′ 11″ S. of zenith.

5. Sun's declination on March 7, at Greenwich mean noon, 5° 14′ 8″ S.; on March 8, 4° 50′ 46″ S.; semidiameter, 16′ 7″. Lat. 33° 5′ 44″ N.

6. Moon's declination on March 16, at 0^h, 26° 48′ 39″ N.; at 1^h, 26° 44′ 20″ N., moon's horizontal semidiameter on March 16, at Greenwich mean noon, 14′ 45·1″; on March 16, at Greenwich mean midnight, 14′ 44·9″; horizontal parallax, 54′ 8·1″ and 54′ 7·4″. Lat. 67° 14′ 35″ N.

7. Declination α Cygni, 44° 41′ 59″ N. Lat. 81° 22′ 12″ N.

8. Declination of α Ursæ Majoris, 62° 37′ 28″ N. Lat. 32° 33′ 1″ N.

9. Right ascension mean sun, on Sept. 9, at Greenwich mean noon, $11^h\ 13^m\ 18{\cdot}15^s$. Lat. 38° 24′ N.

10. Sun's declination on April 9, at Greenwich mean noon, 7° 36′ 29″ N., on April 10, 7° 58′ 43″ N.; semidiameter, 15′ 58″. Arc (1) 23° 13′ 15″, arc (2) 88° 17′ 30″, arc (3) 65° 27′ 0″. Lat. 51° 0′ 47″ N.

11. Sun's declination on May 10, at Greenwich mean noon, 17° 38′ 34″ N.; on May 11, 17° 54′ 7″ N.; equation of time, $3^m\ 50{\cdot}2^s$ S., and $3^m\ 52{\cdot}5^s$ E.; semidiameter, 15′ 51″; hour angle, $3^h\ 31^m\ 21^s$. Longitude, 51° 47′ E.

12. Sun's declination on Jan. 15, at Greenwich mean noon, 21° 6′ 42″ S., on January 16, 20° 55′ 23″ S., equation of time, $9^m\ 49{\cdot}1^s$ A, and $10^m\ 9{\cdot}8^s$ A; moon's horizontal semidiameter on January 15, at Greenwich mean midnight, 15′ 2·1″, on January 16, at Greenwich mean noon, 14′ 58·0″; corresponding horizontal parallax, 55′ 10·6″ and 54′ 55·3″. True distance, 121° 20′ 29″; distance at xxi., 120° 32′ 5″; at xxiv., 121° 55′ 25″. Hour angle, $2^h\ 54^m\ 7^s$. Longitude, 64° 55′ 45″ E.

13. Sun's declination on May 17, at Greenwich mean noon, 19° 20′ 53″ N., on May 18, 19° 34′ 14″ N. True bearing E. 20° 34′ 45″ N. Variation, 30° 22′ 45″ W.

14. Sun's decl. on March 6, at Greenwich mean noon, 5° 37′ 27″ S., on March 7, 5° 14′ 8″ S.; semidiameter, 16′ 8″. True bearing N. 131° 9′ 15″ E. Variation, 25° 59′ 15″ E.

15. Moon's Greenwich meridian passage on March 10, $3^h\ 13^m$ mean time on March 9, $2^h\ 26^m$; moon's semidiameter, 15′ 37″; equation of time, 11^m S. from mean time; high water $9^h\ 1^m$ P.M. and $8^h\ 37^m$ A.M.

Questions.—No. IV.

1. Required the course and distance from A to B.

Lat. A ...... 60° 25′ S. Long. A ...... 35° 22′ E.
" B ...... 64 12 S. " B ...... 30 10 E.

2. Sailed from Ushant due west 492·5 miles: required the latitude and longitude in.

3. May 1, at noon, a point of land in latitude 51° 10′ S., and longitude 3° 15′ E., bore by compass S.S.W. ½ W. distant 25 miles, variation 2¾ E., afterwards sailed as by the following log account: required the latitude and longitude in.

Hours.	Knots.	$\frac{1}{10}$ths.	Course.	Wind.	Leeway.	
1	3	2	S.S.E. ½ E.	S.W.	2¼	A.M.
2	3	4				
3	3	5				
4	4	0				
5	4	2				
6	4	3	W.N.W.	Do.	2½	Variation 2¾ E.
7	4	4				
8	4	5				
9	5	2				
10	5	6				
11	5	7				
12	6	1				
						H.M.S. May 2, 1870.
1	7	2	South.	West.	¼	P.M.
2	7	1				
3	7	3				
4	8	3				
5	8	4				
6	7	5	S.E. ¾ E.	S.S.W.	1½	A current set the ship N.W. ½ W. 20 miles.
7	7	2				
8	7	1				
9	6	7				
10	6	5				
11	6	3				
12	6	0				

4. What bright star will pass the meridian of the Land's End the first after $6^h\ 42^m$ A.M. mean time on August 17, and how far N. or S. of the zenith?

5. August 18, in longitude 110° 32′ E., the observed meridian altitude of the sun's lower limb was 50° 25′ 10″, zenith N. of the sun, the index correction was −2′ 50″, and the height of the eye above the sea was 15 feet: required the latitude.

6. August 18, at $8^h\ 0^m$ A.M. mean time nearly, in longitude 92° 10′ W., the observed meridian altitude of the moon's lower limb was 26° 42′ 10″, zenith S. of the moon, the index correction was −3′ 40″, and the height of the eye above the sea was 14 feet: required the latitude.

7. December 7, the observed meridian altitude of the fixed star α Arietis was 40° 25′ 10″, zenith N. of the star; the index correction was −2′ 10″, and the height of the eye above the sea was 18 feet: required the latitude.

8. December 7, the observed meridian altitude of α Ursæ Majoris, under the N. Pole was 11° 10′ 10″; the index correction was +3′ 20″, and the height of the eye above the sea was 19 feet: required the latitude.

9. December 7, 1835, at $1^h\ 20^m$ A.M., in long. 78° 30′ E., the observed altitude of α Ursæ Minoris (Polaris), was 50° 40′ 15″; the index correction was −5′ 10″, and the height of the eye above the sea was 12 feet: required the latitude.

10. July 31, observed the following double altitude of the sun:

Mean time nearly.	Chronometer.	Obs. alt. sun's L. L.	True bearing.
$11^h\ 58^m$ A.M.	$0^h\ 0^m\ 10^s$	57° 29′ 45″	S. 3° E.
0 4 P.M.	0 6 17	57 29 30	S. 3° 20′ W.

The run of the ship was none, dip none, the index correction was +0′ 30″: required the true latitude, the latitude by account being 51° N., and the longitude 1° W.

11. May 14, at 9^h 30 A.M., in lat. 50° 48′ N., and long. by account 2° W., a chronometer showed 9 $26^m\ 18^s$, and the observed altitude of the sun's lower limb was 46° 48′ 7″, the index correction was +3′ 10″, and the height of the eye above the sea was 10 feet: required the true longitude.

May 1, at Greenwich mean noon, the chronometer was slow on Greenwich mean time $4^m\ 2^s$, and its daily rate was $3{\cdot}5^s$ losing.

12. September 3, at $6^h\ 32^m$ P.M. mean time nearly, in lat. 30° 10′ N., and long. by account 36° 10′ W., the following lunar observation was taken:

	Obs. alt. α Pegasi (Markab) E. of meridian.	Obs. alt. moon's L. L.	Obs. dist. F. L.
	9° 50′ 40″	18° 10′ 50″	55° 46′ 20″
Index cor.	−1 10	+1 30	−0 30

The height of the eye above the sea was 15 feet: required the true longitude.

13. August 23, at $7^h\ 0^m$ P.M. mean time nearly, in lat. 50° 48′ N., and long. by account 140° 25′ E., the sun set by compass W. 5° 10′ S.: required the variation.

14. August 23, at $5^h\ 50^m$ A.M. mean time nearly, in lat. 51° 10′ N., and long. 135° 40′ W., the sun-bearing by compass was S. 92° 10′ E., and the observed altitude of its lower limb was 7° 40′ 50″, the index correction was −2′ 50″, and the height of the eye above the sea was 15 feet: required the variation.

15. Required the mean time of high water at A on Aug. 2, A.M. and P.M.

Change tide... $3^h\ 40^m$ app. time. Long. A... 70° W.

Elements from Nautical Almanac and Answers.

1. S. 32° 32′ 15″ W. 269·3′.

2. Lat. in 48° 28′ N. Long. in 17° 25′ 48″ W.

3. Corrected courses, N.E.b.E.¼E. 25′ departure course; S.S.E. 18·3′; N.¾W. 35·8′; S.S.W.½W. 38·3′; S.E.½S. 47·3′; N.b.W.¾W. 20′. Lat. in 51° 29′ 54″ S. Long. in 4° 0′ 18″ E.

4. α Tauri 33° 54′ S. of zenith.

5. Sun's declination on Aug. 17, at Greenwich mean noon, 13° 35′ 52″ N., on Aug. 18, 13° 16′ 40·3″ N.; semidiameter, 15′ 49″. Latitude, 52° 48′ 51″ N.

6. Moon's declination on August 18, at 2^h Greenwich mean time, 24° 12′ 3″ N., at 3^h, 24° 17′ 6″ N.; moon's horizon semidiameter on August 18, at Greenwich mean noon, 14′ 51·6″, at midn., 14′ 54·5″; corresponding horizontal parallax, 54′ 31·8″ and 54′ 42·7.″ Latitude, 38° 10′ 36″ S.

7. Declination of α Arietis 22° 41′ 5″ N. Latitude, 72° 23′ 24″ N.

8. Declination of α Ursæ Majoris, 62° 37′ 57″ N. Latitude, 38° 26′ 29″ N.

9. Right ascension mean sun on Dec. 6, at Greenwich mean noon, $16^h\ 58^m\ 12{\cdot}52^s$. Latitude, 50° 16′ N.

10. Sun's declination on July 31, at Greenwich mean noon, 18° 39′ 18″ N., on August 1, 18° 24′ 47″ N.; semidiameter, 15′ 47″. Latitude, 50° 53′ 30″ N.

11. Sun's declination on May 13, at Greenwich mean noon, 18° 16′ 32″ N., on May 14, 18° 31′ 19″ N.; corresponding equation of time, $3^m\ 45{\cdot}9^s$ S. and $3^m\ 55{\cdot}9^s$ S.; semidiameter, 15′ 50″; hour-angle, $21^h\ 36^m\ 45^s$ W. Longitude, 0° 26′ 0″ E.

12. Right ascension mean sun September 3, at Greenwich mean noon, $10^h\ 47^m\ 36{\cdot}37^s$; right ascension α Pegasi, $22^h\ 56^m\ 35{\cdot}2^s$; declination, 14° 19′ 23″ N.; moon's horizontal semidiameter on September 3, at Greenwich mean noon, 15′ 51·7″, at midnight, 15′ 48·3″; corresponding horizontal parallax, 58′ 12·5″ and 57′ 59·9″; true distance, 55° 32′ 23″; distance from *Nautical Almanac*, at vi., 56° 49′ 7″, at ix., 55° 19′ 51″; hour-angle, $18^h\ 11^m\ 57^s$ W. Longitude, 33° 48′ 0″ W.

13. Sun's declination on August 22, at Greenwich mean noon, 11° 57′ 49″ N., on August 23, 11° 37′ 37″ N.; true bearing, W. 18° 38′ 45″ N. Variation, 23° 48′ 45″ E.

14. Sun's declination on August 23, at Greenwich mean noon, 11° 37′ 37″ N., on August 24, 11° 17′ 14″ N.; semidiameter, 15′ 51″; true bearing, N. 81° 6′ 30″ E. Variation, 6° 43′ 30″ W.

15. Moon's Greenwich meridian passage August 2, 6^h 38^m, August 1, 5^h 46^m; semidiameter, 16′ 10″; equation of time, 6^m S, from mean time. High water, 9^h 12^m A.M., and 9^h 38^m P.M.

Questions.—No. V.

1. Required the course and distance from A to B.

Lat. A... 65° 25′ S.		Long. A... 3° 28′ W.
„ B... 73 42 S.		„ B... 4 2 E.

2. Required the course and distance from C to D.

Lat. C... 70° 15′ N.		Long. C... 15° 25′ E.
„ D... 70 15 N.		„ D... 20 25 E.

3. October 23, at noon, a point of land in latitude 34° 28′ S., and longitude 18° 28′ E., bore by compass N.W. distant 10 miles (variation of compass 2¼ W.), afterwards sailed as by the following log account: required the latitude and longitude in on October 24, at noon.

Hours.	Knots.	$\frac{1}{10}$ths.	Course.	Wind.	Leeway.	
1	5	4	N.b.E. ½ E.	N.W. ½ W.	2¼	
2	5	2				
3	5	8				
4	6	1				
5	6	5	S.S.W.	W.N.W.	¼	
6	7	3				Variation 2¼ W.
7	7	0				
8	7	2				
9	6	8	N.W. b. W.	S.E.	0	
10	6	5				
11	6	1				
12	5	8	S. b. W. ¾ W.	S.E.	2¼	
						—— Oct. 24, 1870.
1	6	0				
2	6	5				
3	6	8				
4	6	4	N.N.E.	N.W.	2	A current set the ship the last 5 hours N.W. 2 miles an hour by compass.
5	6	0				
6	6	5				
7	6	8				
8	7	2	N.W.	East.	0	
9	7	6				
10	7	9				
11	8	1				
12	8	5				

4. At what time will the star α Aquilæ (Altair) pass the meridian of the Land's End on December 8, and how far N. or S. of the zenith?

5. December 10, in long. 55° 20′ E., the observed meridian altitude of the sun's lower limb was 25° 52′ 5″ (zenith N.), the index correction was −2′ 10″, and the height of the eye above the sea was 17 feet: required the latitude.

6. August 10, at $6^h\ 40^m$ P.M. mean time nearly, in long. 50° 17′ E., the observed meridian altitude of the moon's lower limb was 45° 47′ 39″ (zenith N. of the moon), the index correction was −3′ 18″, and the height of the eye above the sea was 24 feet: required the latitude.

7. October 15, the observed meridian altitude of α Aquilæ was 50° 25′ 30″ (zenith N.), the index correction was −3′ 20″, and the height of the eye above the sea was 13 feet: required the latitude.

8. October 16, the observed meridian altitude of α Ursæ Majoris (Dubhe) under the North Pole was 5° 26′ 10″, the index correction was −2′ 10″, and the height of the eye above the sea was 17 feet; required the latitude.

9. March 17, 1837, at $9^h\ 43^m$ P.M., in long. 93° 14′ W., the observed altitude of α Ursæ Minoris (Polaris) was 32° 49′ 14″, the index correction was +7′ 49″, and the height of the eye above the sea was 12 feet: required the latitude.

10. March 14, the following double altitude of the sun was observed:

Mean time nearly.	Chronometer.	Obs. alt. sun's L. L.	True bearing.
$1^h\ 5^m$ P.M.	$8^h\ 2\ 25^s$	41° 20′ 45″	S.S.W.½W.
5 6 P.M.	12 3 30	7 29 30	W. by S.½S.

The run of the ship in the interval was N.E. 18 miles, the index correction was −3′ 20″, and the height of the eye above the sea was 23 feet; required the true latitude at the second observation, the latitude by account being 45° N., and the long. 50° 20′ W.

11. February 10, at $9^h\ 20^m$ P.M. mean time nearly, in lat. 28° 20′ N. and longitude by account 31° 2′ W., a chronometer showed $11^h\ 16^m\ 25^s$, and the observed altitude of the star α Leonis (Regulus) was 41° 55′ 10″ E. of the meridian, the index correction was +1′ 20″, and the height of the eye above the sea was 25 feet: required the true longitude.

On February 1, at Greenwich mean noon, the chronometer was *fast* on Greenwich mean time $5^m\ 20{\cdot}6^s$, and its daily rate was $2{\cdot}7^s$ *losing*.

12. April 27, at $2^h\ 30^m$ A.M. mean time nearly, in lat. 45° 20′ N., and longitude by account 46° W., the following lunar observation was taken:

	Obs. alt. α Virginis W. of meridian.	Obs. alt. moon's L. L.	Obs. dist. F. L.
	16° 30′ 50″	15° 38′ 56″	98° 2′ 40″
Index cor.	+2 20	+5 52	+1 5

The height of the eye above the sea was 12 feet: required the true longitude.

13. June 15, at 8^h 10^m P.M. mean time, in lat. 50° 48′ N., and long. 73° 19′ E., the sun set by compass W. 30° 29′ N.: required the variation.

14. June 15, at 9^h 39^m A.M. mean time nearly, in lat. 50° 48′ N., and long. 99° 29′ E., the compass bearing of the sun was S. 38° 19′ 50″ E., and the observed altitude of the sun's lower limb at the time was 49° 58′ 37″, the index correction was +10′ 43″, and the height of the eye above the sea was 12 feet: required the variation.

15. Required the time of high water at A on February 17, A.M. and P.M.

Change tide at A... 11^h 42^m P.M. app. time. Long. A... 2° W.

Elements from Nautical Almanac and Answers.

1. S. 17° 19′ 15″ E. 520·6′.

2. E. 101·3′.

3. Corrected courses E.b.S.¾S. 10′ departure course; N.b.E. ½E. 22·5′; S.½E. 28′; W.¾N. 19·4′; S.b.W.¾W. 25·1′; N.b.E. ¾E. 25·7′; W.b.N. ¾N. 39·3′; W.b.N.¾N. 10′. Latitude in 34° 17′ 54″ S. Longitude in 17° 32′ E.

4. At 2^h 34^m and 41° 37′ S. of zenith.

5. Sun's declination on Dec. 9, at Greenwich mean noon, 22° 51′ 14″ S.; on Dec. 10, 22° 56′ 49″ S.; semidiameter, 16° 16″. Lat. 41° 3′ 48″ N.

6. Moon's declination on Aug. 10, at 3^h, 23° 15′ 12″ S.; at 4^h, 23° 24′ 37″ S.; moon's horizontal semidiameter on August 10, at Greenwich mean noon, 15′ 43·8″; on August 10, at Greenwich mean midnight, 15′ 51·3″; corresponding horizontal parallax, 57′ 43·5″ and 58′ 11·0″. Lat. 20° 7′ 1″ N.

7. Declination of α Aquilæ 8° 26′ 42″ N. Lat. 48° 8′ 53″ N.

8. Declination of α Ursæ Majoris 62° 37′ 28″ N. Lat. 32° 33′ 1″ N.

9. Right ascension mean sun, on March 17, at Greenwich mean noon, 23^h 39^m $24{\cdot}25^s$. Lat. 33° 47′ N.

10. Sun's declination on March 14, at Greenwich mean noon, 2° 29′ 26″ S.; on March 15, 2° 5′ 46″ S.; semidiameter, 16′ 6″. Arc (1) 60° 12′ 45″; arc (2) 91° 26′ 30″; arc (3) 46° 23′ 30″. Lat. 43° 59′ 23″ N.

11. Sun's right ascension on February 10, at Greenwich mean noon, 21^h 21^m $24{\cdot}83^s$; right ascension α Leonis, 9^h 59^m 43^s; decl., 12° 45′ 40″ N. Hour-angle, 20^h 43^m 40^s. Long. 27° 50′ 45″ W.

12. Right ascension mean sun on April 26, at Greenwich mean noon, 2^h 17^m $6{\cdot}41^s$; right ascension of α Virginis, 13^h 16^m 38^s; decl., 10° 18′ 40″ S.; moon's horizontal semidiameter on April 26, at Greenwich mean midnight,

16′ 8·3″; on April 27, at Greenwich mean noon, 16′ 8·4″; corresponding horizontal parallax, 59′ 13·5″ and 59′ 13·7″. True distance, 97° 30′ 29″; distance at xv., 96° 0′ 34″; at xviii., 97° 46′ 47″. Hour-angle, $3^h\ 34^m\ 26^s$ W. Long. 45° 19′ 30″ W.

13. Sun's declination on June 15, Greenwich mean noon, 23° 19′ 50″ N.; on June 16, 23° 22′ 11″ N. True bearing, W. 38° 48′ 45″ N. Variation, 8° 19′ 45″ E.

14. Sun's declination on June 14, at Greenwich mean noon, 23° 17′ 5″ N.; on June 15, 23° 19′ 50″ N.; semi., 15′ 46″. True bearing, N. 119° 53′ E. Variation, 21° 47′ 10″ W.

15. Moon's meridian passage on February 17, $10^h\ 21{\cdot}9^m$; on February 16, $9^h\ 32{\cdot}2^m$; semidiameter, 14′ 43″. Equation of time, 14^m S. from mean time. High water, $9^h\ 32^m$ A.M. and $9^h\ 57^m$ P.M.

THE END.

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